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A COURSE IN ELECTRICAL ENGINEERING

VOLUME II

ALTERNATING CURRENTS

ELECTRICAL ENGINEERING TEXTS

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ELECTRICAL ENGINEERING TEXTS

A COURSE IN ELECTRICAL ENGINEERING

VOLUME II
ALTERNATING CURRENTS

BY

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SECOND EDITION

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PREFACE TO THE SECOND EDITION

The first edition of this volume, published in 1922, was planned to meet the needs for courses in alternating-currents which are semi-elementary in character, such as are given in the engineering schools to non-electrical students and in the advanced courses of certain industrial schools. Almost immediately on its publication, however, it was adopted by a large number of engineering schools and universities as a text for their electrical engineering courses. Moreover, it is probably used more for students who are specializing in electrical engineering than by any other class of students.

Also, since the publication of this volume six years ago, many changes and great progress have occurred in electrical engineering. The wide use of electron tubes in nearly every branch of electrical engineering has now made the principles of their operation one of the fundamentals of electrical engineering. Many radical changes and improvements in electrical apparatus and methods have taken place. The standards of electrical engineering education have been raised, due in large measure to the insistent demands of the industry for men with sound training and high analytical ability. These many factors made a revision of the first edition desirable, and this second edition is designed to meet, so far as possible, the changed conditions. For example, some of the more special properties of alternating-current circuits are added to Chapter II. Since the use of complex quantities is now taught in nearly every electrical engineering course of standing, including courses to non-electrical students, Chapter III on complex quantities, which is entirely new, has been added. The applications of complex quantities to polyphase systems is now given in Chapter V, and also to the solutions of various types of problems as they occur throughout the text. The fundamental principles and some of the simpler uses of electron tubes are given in Chapter XIII, a new chapter. The analysis and

treatment of alternating-current machinery have been carried further than in the first edition and the book has been brought up to date as regards the latest developments in alternating-current apparatus and methods.

To meet better the needs of teachers, the number of problems has been doubled practically.

The author is indebted to a number of his fellow teachers in electrical engineering for many suggestions. Prof. F. C. Caldwell of The Ohio State University and Prof. L. A. Doggett of Pennsylvania State College have been particularly helpful in this respect. Thanks are also due to Prof. Robert F. Field of the Cruft Laboratory, Harvard University, for preparing Chapter XIII on electron tubes. The author is particularly indebted to Prof. H. E. Clifford, Consulting Editor, whose advice and supervision were invaluable in the preparation of this volume.

C. L. D.

Harvard University, Cambridge, Mass.
August, 1928.

PREFACE TO THE FIRST EDITION

This volume is intended for those who have such a knowledge of direct currents as is given by Volume I. It presupposes no knowledge of alternating currents. The first two chapters are devoted to the development of the fundamental laws of alternating currents and alternating-current circuits. Subsequent chapters consider the application of these fundamental laws to alternating-current measurements, to polyphase circuits, to alternating-current machinery, and to power transmission. A chapter on illumination and photometry has been included, as a brief discussion of the underlying principles of light and of light measurements is important in a general course in electrical engineering.

The development of the various alternating-current formulas and of the operation of various types of machinery, transmission lines, etc., are based on the fundamental laws of electricity and magnetism as set forth in Volume I. Mathematical developments are occasionally introduced, as supplementary to the descriptive matter. As in Volume I, numerous illustrative problems and methods of making laboratory tests are given throughout the text.

This volume is intended to be elementary in character and to act as a stepping stone to the more advanced texts of this series. In many cases rigorous and detailed analysis is not given, particularly in the chapter on alternating-current measurements and in the discussion of certain types of alternating-current apparatus. A thorough analysis of these subjects is found in "Electrical Measurements" by F. A. Laws, and "Principles of Alternating Current Machinery" by R. R. Lawrence, both of which volumes are included in this series of Electrical Engineering Texts.

The author is indebted to various manufacturing companies for their cooperation in supplying material and illustrations for

the text; to Professor R. R. Lawrence of the Massachusetts Institute of Technology for his careful review of the manuscript and his many helpful suggestions given during its preparation; and particularly to Professor H. E. Clifford of The Harvard Engineering School, for his helpful advice during the preparation of the manuscript and for the thorough manner in which he has edited the material contained in this volume.

C. L. D.

Harvard University, Cambridge, Mass.
Jan., 1922.

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A COURSE IN
ELECTRICAL ENGINEERING

VOLUME II
ALTERNATING CURRENTS

CHAPTER I

ALTERNATING CURRENT AND VOLTAGE

1. General Field of Use of Alternating Current.—Over 90 per cent. of the electrical energy generated at the present time is generated as alternating current. This is not due primarily to any superiority of alternating over direct current in its applicability to industrial and domestic uses. In fact, there are many instances where direct current is absolutely necessary for industrial purposes, but even in these cases the energy is often generated as alternating current.

Some of the reasons for generating energy as alternating current are:

Alternating current can be generated at comparatively high voltages, and these voltages can be readily raised and lowered by means of static transformers. This permits the economical transmission of alternating current over considerable distances by using high transmission voltages. These high voltages can be reduced efficiently at the receiving end of the transmission line. Direct-current voltages cannot be raised and lowered on an industrial scale without the use of rotating commutators, and the permissible voltage per commutator is low. The voltage of direct-current circuits, therefore, cannot be changed economically.

Alternating-current generators can be built in large units running at high speeds, are suited to turbine drive, and the cost per kilowatt of such alternators is low. The largest single unit

today (1928) has a rating of about 160,000 kv-a.[1] Owing to commutation difficulties, direct-current generators cannot be built in large units, particularly for high speeds. At 1,000 r.p.m., it is difficult to build a direct-current generator having a rating of even 1,000 kw. On the other hand, 5,000-kw. alternators, operating at speeds of 3,600 r.p.m., are not uncommon.

For constant-speed work, the alternating-current induction motor is cheaper in first cost and in maintenance than the direct-current motor. This is due to the fact that the induction motor has no commutator. It is occasionally desirable, therefore, to generate power as alternating current in order to be able to use induction motors.

The high transmission efficiencies obtainable with alternating current make it possible to generate electrical energy in large quantities in a single station and to distribute it over a comparatively large territory. The large boilers, automatic stokers, superheaters, recording instruments, etc., which are possible in large stations, result in high boiler-room efficiency. Large turbines have an economy which may be three or four times as good as that of the steam units in a small plant. The generator has an efficiency of 95 to 96 per cent. in the larger sizes (see p. 197). Then, again, as the boilers and large turbo-units require few attendants per kilowatt, the labor and superintendence charges per kilowatt are small.

For these reasons, it is often more economical to generate power with large units, to transmit it long distances, and even to convert it into direct current than to generate the direct current at the place where it is to be utilized.

It must be remembered, however, that the reduced generating costs may be balanced by the distribution costs resulting from high investment charges in lines, cables, substations, machinery, etc., in addition to the labor and maintenance costs of this distribution system.

Alternating current owes its importance to the fact that it can be generated economically with large units. Its voltage can be readily raised and lowered, so that energy can be transmitted

[1] This unit is being constructed by the General Electric Company for the New York Edison Company. It operates at 25 cycles, 1,500 r.p.m., 11,000 volts, three phase.

economically for considerable distances. Alternating-current motors for constant-speed work are usually preferable to direct-current motors.

2. Sine Waves.—It was shown in Vol. I (Chap. X) that when a single coil rotates at constant speed in a uniform field (Fig. 1) an alternating e.m.f. is generated. This e.m.f. is zero when the plane of the coil is perpendicular to the field and reaches its maximum value when the plane of the coil is parallel to the field (see Vol. I. p. 265, Fig. 211). The successive values of the

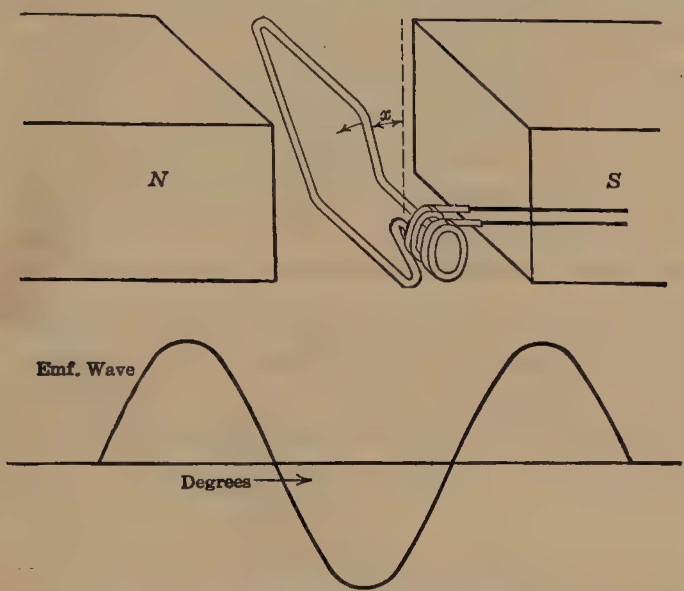


FIG. 1.—Generation of alternating e.m.f.

e.m.f. may be represented by a smooth curve called a *sine wave*, since the values of the e.m.f. are proportional to the sine of the angle x which the coil makes with the vertical (Fig. 1). This will be discussed more in detail later.

For various reasons, commercial generators do not give exact sine waves of e.m.f. In fact, the e.m.fs. of some generators differ materially from a sine wave. Still, most commercial generators have waves of e.m.f. which are sufficiently close to a sine wave to warrant their being treated as such.

If a wave is not a sine wave, it may be resolved into a series of sine waves of fundamental and higher frequencies. Each one of these components may then be dealt with as a sine wave.

The waves ordinarily encountered in practice are approximately sine waves and may be treated by simple methods of analysis. The formulas and equations which follow apply to sine waves of current and voltage, unless otherwise specified.

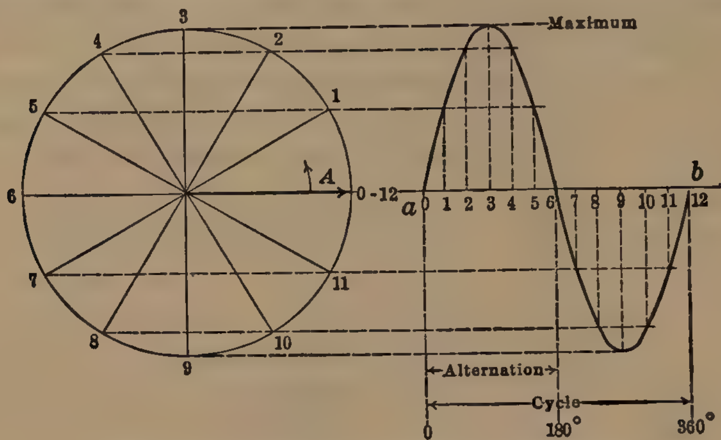


FIG. 2.—Graphical construction of sine wave.

The sine wave may be produced graphically as follows: Draw a circle (Fig. 2) whose radius A is equal to the maximum value of the sine wave. Divide the circumference of this circle into any number of equal parts, in this case 12, and number them 0,

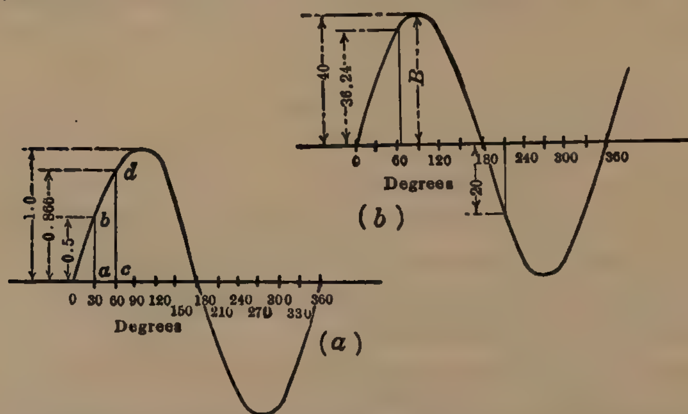


FIG. 3.—Numerical values of the ordinates of sine waves for definite angles.

1, 2 . . . 12. Also, draw a horizontal line ab, which, if extended, would pass through the center of the circle. Divide ab into the same number of equal parts as there are on the circumference of the circle, and give the points corresponding numbers. Erect an ordinate or perpendicular at each point.

Project the points on the circle horizontally until they meet perpendiculars having corresponding numbers. A smooth curve drawn through the intersections will be a sine wave.

The sine wave may also be plotted from a table of sines (Appendix E, p. 536). Mark a horizontal axis (Fig. 3 (*a*)) in degrees. At each point erect an ordinate equal to the sine of the corresponding angle. Thus, at 30° the ordinate *ab* is 0.5; at 60° the ordinate *cd* is 0.866; at 90° it is 1.0; etc. The wave passes through zero at 180°, because the sine of 180° is zero. When the angle becomes greater than 180°, the sine becomes negative and the wave falls below the line, as the sine is negative between 180° and 360° (see p. 534). The above is equivalent to plotting the sines of the angle x (Fig. 1), x being the angle which the plane of the rotating coil makes with the vertical at any instant.

If the wave in question has a maximum value of B (Fig. 3 (*b*)) instead of unity, the value of the ordinate at any point may be found by multiplying B into the sine of the corresponding angle. That is,

$$y = B \sin x \tag{1}$$

where x is in degrees.

Example.—Find the ordinates of a sine wave at points corresponding to 65° and 210°, the maximum ordinate being 40 units (Fig. 3 (*b*)).

From page 537, sin 65° = 0.906.

$$40 \times 0.906 = 36.24. \quad \textit{Ans.}$$

$$\sin 210^\circ = -(\sin 210^\circ - 180^\circ) = -\sin 30^\circ = -0.5. \quad \text{(p. 534).}$$

$$40 \times (-0.5) = -20. \quad \textit{Ans.}$$

These values are shown in Fig. 3 (*b*).

3. Cycle; Frequency.—When the coil (Fig. 1) has completed one revolution, it has passed one pair of poles (a north and a south pole), and it has traversed 360 space-degrees. The voltage wave has gone through one complete *cycle* of values, and the wave is now ready to repeat itself. This is illustrated in Fig. 4.

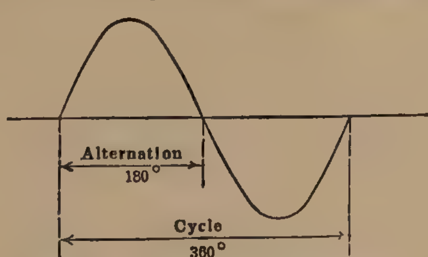


FIG. 4.—Alternation and cycle.

Having gone through one complete cycle, the voltage has gone through 360 electrical time-degrees. In a two-pole machine, therefore, *one space-degree is the same as one electrical time-degree.*

When the voltage has completed only ½ cycle, or 180°, it has gone through *one alternation.*

Assume that the coil in Fig. 1 is making 60 revolutions per second, or 3,600 r.p.m. Sixty complete cycles will be generated each second. A two-pole, 60-cycle generator, therefore, must have a speed of 3,600 r.p.m.

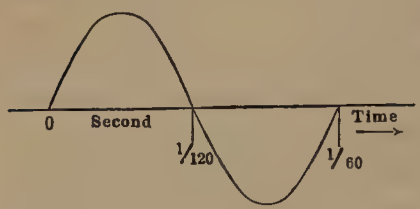


FIG. 5.—Sine wave as function of time.

The abscissas may be graduated in *time* as well as in degrees. For example (Fig. 5), the time corresponding to 360° is 1/60 sec., and the time corresponding to 180° is 1/120 sec.

Alternating-current waves may be plotted with either *time* or *degrees* as abscissas.

Figure 6 (*a*) shows a four-pole machine. A single conductor *a* of a coil is shown rotating, rather than the complete coil. As

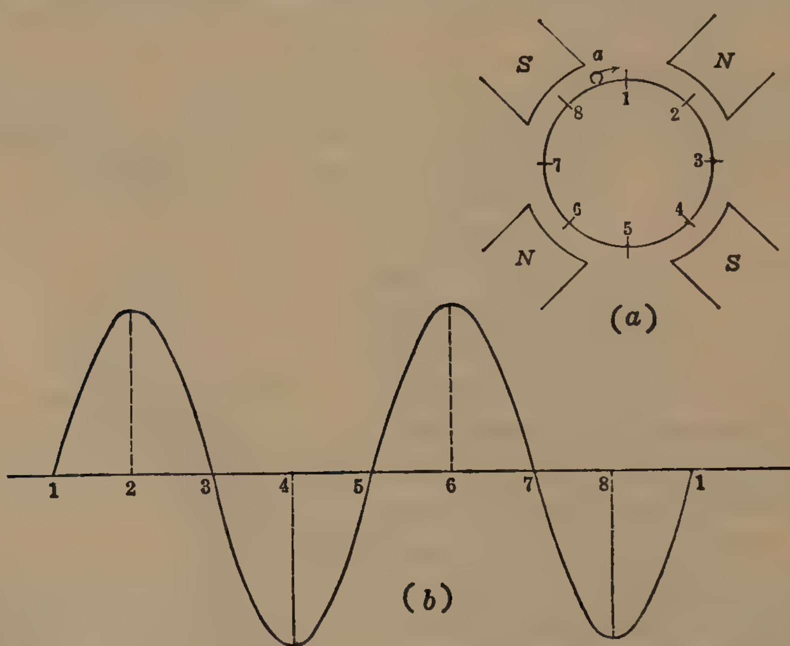


FIG. 6.—Two cycles per revolution in four-pole alternator.

soon as this conductor has passed a north and a south pole, that is, after it has passed from 1 to 5, it has completed one electrical cycle or 360 electrical time-degrees, as is shown in Fig. 6 (*b*).

Mechanically, it has completed one-half a revolution, or 180 space-degrees, so that in one revolution, or 360 space-degrees, the e.m.f. in the conductor will have completed two cycles and will have gone through 720 electrical time-degrees. In this case, therefore, one space-degree corresponds to two electrical time-degrees. That is, for every space-degree that the coil passes through, the voltage wave completes two electrical time-degrees. This conductor needs to make only 30 r.p.s. or 1,800 r.p.m. in order to generate a 60-cycle electromotive force. Likewise, for a 25-cycle electromotive force, this conductor needs to revolve at only 12.5 r.p.s. or 750 r.p.m. For a given frequency, as the number of poles increases, the mechanical speed decreases proportionately. The relation among speed, poles, and frequency may be written in the form of an equation:

$$f = \frac{P \times S}{2 \times 60} = \frac{P \times S}{120} \tag{2}$$

where f is the frequency in cycles per second, P is the number of poles, and S is the speed in r.p.m.

The table shows the relation of speed, frequency, and number of poles for a few typical cases.

Poles	Speed, r.p.m.	
	60 cycles	25 cycles
2	3,600	1,500
4	1,800	750
6	1,200	500
8	900	375
40	180	75

Example.—A 60-cycle, engine-driven alternator has a speed of 120 r.p.m. How many poles has it?

Using Eq. (2) and solving for P,

$$P = \frac{120f}{S} = \frac{120 \times 60}{120} = 60 \text{ poles.} \quad \textit{Ans.}$$

In practice, nearly all alternators have stationary armatures and rotating fields, and the above equations apply.

4. Commercial Frequencies.—In this country, frequencies are standardized at 60 cycles and at 25 cycles per second, although other frequencies are used. In California and in Mexico, for example, 50 cycles is used on some of the large transmission systems. In the early days of alternating-current development, 133 cycles was common, but few, if any plants now generate at this frequency. The principal advantage of higher frequencies is that transformers require less iron and copper and so are lighter and cheaper. The flicker of lamps is not perceptible at 60 cycles, but at 25 cycles it is very evident. On the other hand, the voltage drop in transmission lines and in apparatus varies almost directly as the frequency, so that better voltage regulation throughout the system is obtained with low frequency. Power apparatus, such as induction motors, synchronous converters, alternating-current commutator motors, etc., operates better at low than at high frequencies. With one or two exceptions, however, the operation is satisfactory at 60 cycles per second. A power and lighting company would ordinarily operate at 60 cycles per second, because the flicker of lamps at 25 cycles per second is objectionable and the transformers at this lower frequency are heavier and more costly than they are at the higher frequency. On the other hand, an electric company generating strictly for power purposes would ordinarily use 25 cycles. This frequency is used by the New York, New Haven and Hartford Railroad for its electric locomotives; on the Norfolk and Western Railway for operating electric locomotives; and by the Boston Elevated Railway Company for transmitting high-voltage power to its direct-current substations. In Europe, frequencies as low as 15 and even 12.5 cycles per second are common.

5. Alternating-current Ampere.—Figure 7 (*a*) shows an alternating-current sine wave, having a maximum value of 1.414 amp. At first thought, it might seem that the value in amperes of such a wave should be based on the *average* value. If the wave over one complete cycle is considered, the average value is zero, as there is just as much negative as positive current. A direct-current ammeter, if connected to measure this current, would indicate zero, as such an instrument reads *average* values.

The value of an alternating current is not based on its average value but on its *heating* effect and may be defined as follows:

An alternating-current ampere is that current which, flowing through a given ohmic resistance, will produce heat at the same rate as a direct-current ampere.

Assume that a resistance unit is immersed in a calorimeter and that when a direct-current ampere is sent through this resistance the temperature of the water is raised 20° in 10 min. An alternating-current ampere, if sent through this same resistance unit, will raise the temperature of the water by the same amount in the same time, other conditions such as radiation, etc., being the same. That is, both currents produce heat at the same rate.

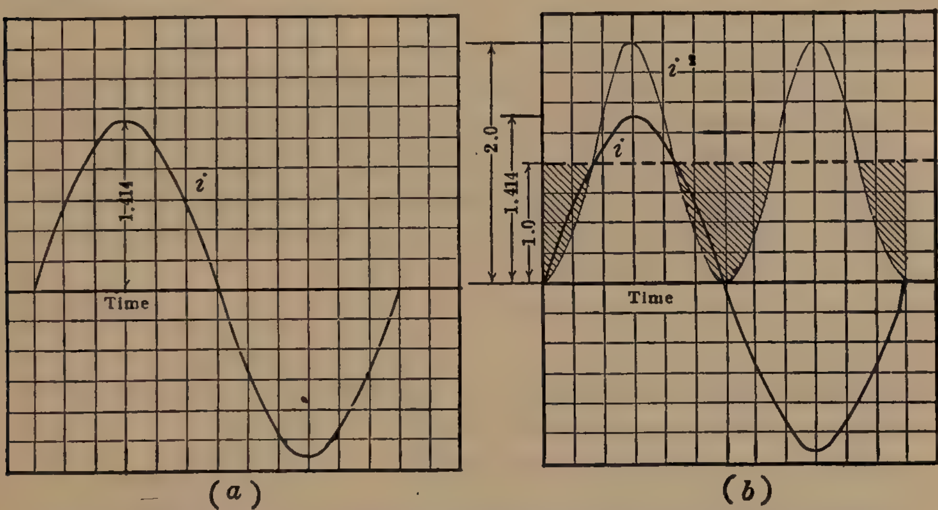


FIG. 7.—Maximum and effective values of sine-wave alternating current.

The heating effect varies as the *square* of the current ($= i^2R$). The value in amperes of the wave of current in Fig. 7 (*a*), therefore, must be based upon its *squared* values. Figure 7 (*b*) shows the current wave of Fig. 7 (*a*) plotted, together with its squared values. That is, each ordinate of the i wave is squared, and these values plotted to give the i^2 wave shown. The maximum value of this new wave will be 2.0 $(= \overline{1.414})^2$, since the maximum value of the original current wave is 1.414 or $\sqrt{2}$. The squared wave also lies entirely above the zero axis, because the square of a negative value is positive (see Par. 7).

This squared wave has a frequency twice that of the original wave and has its horizontal axis of symmetry at a distance of 1.0 unit above the zero axis, as shown in Fig. 7 (*b*). The average

value of this squared wave is 1.0 amp., as shown by the dotted line, because the areas above the dotted line will just fit into the shaded valleys below the dotted line. If, therefore, an equivalent rectangle were made from this wave, its height would be 1.0 unit. This value, 1.0, is the *average of the squares* of the current wave. Average heating varies as the average of the squares of the current, so this procedure for determining the ampere value of the wave of Fig. 7 (*a*) is correct.

To obtain the correct value of the current in amperes, the square root of the average square must be taken. That is, I (in amperes) $= \sqrt{1.0} = 1.0$ amp. This value of the current is called the *root-mean-square* (r.m.s.) or *effective* value of the current.

An alternating-current ampere, sine wave, which produces heat at the same rate as a direct-current ampere, therefore has a *maximum* value of 1.414 ($= \sqrt{2}$) amp. In fact, for any sine-wave current, the ratio of the *maximum* to the *effective* value is equal to $\sqrt{2}$ or 1.414. The ratio of effective to maximum value is $1/1.414 = 0.707$.

To obtain the *effective* value of *any* current wave, not necessarily a sine wave:

a. Plot a wave whose ordinates are equal to the squares of the ordinates of the given current wave.

b. Find the average value of this squared wave by obtaining the area of its loops with a planimeter and dividing this area by the base; or by averaging the ordinates.

c. Find the square root of this average.

The same result may be obtained by erecting equidistant ordinates on the original wave, averaging their squares, and taking the square root of this average. This will give the *r.m.s.* value.

If a sine wave of current be *averaged* in the ordinary manner for ½ *cycle*, it will be found that this average is equal to $2/\pi$ or 0.637 times the maximum value. The ratio of *effective* to *average* value is then $0.707/0.637 = 1.11$, and the ratio of average to effective value is 0.9. It is sometimes necessary to know the average value, and the ratio of effective to average value enters into computations of induced e.m.fs. in alternators, transformers, and other types of alternating-current machinery.

The ratio of effective to average value is called the *form factor* of the wave. The form factor of a sine wave is 1.11. The maximum, effective, and average values for a sine wave of voltage, whose r.m.s. value is 100 volts, are shown in Fig. 8.

6. Equation of Sine Wave of Current.—If ωt is substituted for x in Eq. (1) (Par. 2), the equation of a sine wave of alternating current is given by

$$i = I_{max} \sin \omega t \tag{3}$$

where i is the value of the current at any time t, I_{max} is the maximum value of the current, and $\omega = 2\pi f$. The term ω is equal to

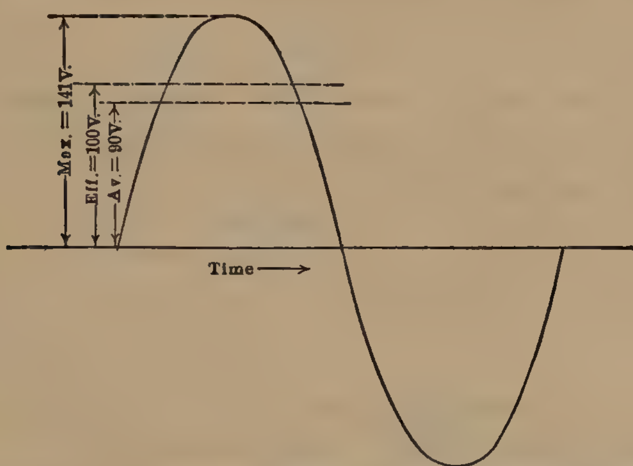


Fig. 8.—Relation of maximum, effective, and average values of sine wave.

2π times the frequency f and is the *angular velocity* in radians per second of the rotating vector which may be used to construct the wave (Appendix, p. 531).

For example, if the vector A (Fig. 2) be considered as rotating in a counterclockwise direction and taking successive positions 1, 2, 3, etc., it will produce 1 cycle for each revolution. In each revolution, it goes an angular distance of 2π radians. If it rotates sixty times a second, its angular velocity is $2\pi 60$ or 377 radians per second. The wave produced from this rotating vector has a frequency of 60 cycles per second. Hence, for a 60-cycle wave, $\omega = 377$.

Similarly, the equation of a sine wave of electromotive force will be given by

$$e = E_{max} \sin \omega t. \tag{4}$$

Example.—What is the equation of a 25-cycle current, sine wave, having an effective value of 30 amp., and what is the value i' of the current when the time is 0.005 sec.? The wave crosses the time axis in a positive direction when the time is equal to zero.

$$I_{max} = 30\sqrt{2} = 42.4 \text{ amp.}$$
$$2\pi 25 = 157 = \omega$$
$$i = 42.4 \sin 157\, t. \quad \textit{Ans.}$$
$$i' = 42.4 \sin 157 \times 0.005 = 42.4 \sin 0.785 \text{ radian}$$
$$2\pi = 6.28 \text{ radians} = 360° \text{ (p. 531)}$$

$\frac{0.785}{6.28} \times 360° = 45°$. As the wave completes 360° in 1/25 or 0.04 sec., in 0.005 sec., it will have completed 0.005/0.040 = 1/8 cycle.

$$\frac{360°}{8} = 45°$$

$$i' = 42.4 \sin 45° = 42.4 \times 0.707 = 30 \text{ amp.} \quad \textit{Ans.}$$

7. Equation of the Current Squared Wave.—Let the equation of a current wave be

$$i = I_{max} \sin \omega t. \tag{I}$$

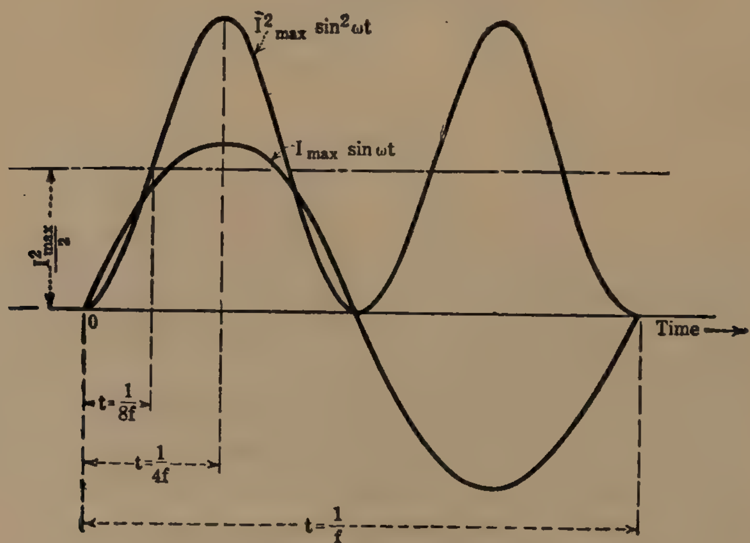


Fig. 9.—Current and current-squared waves.

Let it be required to find the equation of the current squared wave.

Squaring (I),

$$i^2 = I^2_{max} \sin^2 \omega t \tag{II}$$

$$\sin^2 \omega t = \cos^2 \omega t - \cos 2\,\omega t \text{ (Eq. (41), p. 535)} \tag{III}$$

Adding $\sin^2 \omega t$ to both sides of (III),

$$2 \sin^2 \omega t = \cos^2 \omega t + \sin^2 \omega t - \cos 2\omega t$$
$$= 1 - \cos 2\omega t \text{ (see Eq. (32), p. 59)} \qquad \text{(IV)}$$

Hence, from (II) and (IV),

$$i^2 = I^2_{max} \frac{1 - \cos 2\omega t}{2}.$$

This is a sine wave having a frequency $2f$, where f is the frequency of the current and is given by $f = \omega/2\pi$. When $t = 0$, $i^2 = 0$; when $t = \pi/4\omega = \pi/8\pi f = 1/8f$, $i^2 = I^2_{max}/2$ (see Fig. 9). i^2 is a maximum when $2\omega t = \pi$ radians $= 180°$.

$$t = \frac{\pi}{2\omega} = \frac{1}{4f} \quad \text{(Fig. 9)}.$$

Under these conditions, $i = I^2_{max}$. It follows that the axis of the wave is at a distance $I^2_{max}/2$ above the axis of reference.

8. Scalars and Vectors.—Quantities, in general, are divided into two classes, scalars and vectors.

A scalar is a quantity which is completely determined by its magnitude alone. Examples of scalar quantities are dollars, energy, gallons, mass, temperature, etc. Such quantities are added algebraically. For example, two dollars plus five dollars equals seven dollars.

A vector has direction as well as magnitude. A common example of a vector is force. When a force is under consideration, not only its magnitude but its direction as well must be considered. When two or more forces are added, they are not necessarily added algebraically but must be combined in such a way as to take into consideration their directions as well as their magnitudes.

Figure 10 (a) shows two forces acting at the point O and represented by the vectors F_1 and F_2. The length of each of these vectors, to scale, is equal to the *magnitude* of the force which it represents. The direction of each of these vectors shows the *direction* in which the force acts. β is the angle between F_1 and F_2. Their sum, F_0, or the single force which would have the same effect on their point of application, O, as F_1 and F_2 acting in conjunction, is called their *resultant*. F_0 is one diagonal of the parallelogram having F_1 and F_2 as adjacent sides.

Figure 10 (b) shows a triangle having F_1 and F_2 as two of its sides, F_1 and F_2 being, respectively, parallel o, and acting in the same directions as, F_1 and F_2 of Fig. 10 (a). The exterior angle

between F_1 and F_2 is, therefore, equal to β. The third side of the triangle F_0 is equal in magnitude and direction to F_0 of Fig. 10 (a). The resultant of two vectors, therefore, may be found by means of a triangle properly constructed, of which two sides are the two component vectors and the third side is their sum. Such a triangle is called a *triangle of forces.* It is usually simpler to use the triangle of forces than to use the parallelogram of forces.

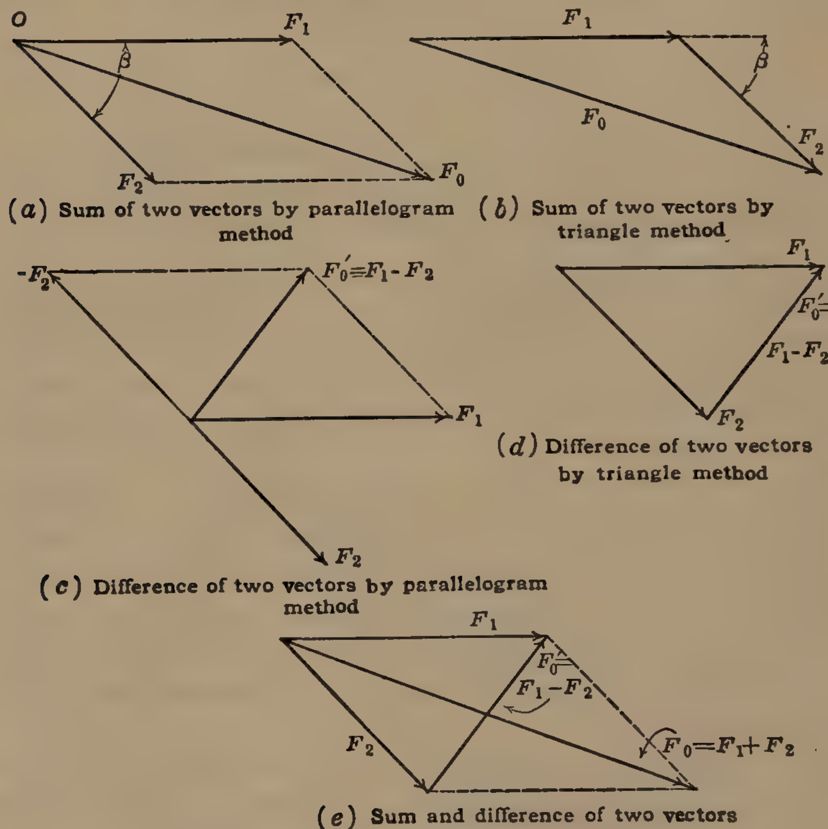


FIG. 10.—Sum and difference of two vectors.

To subtract one vector from another, reverse this vector and add it vectorially to the second vector. For example, in Fig. 10 (c) it is desired to subtract F_2 from F_1. F_2 is reversed, giving $-F_2$. F_0' the vector sum of F_1 and $-F_2$, found by completing the parallelogram, is equal to $F_1 - F_2$. Vectors may be subtracted by the triangle method, as shown in Fig. 10 (d). The vector F_0', connecting the ends of the two vectors F_1 and F_2 whose difference is desired, is their vector *difference.*

If a parallelogram (Fig. 10 (*e*)) having vectors F_1 and F_2 as adjacent sides, be completed, one diagonal F_0 of the parallelogram is the vector *sum* of F_1 and F_2. The other diagonal F_0' of the parallelogram is the vector *difference* of F_1 and F_2.

A vector is often indicated by placing a dot under its symbol. For example, in Figs. 10 (*a*) and 10 (*b*),

$$\underset{\bullet}{F}_0 = \underset{\bullet}{F}_1 + \underset{\bullet}{F}_2$$

shows that F_0 is the *vector* sum of F_1 and F_2 and not their algebraic sum.

When more than two vectors are added, the resultant of two is first found, and this resultant is combined with a third vector, etc. This is illustrated in Fig. 11, in which three vectors F_1, F_2, and F_3 are added.

F_1 and F_2 are first combined, and the resultant F' is found. F' is then combined with F_3, giving F_0 as the sum of all three vectors, F_1, F_2, and F_3. That is,

$$\underset{\bullet}{F}_0 = \underset{\bullet}{F}_1 + \underset{\bullet}{F}_2 + \underset{\bullet}{F}_3.$$

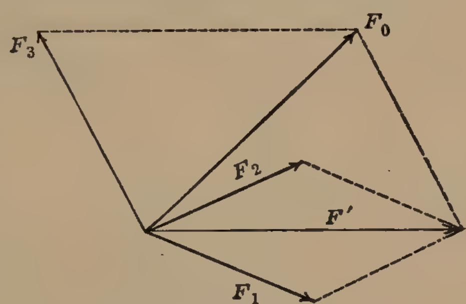


FIG. 11.—Sum of three vectors.

F' is an intermediate vector and, therefore, does not appear in the ultimate result.

9. Ohm; Volt.—If a resistance of 1 ohm, as measured with direct current, has no inductance and is so designed that alternating current in flowing through it does not produce any secondary effects, such as eddy currents or skin effect, it offers a resistance of 1 *ohm* to alternating current.

When an alternating-current ampere flows through such a resistance, the drop across its terminals is equal to 1 alternating-current *volt*. Hence, the relation between *maximum* and *effective* volts is the same as the relation between *maximum* and *effective* amperes. For a sine wave, the maximum voltage is $\sqrt{2}$, or 1.414, times the effective voltage.

10. Phase Relations.—The current and voltage in the ordinary alternating-current system have the same fundamental frequency under normal operating conditions, although they do not necessarily pass through their corresponding zero values at the same

instant. Figure 12 (*a*) shows two sine-wave currents, one having an effective value of 8 and the other of 12 amp. Their respective maximum values are, accordingly, $8\sqrt{2}$ or 11.3 amp. and $12\sqrt{2}$ or 17.0 amp. Both currents pass through zero, increasing positively, at the same instant and are, therefore, said to be *in phase* with each other.

Figure 12 (*b*) shows two sine-wave currents of 8 and 12 amp., respectively, but not passing through zero at the same instant.

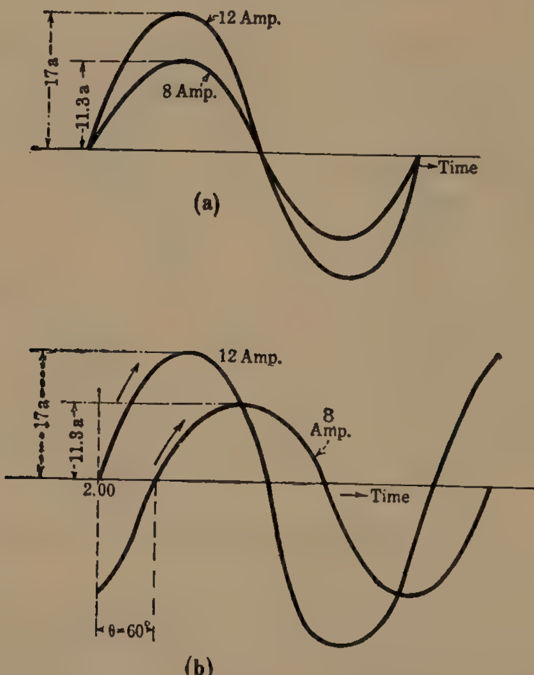


Fig. 12.—Phase relations of alternating currents.

The 8-amp. current passes through zero, increasing positively, later than does the 12-amp. current. It must be remembered that time is increasing from left to right. If the 12-amp. current is passing through its zero value at 2.00 o'clock, the 8-amp. current is passing through its corresponding zero value some time later, for any value of time to the right of 2.00 is later than 2.00 o'clock. The 8-amp. current, therefore, *lags* the 12-amp. current.

The time of lag shown in Fig. 12 (*b*) corresponds to 60° and is represented by the angle θ. The 8-amp. current, therefore, lags the 12-amp. current by an angle θ or by 60°. Or the 12-amp.

current may be said to *lead* the 8-amp. current by an angle θ or by 60°.

In Fig. 12 (*a*), the two currents are *in phase* with each other. In Fig. 12 (*b*), the two currents have a *phase difference* of 60°.

These phase differences may exist between currents and voltages, between two or more voltages, or between two or more currents.

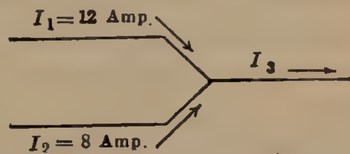


FIG. 13.—Alternating currents meeting at junction.

11. Addition of Currents.—Figure 13 shows two currents, having effective values of 8 and 12 amp., respectively, uniting to flow in a common wire. If these two currents were direct currents, then by Kirchhoff's first law (see Vol. I, p. 88), the current I_3 could have only two possible numerical

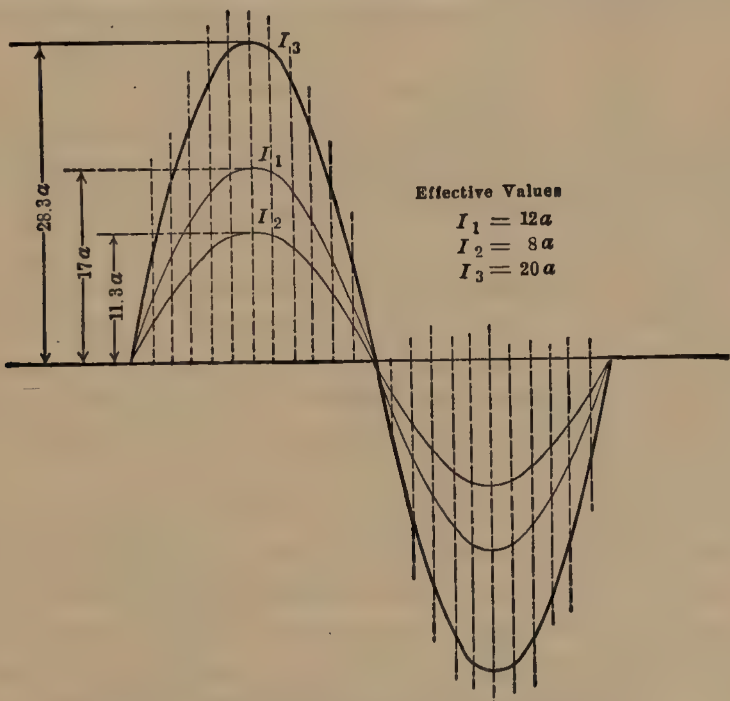


FIG. 14.—Addition of two currents in phase.

values, $12 + 8 = 20$ amp., if the two currents flow in the same direction, and $12 - 8 = 4$ amp., if they flow in opposite directions.

If the two currents (Fig. 13) are alternating, their sum I_3 may be equal numerically to *any* value *from* 20 *amp. to*

4 *amp.*, depending on the phase relation existing between I_1 and I_2.

Figure 14 shows these two currents plotted *in phase* with each other. Their sum I_3 is found by adding their ordinates at each instant. The resulting current obtained in this manner will be a sine wave and will have a *maximum* value of 28.3 amp. corresponding to an effective value of $28.3/\sqrt{2} = 20$ amp. That is, when two currents are in phase, their sum is found arithmetically.

Figure 17 corresponds to the condition of Fig. 12 (*b*), where the two currents differ in phase by 60°. Their sum is found in the same manner as in Fig. 14 by adding the two, point by point, and obtaining the resulting current I_3. The resultant I_3 will not have a maximum value of 28.3 amp., as it did when the currents were in phase, but its maximum value will be less, actually being 24.7 amp. This corresponds to an effective value of 17.45 amp. for the sum of the two, rather than of 20 amp. as before. *Therefore, the sum of any number of alternating currents depends upon their phase relations as well as upon their magnitudes.*

If voltages rather than currents be added, it obviously follows that their sum depends upon their phase relations as well as upon their magnitudes.

12. Vector Representation of Alternating Quantities.—It was shown in Fig. 2 that a sine wave could be drawn by projecting a rotating radius, in its successive positions, to meet corresponding equally spaced ordinates. The value of the current or voltage may be found at any instant by projecting a radius upon a vertical line.

This is illustrated in Fig. 15. A certain current has a maximum value I_{max}. This value I_{max} is laid off as a radius, and this radius rotates at a speed in revolutions per second equal to the frequency of the current. For example, if the current I_{max} has a frequency of 60 cycles, the radius I_{max} must make 60 complete revolutions per second, in a counterclockwise direction. Counterclockwise rotation has been adopted internationally as the positive direction of rotation.

When the radius I_{max} is at the right-hand horizontal position, the value of the current is zero. When I_{max} has advanced 30°, the point *b* on the current wave has been reached. The value of the current at this instant is *ab*, or, which is the same thing, the

current value is given by the distance $a'b'$, the projection of I_{max} upon the *vertical* axis. At this particular instant, the distance $ab = a'b' = I_{max}/2$, since sin 30° = 0.5.

Consider two currents I_1 and I_2 (Fig. 16) having effective values of 12.0 and 8.0 amp., respectively. The current I_2, whose

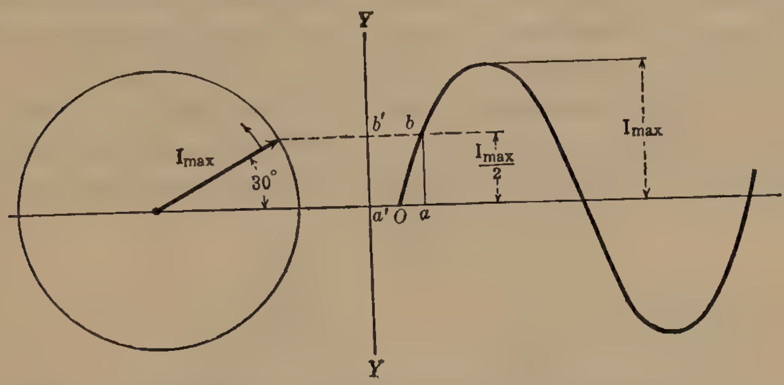


FIG. 15.—Instantaneous values of current from rotating vector.

maximum value is 11.3 amp., lags current I_1, whose maximum value is 17.0 amp., by 60°. When the radius I_1 is in the horizontal position, the value of I_1 is zero at this instant. At this same instant, the radius I_2 will not have reached its horizontal

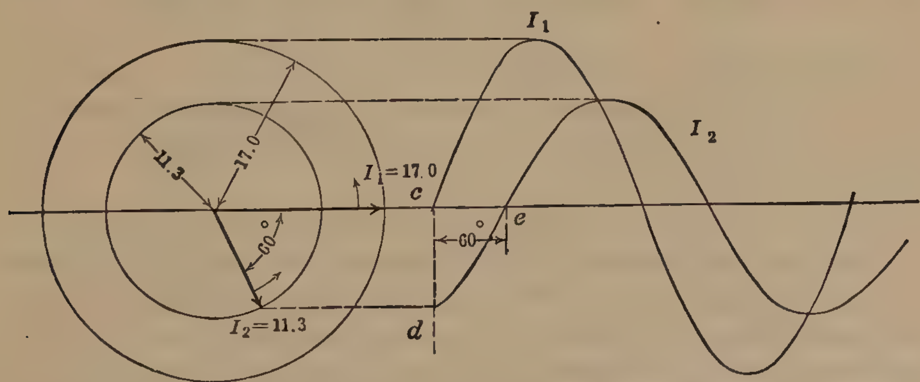


FIG. 16.—Current waves produced by two current vectors differing in phase by 60°.

position, the value of the current being represented by *cd* (Fig. 16). In fact, the radius I_2 does not reach its horizontal or zero position until I_1 has advanced 60° beyond the horizontal. Further, the horizontal distance *ce* is 60°, the same as the phase angle between the two rotating vectors.

These two current waves, therefore, can be constructed in their proper phase relation by means of two rotating vectors having lengths of 17.0 and 11.3 amp., having equal angular velocities, and differing in phase by 60° (Fig. 16).

13. Vector Addition of Sine Waves.—Assume that it is desired to add the two currents of Fig. 16. This may be done by adding the ordinates of the two curves at each point, as in Fig. 17, and plotting a new curve, I_3. This new curve is the sum of the two currents whose maximum values are 17.0 and 11.3 amp. and effective values 12 and 8 amp., respectively, and the maximum value of this resultant, if measured accurately, will be 24.7 amp.

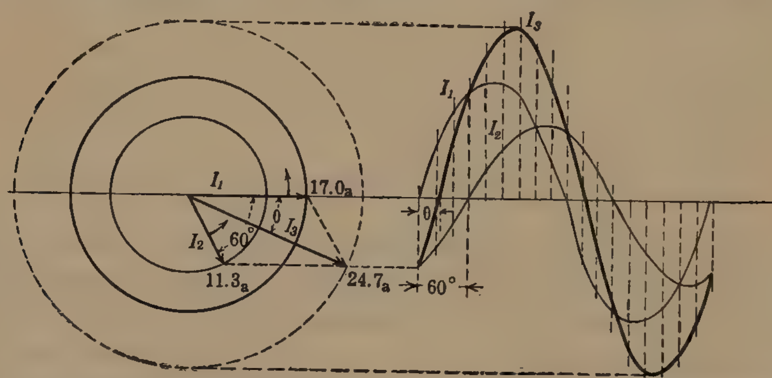


Fig. 17.—Relation of vector addition of vectors to scalar addition of ordinates.

This corresponds to an effective value of 17.45 amp. The sum, therefore, of two sine-wave alternating currents, having effective values of 12 and 8 amp., respectively, and differing in phase by 60°, is 17.45 amp.

If the rotating vectors (Fig. 17) be added vectorially by completing the parallelogram, a third vector I_3 results. This vector I_3 will be found to have a length of 24.7 amp., the exact value of the maximum of the resultant current wave as just found. If a wave be plotted using I_3 as the rotating vector, projecting as before, it will coincide with I_3 as obtained by the addition of the ordinates for the 12- and 8-amp. waves. The angle θ by which the radius vector I_1 leads I_3 equals the angle θ by which the current wave I_1 leads the current wave I_3.

Hence, this problem can be solved without going through the somewhat lengthy process of plotting the waves and adding their ordinates. It is merely necessary to lay off the maximum

values of the waves 60° apart and add them vectorially, just as forces are combined. The resulting vector will be the maximum value of the wave obtained by adding the waves of I_1 and I_2.

In practice, one generally has to do with effective rather than maximum values. If the effective values of the waves be added in this same manner, their vector sum is the sum of the two alternating currents in effective amperes. This is illustrated in

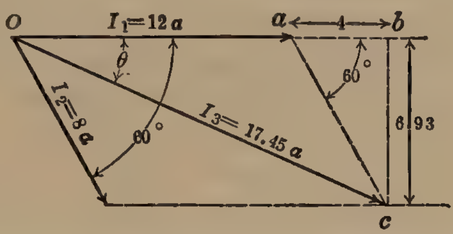


FIG. 18.—Vector addition of currents, using effective values.

Fig. 18, where the 12- and 8-amp. vectors are laid off 60° apart, the 12-amp. vector leading. By completing the parallelogram, the resultant current *Oc* is obtained. This has a value of 17.45 amp. Its value is readily found as follows:

Project *ac* upon *Ob*, where $ac = 8$

$$ab = ac \cos 60° = 4.00$$
$$bc = ac \sin 60° = 6.93$$
$$Oc = \sqrt{(12 + 4.00)^2 + (6.93)^2} = 17.45 \text{ amp.} \qquad \textit{Ans.}$$

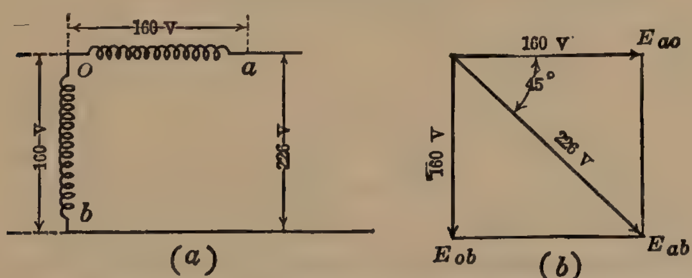


FIG. 19.—Vector addition of two equal voltages having 90° phase difference.

The angle θ can be readily determined.

$$\tan \theta = \frac{6.93}{12 + 4} = 0.433$$
$$\theta = 23.4°$$

Example.—Each of two alternator coils *Oa* and *Ob* (Fig. 19 (*a*)) is generating an e.m.f. of 160 volts. These voltages differ in phase by 90°. Deter-

mine the voltage across their open ends if they are connected together at O as shown.

Let E_{ao} and E_{ob} (Fig. 19 (*b*)) represent the respective voltages across coils aO and Ob. Combining these two vectorially, the voltage E_{ab} is obtained. As E_{ao} and E_{ob} are at right angles, their resultant is readily found.

$$E_{ab} = \sqrt{E_{ao}^2 + E_{ob}^2} = \sqrt{160^2 + 160^2} = 226 \text{ volts.} \quad \textit{Ans.}$$

It must be kept constantly in mind that alternating voltages and currents must be combined vectorially.

The only occasions when arithmetical addition is permissible are when the voltages or the currents are in phase.

CHAPTER II

ALTERNATING-CURRENT CIRCUITS

14. Alternating-current Power.—The power in a direct-current circuit under steady conditions is always given by the product of the volts across the circuit and the current in amperes flowing in the circuit. This same rule applies to alternating-current circuits, provided that only *instantaneous* values of amperes and volts are considered. The *average power*, however, is not necessarily the product of the effective volts and effective amperes, the values which are ordinarily measured with instruments.

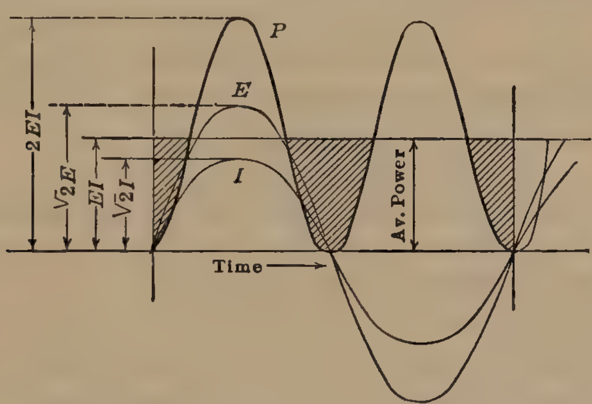


FIG. 20.—Power curve; current and voltage in phase.

Figure 20 shows a voltage wave and a current wave in phase with each other. To obtain the power at any instant, the amperes and the volts at that instant are multiplied together, and a new curve P may be plotted, the ordinates being the instantaneous products of E and I. The curve P then gives the power in the circuit at any instant. The ordinates of this power curve will *always* be positive when E and I are in phase, because the voltage and the current are both positive together during the first half-cycle and are both negative together during the second half-cycle, and the product of two negative quantities is positive. Quite apart from the mathematical reason, it is true that the sign

of the power does not change if both current and voltage are reversed. For example, if a direct-current voltage impressed across a resistance be reversed, the current also reverses. The power dissipated in the resistance does not change, for it is well known that the power dissipated in a constant resistance with fixed voltage is constant, irrespective of the polarity. That is, the power is positive as long as the voltage and the current act in conjunction.

Under the conditions shown in Fig. 20, the current and the voltage act in conjunction throughout the cycle, and the ordinates of the power curve are always positive.

It will be noted that this power curve is a sine wave having double the frequency of either the voltage or the current. In fact, this power wave is identical in character with the current squared waves of Figs. 7 and 9. For every cycle of either voltage or current, the power wave touches the zero axis twice, so that in such a circuit the power is zero twice during each cycle. Since the peaks of the voltage and current waves occur at the same instant, the corresponding peak of the power curve is

$$(\sqrt{2}E)\ (\sqrt{2}I) = 2EI$$

where E and I are the *effective* values of voltage and current.

Although the power varies over wide limits during the cycle, its effect will be determined by its average value. That is, the energy over a complete cycle is equal to the average power (or average ordinate of the power curve) multiplied by the time required to complete a cycle. The average power is determined as follows:

The horizontal axis of symmetry of the power curve is at a distance EI above the zero axis. Consequently, EI must be the *average* value of the power, since the upper half-waves will just fill the shaded valleys below the axis of symmetry of the power curve. When the current and the voltage are *in phase*, the average power is their product, as with direct currents.

Example.—An incandescent-lamp load takes 30 amp. from 115-volt, 60-cycle mains. (In this type of load, the current and voltage are substantially in phase.) How much power do the lamps consume?

$$P = EI = 115 \times 30 = 3{,}450 \text{ watts.} \quad \textit{Ans.}$$

Figure 21 shows the current and voltage 90° out of phase, or in quadrature, the voltage leading. Let it be required to determine the power curve for this condition. At points *a*, *b*, *c*, *d*, and *e*, either the current or the voltage is zero, and the power must be *zero* at each of these points. Between *a* and *b* the voltage is positive and the current is negative. The product of a positive and a negative quantity is negative. Also, the voltage and the current are acting in opposition. Hence, the power between points *a* and *b* must be negative. This means that the circuit is *giving* power to the source of supply. Between points *b* and *c* both current and voltage are positive and, therefore, are acting in conjunction. Hence, the power between these two points must be positive. Between *c* and *d* the current is positive, but the voltage is now negative. The power is again negative, therefore, between these two points. Between *d* and *e* both the current and the voltage are negative, and the power now becomes positive.

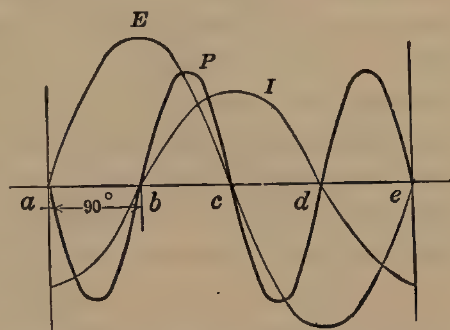


Fig. 21.—Power curve; current and voltage in quadrature, current lagging.

This power curve is a sine wave having double the frequency of either the current or the voltage. Its axis of symmetry coincides with the axis of current and voltage. There must be as much of the power curve above the zero axis as there is below that axis, or the positive power above the axis must be equal to the negative power below the axis. That is, all the positive power received *from* the source is returned *to* the source of supply. The net power, therefore, is *zero*. When current and voltage differ in

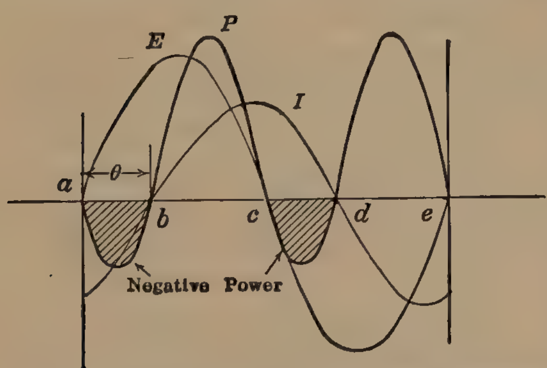


Fig. 22.—Power curve; current and voltage out of phase by angle θ.

phase by 90°, or are in quadrature, the average power is *zero*. If the current *leads* the voltage by 90°, the average power is *zero*, as is shown later in Fig. 31 (p. 33).

If current and voltage are out of phase by an angle less than 90°, but greater than 0°, the resulting power curve P is that indicated in Fig. 22. At points a, b, c, d, and e, either the voltage or the current is zero, and the power is zero at each of these points. Between a and b, and between c and d, the current and voltage are in opposition, and the power is negative. Between b and c, and between d and e, they are in conjunction, and the power is positive. It will be noted that there is more positive power than negative power. The average power is not zero but is positive and is less than the product of E and I. It will be shown later that this power

$$P = EI \cos \theta \qquad (5)$$

where θ is the phase angle between voltage and current. Cos θ is called the *power factor* of the circuit. P is the *true watts* and EI the *apparent watts*, or *volt-amperes*.

The power factor

$$\text{P. F.} = \cos \theta = \frac{\text{true watts}}{\text{apparent watts}} = \frac{P}{EI}. \qquad (6)$$

The power factor can never be greater than unity.

15. Circuit Containing Resistance Only.—Figure 23 shows an alternating-current circuit containing resistance only. A potential difference of E volts is impressed across the resistance R. In virtue of this voltage, a current having the equation $i = I_{max} \sin \omega t$ flows, where ω is the angular velocity of the rotating vector in radians per second (see p. 11, Par. 6, Eq. (3)). As one revolution of the rotating vector corresponds to 2π radians, the vector must complete $2\pi f$ radians per second, where f is the frequency. Hence, $\omega = 2\pi f$. (For 60 cycles, $\omega = 377$; for 25 cycles, $\omega = 157$.)

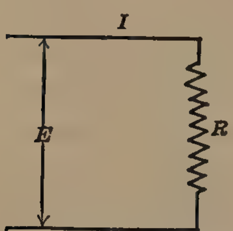


FIG. 23.—Circuit containing resistance only.

From the definition of an alternating-current volt (Par. 9),

$$e = Ri = RI_{max} \sin \omega t.$$

The current and the voltage have the same frequency $\omega/2\pi$. They are also in phase, for when $t = 0$, $\sin \omega t = 0$, and both the

current and voltage waves are crossing the zero axis simultaneously and increasing positively, as shown in Fig. 24 (a).

If effective values are used, $E = IR$. Figure 24 (b) shows the vector diagram for this circuit, using effective values. The IR drop is in phase with the current I and is equal to the voltage E, since no other voltage exists in the circuit.

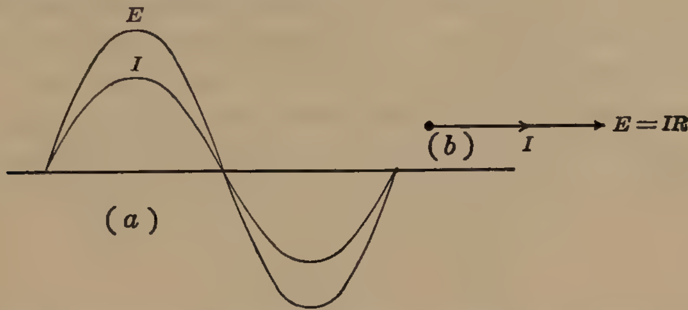


FIG. 24.—Current and voltage waves in phase, and vector diagram.

As the current and the voltage are in phase, the power

$$P = EI \tag{7}$$

as is shown in Fig. 20. Also,

$$P = I^2R$$

It will be observed that with resistance only, the alternating-current circuit follows the same laws as the direct-current circuit, in regard to the relation existing among voltage, current, resistance, and power.

16. Circuit Containing Inductance Only.—It was shown in Vol. I (Chap. VIII) that inductance always *opposes* any *change*

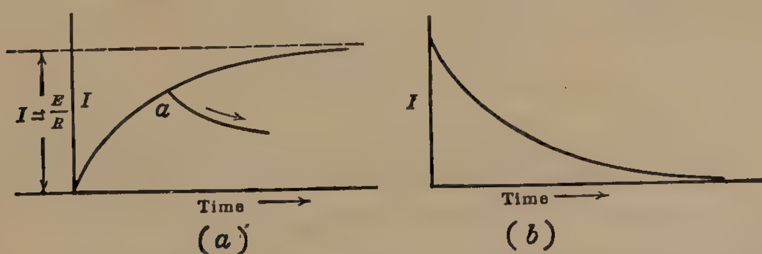


FIG. 25.—Increase and decrease of current in an inductive circuit.

in the current flowing in a circuit. For example, when the current starts to increase in an inductive circuit, the electromotive force of self-induction opposes this increase. This is illustrated in Fig. 25 (a), which shows the rise of current in a direct-current

circuit containing resistance and inductance, when a steady voltage is impressed. The current rises *slowly* to its ultimate value.

On the other hand, when the current attempts to decrease in the circuit, the inductance tends to prevent this decrease, as is shown in Fig. 25 (*b*). In other words, if inductance is present in a circuit, it always opposes any change in the current. With a *steady* direct current, however, the inductance has no effect.

If, in Fig. 25 (*a*), the voltage across the inductance be lowered when the current reaches point *a*, the current will not reach its Ohm's-law value. This same effect occurs in alternating-current circuits. With inductance in the circuit, the current does

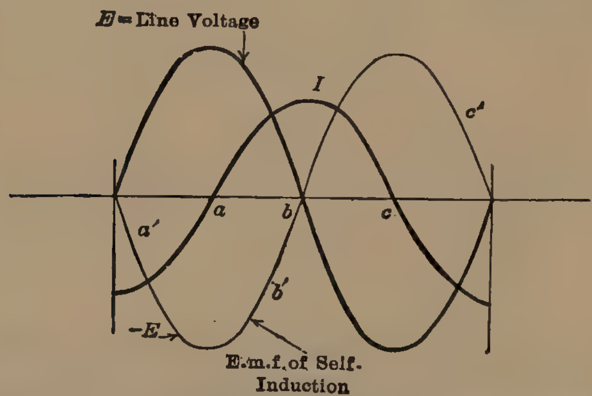


FIG. 26.—Current and voltages existing in alternating-current circuit containing inductance only.

not have time to reach its Ohm's-law value before the voltage begins to decrease either positively or negatively. The current change is opposed by the electromotive force of self-induction, which at any instant is equal to $-L\frac{di}{dt}$, where L is the inductance in henrys and di/dt is the rate in amperes per second at which the current is changing at that instant. The minus sign signifies that this voltage is opposing the change in the current.

Figure 26 shows a current wave I. Starting at a, the current is *changing* at its maximum rate in a positive direction. At this instant, therefore, the electromotive force of self-induction must be at its negative maximum value. At point b, the top of the current wave is horizontal, and, therefore, at this instant the current is not changing at all. Hence, the electromotive force of

self-induction is zero. At c the current is changing at its maximum rate negatively, and the electromotive force of self-induction must be maximum positive, because of the negative sign in the formula. Continuing in this way, the voltage curve $a'b'c'$ is obtained. It will be observed that this wave is a sine wave and is lagging the current by 90°.

This is the only voltage in the circuit which *opposes* the change of current. It corresponds to the back electromotive force of a motor. The line, in the case of the motor, must supply a voltage opposite and equal to the back electromotive force before any current can flow into the armature. This same condition exists in the alternating-current circuit. Before any current can flow into a circuit containing inductance, but no resistance, a voltage opposite and equal to the electromotive force of self-induction must be supplied by the line.

In Fig. 26, therefore, the voltage E, which is the line voltage, is opposite and equal to the electromotive force of self-induction. It will be noted that the impressed voltage *leads* the current by 90°, or the current *lags* this voltage by 90°. With inductance only in the circuit, the current *lags* the impressed voltage by 90°. (In practice, it is impossible to obtain a pure inductance, as inductance must necessarily be accompanied by a certain amount of resistance.)

The above may also be proved as follows: Let the current be given by $i = I_{max} \sin \omega t$. The e.m.f. of self-induction

$$e' = -L\frac{di}{dt} = -L\omega I_{max} \cos \omega t$$

$$= L\omega I_{max} \sin (\omega t - 90°) \qquad \text{(I)}$$

is a sine wave lagging 90° with respect to $I_{max} \sin \omega t$.

The equation of the line voltage which balances this e.m.f.,

$$e = L\omega I_{max} \sin (\omega t + 90°) \qquad \text{(II)}$$

is a sine wave *leading* the current $I_{max} \sin \omega t$ by 90°.

The choking effect of inductance is obviously proportional to the frequency and to the inductance. To express this choking effect in ohms, the self-inductance in henrys must be multiplied by $\omega = 2\pi f = 6.28f$, where f is the circuit frequency.

This follows from Eq. (II). The maximum value of e occurs when $\sin(\omega t + 90°)$ is unity. Hence,

$$E_{max} = L\omega I_{max}$$

$$I_{max} = \frac{E_{max}}{L\omega} \qquad \text{(III)}$$

Dividing both sides of (III) by $\sqrt{2}$,

$$\frac{I_{max}}{\sqrt{2}} = \frac{(E_{max})}{(\sqrt{2})}\frac{1}{L\omega}$$

$$I = \frac{E}{L\omega} = \frac{E}{2\pi fL} \qquad (8)$$

where I and E are effective values of current and voltage.

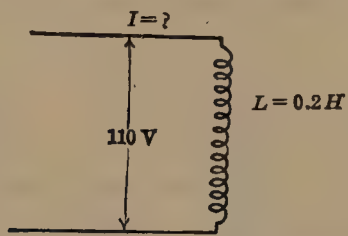


Fig. 27.—Circuit containing inductance only.

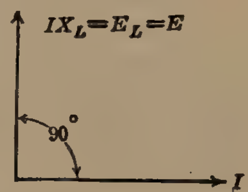


Fig. 28.—Vector diagram for circuit containing inductance only.

That is, $2\pi fL$ is the resistance to the flow of current offered by inductance and is called the *inductive reactance* of the circuit. It is denoted by X_L and is expressed in *ohms*.

The impressed voltage is

$$E = 2\pi fLI = IX_L \qquad (9)$$

Example.—Figure 27 shows a pure inductance of 0.2 henry connected across 110-volt, 60-cycle mains. What current flows?

$$X_L = 2\pi\ 60 \times 0.2 = 377 \times 0.2 = 75.4 \text{ ohms}$$

$$I = \frac{110}{75.4} = 1.46 \text{ amp.} \quad \textit{Ans.}$$

Figure 28 shows a vector diagram for an inductive circuit in which the impressed voltage leads the current by 90°.

17. Circuit Containing Capacitance Only.—When a direct-current voltage is impressed across the plates of a perfect condenser (Vol. I, Chap. IX), there is an initial rush of current which charges the condenser to line potential. After this there is no further flow of current if the line voltage remains constant. If the con-

denser plates now are short circuited, making the voltage across the plates zero, current flows out of the condenser.

Figure 29 (a) shows an alternating voltage E impressed across the plates of a condenser C. When the voltage starts from its zero value at a (Fig. 29 (b)) and increases positively, current flows into the condenser. This current, therefore, is positive. As long as the voltage across the condenser plates continues to increase, current must flow into the condenser from the positive wire, and this current will be positive in sign. When point b is reached, the increase of voltage ceases, and the current becomes

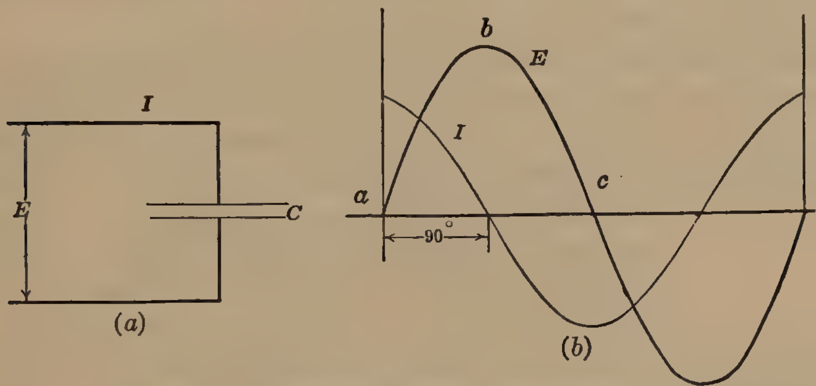


FIG. 29.—Circuit containing capacitance only.

zero. Between b and c the voltage is decreasing so that current is flowing *out* of the condenser into the positive line, and as the current flow has reversed, the sign of the current is now negative. After E passes through zero at c, the e.m.f. is negative and charges the condenser in the opposite direction, so that the current still remains negative. This continues until the voltage reaches its negative maximum. At this point, the current reverses and again becomes positive.

An examination of Fig. 29 shows that when an alternating voltage is impressed across a condenser, the current into the condenser leads the voltage by 90°. This is illustrated by Fig. 30, in which the relation is shown vectorially.

These relations of current and voltage in a condenser circuit may also be proved as follows:

Let e be the instantaneous voltage across the condenser, C the capacitance in farads, and q the charge in coulombs at any instant.

Let $i = I_{max} \sin \omega t$ be the equation of the current.

$$e = \frac{q}{C}$$

$$q = \int idt = \int I_{max} \sin \omega tdt$$

$$e = \frac{q}{C} = \frac{I_{max}}{C}\int \sin \omega tdt = \frac{I_{max}}{C\omega}(-\cos \omega t)$$

$$= \frac{I_{max}}{C\omega} \sin (\omega t - 90°). \qquad \text{(I)}$$

This equation (I) shows that the sine wave of voltage lags the current wave by 90°.

It will be seen from the foregoing that alternating current does not actually flow conductively through the insulation of the condenser. A perfect condenser offers an *infinite resistance* to alternating as well as to direct current. With alternating current, however, the condenser is alternately charged and discharged, so that a quantity of electricity flows into the positive plate and then out again, etc. It is this quantity of electricity which flows to charge and to discharge the condenser which constitutes the alternating current. An ammeter placed in the line to such a condenser indicates a current. The more rapidly the voltage alternates the greater the quantity of electricity charged and discharged per second and therefore, the greater the flow of current. Hence, the current must be proportional to the frequency. This is further shown by Eq. (10).

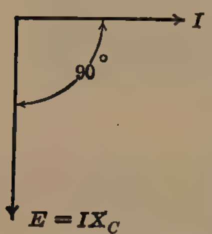


FIG. 30.—Vector diagram for circuit containing capacitance only.

In Eq. (I), the current reaches its maximum value when $\sin (\omega t - 90°) = 1.0$

Hence,

$$E_{max} = \frac{I_{max}}{C\omega} \qquad \text{(II)}$$

and

$$I_{max} = E_{max}C\omega \qquad \text{(III)}$$

Dividing both sides of (III) by $\sqrt{2}$,

$$\frac{I_{max}}{\sqrt{2}} = \frac{E_{max}}{\sqrt{2}}C\omega \qquad \text{(IV)}$$

$$I = EC\omega = E(2\pi fC) \qquad \text{(10)}$$

where I and E are effective values, and C is in *farads.* The current is proportional to the voltage and capacitance as well as to the frequency.

Equation (10) may also be written

$$I = \frac{E}{1/2\pi fC} = \frac{E}{X_C}. \tag{11}$$

X_C is called the *condensive* or *capacitive reactance* of the circuit in ohms and is equal to $1/(2\pi fC)$.

Also,

$$E = \frac{I}{2\pi fC} = IX_C. \tag{12}$$

Example.—What is the condensive reactance of a 10-μf. condenser at 60 cycles per second, and how much current will it take from 110-volt, 60-cycle mains?

$$10\ \mu\text{f.} = 0.00001 \text{ farad.}$$

$$X_C = \frac{1}{2\pi 60 \times 0.00001} = \frac{100{,}000}{2\pi 60} = 265 \text{ ohms.} \quad \textit{Ans.}$$

$$I = {}^{110}\!/\!_{265} = 0.415 \text{ amp.} \quad \textit{Ans.}$$

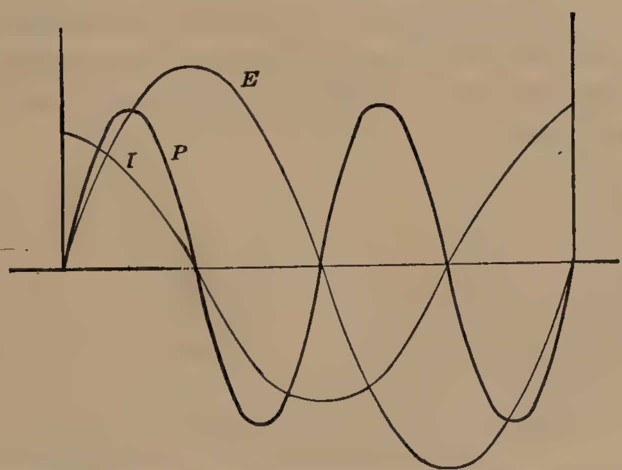


FIG. 31.—Voltage, current, and power curves; circuit containing capacitance only.

The average power in a circuit containing capacitance only is zero.

This may be shown by plotting the power curve from the current and voltage curves, as was done in Fig. 21. This is done in Fig. 31, where P is the curve of power. There is as much of the power curve below as above the zero axis, so that the net power is *zero*, as in a circuit with pure inductance only. When

the power curve is positive, energy is being delivered to the circuit and stored in the condenser; when the power curve is negative, this energy is being given back again to the source. Although the net power is zero, there is a continuous transfer of energy from the source to the condenser and back again to the source.

18. Circuit Containing Resistance and Inductance in Series.—Figure 32 shows a circuit consisting of a resistance R and an inductive reactance X_L connected in series across an alternating circuit whose frequency is f cycles per second. The voltage impressed cross the circuit is E, and a current I flows. Let it be required to determine the relations among I, E, R, and X_L.

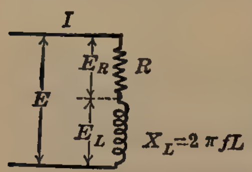


FIG. 32.—Circuit containing resistance and inductance in series.

Figure 33 (*a*) shows a vector diagram for this circuit. As the current I is the same in both X_L and R, it is laid off horizontally to scale. The position of the current vector I is arbitrary (it is given the position shown merely for convenience). From Fig. 24 (*b*) (p. 27), the voltage E_R across the resistance R is *in phase* with the current. It is laid off, therefore, along the current vector (Fig. 33(*a*)).

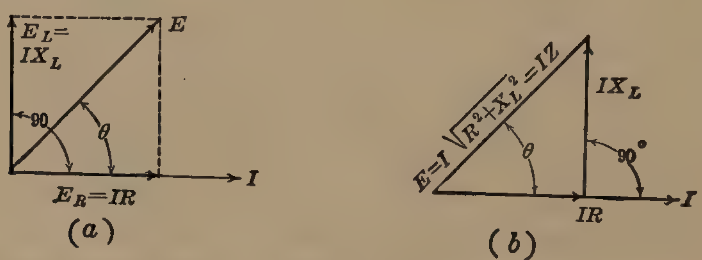


FIG. 33.—Vector diagram for series circuit containing resistance and inductance.

From Fig. 28 (p. 30), the voltage E_L across the inductance *leads* the current I by 90° and is equal to IX_L (Fig. 33 (*a*)).

The line voltage E must be the vector sum of these two voltages. Hence, the parallelogram is completed, and the diagonal, which is the vector sum of E_R and E_L, gives the line voltage E. The same result is obtained if IX_L is laid off perpendicular to I at the end of the vector IR, using a triangle rather than a parallelogram, as shown in Fig. 33 (*b*).

As a right triangle is formed by these three voltages, the hypotenuse

$$E = \sqrt{(IR)^2 + (IX_L)^2}$$
$$= \sqrt{I^2(R^2 + X_L^2)} = I\sqrt{R^2 + X_L^2}$$

and

$$I = \frac{E}{\sqrt{R^2 + X_L^2}} = \frac{E}{\sqrt{R^2 + (2\pi fL)^2}} = \frac{E}{Z}. \quad (13)$$

$Z = \sqrt{R^2 + X_L^2}$ is the *impedance* of the circuit and is expressed in ohms. It is ordinarily denoted by Z. Equation (13) corresponds to Ohm's law for the direct-current circuit. The current in an alternating-current circuit is *directly* proportional to the *voltage* across the circuit and *inversely* proportional to the *impedance* of the circuit. That is, if the voltage in volts be divided by the impedance in ohms, the value of the current in amperes is obtained.

Also, the voltage

$$E = IZ. \quad (14)$$

An inspection of Fig. 33 shows that the angle θ by which the current lags the voltage may be determined as follows:

$$\tan\theta = \frac{IX_L}{IR} = \frac{X_L}{R} = \frac{2\pi fL}{R} \quad (15)$$

$$\cos\theta = \frac{IR}{\sqrt{(IR)^2 + (IX_L)^2}} = \frac{R}{\sqrt{R^2 + X_L^2}} = \frac{R}{Z}. \quad (16)$$

Example.—A circuit containing 0.1-henry inductance and 20-ohms resistance in series is connected across 100-volt, 25-cycle mains. (*a*) What is the impedance of the circuit? (*b*) What current flows? (*c*) What is the voltage across the resistance? (*d*) What is the voltage across the inductance? (*e*) Determine the angle by which the voltage leads the current.

$X_L = 2\pi 25 \times 0\ 1 = 157 \times 0.1 = 15.7$ ohms.

(*a*) $Z = \sqrt{(20)^2 + (15.7)^2} = \sqrt{646} = 25.4$ ohms. *Ans.*

(*b*) $I = E/Z = 100/25.4 = 3.94$ amp. *Ans.*

(*c*) $E_R = IR = 3.94 \times 20 = 78.8$ volts. *Ans.*

(*d*) $E_L = IX_L = 3.94 \times 15.7 = 61.8$ volts. *Ans.*

As a check, $\sqrt{(78.8)^2 + (61.8)^2} = 100$ volts.

(*e*) $\tan\theta = X_L/R = 15.7/20 = 0.785$.

From p. 538, $\theta = 38.1°$. *Ans.*

19. Power.—It has already been shown that a pure inductance consumes no power. The inductance of Fig. 32, therefore, con-

sumes no power. All the power expended in the circuit must be accounted for in the resistance. That is,

$$P = I^2R = I(IR).$$

IR is obviously equal to $E \cos \theta$ (Fig. 33). Therefore, the power

$$P = I(IR) = IE \cos \theta = EI \cos \theta.$$

As has already been shown, $\cos \theta$ is the *power factor* of the circuit and is equal to the true power divided by the volt-amperes or apparent power.

$$\text{P. F.} = \frac{P}{EI}.$$

Obviously, the power factor can never exceed 1.0. It is usually less than 1.0.

Example.—How much power is consumed in the foregoing circuit, and what is the power factor?

$$P = I^2R = (3.94)^2 \times 20 = 310 \text{ watts.} \quad \textit{Ans.}$$

$$\text{P. F.} = \frac{P}{EI} = \frac{310}{100 \times 3.94} = 0.787. \quad \textit{Ans.}$$

Also,

$$\cos \theta = \text{P. F.} = R/Z = 20/25.4 = 0.787. \quad \textit{Ans.}$$

20. Circuit Containing Resistance and Capacitance in Series. Figure 34 shows a circuit containing a resistance R and a con-

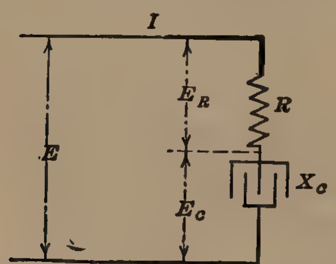


FIG. 34.—Circuit containing resistance and capacitance in series.

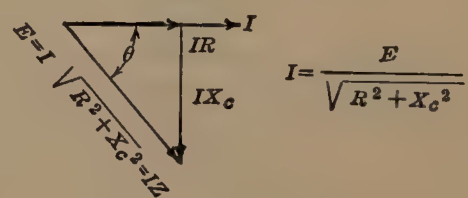


FIG. 35.—Vector diagram for circuit containing resistance and capacitance in series.

densive reactance X_C in series. An alternating voltage E, of frequency f cycles per second, is impressed across this circuit, and a current I flows. Let it be required to determine the relation existing among E, I, R, and X_C.

The current I is the same in both R and X_C and is laid off horizontal in the vector diagram (Fig. 35). The voltage E_R

across the resistance is *in phase* with the current. The voltage E_C across the condensive reactance *lags* the current I by 90° (see Fig. 30, p. 32). The line voltage E is obviously the vector sum of IR and IX_C and is, therefore, the hypotenuse of the right triangle having these two voltages as sides. Hence,

$$E = \sqrt{(IR)^2 + (IX_C)^2} = I\sqrt{R^2 + X_C^2} = IZ \qquad (17)$$

where Z is the *impedance* of the circuit.

Solving for the current I,

$$I = \frac{E}{\sqrt{R^2 + X_C^2}} = \frac{E}{\sqrt{R^2 + (1/2\pi fC)^2}} = \frac{E}{Z}. \qquad (18)$$

The power taken by the circuit is, obviously,

$$P = I^2R = I(IR)$$

as the net power taken by the condenser is zero.

$$IR = E \cos \theta.$$

Therefore, $P = EI \cos \theta$, which is the same expression for power as with inductance and resistance in circuit.

The angle θ may be determined as follows:

$$\tan \theta = \frac{IX_C}{IR} = \frac{X_C}{R} = \frac{1}{2\pi fCR} \qquad (19)$$

$$\cos \theta = \frac{R}{\sqrt{R^2 + X_C^2}} = \frac{R}{\sqrt{R^2 + (1/2\pi fC)^2}} = \frac{R}{Z} = \text{P. F.}$$

C must be expressed in *farads*.

Example.—A capacitance of 20 μf. and a resistance of 100 ohms are connected in series across 120-volt, 60-cycle mains. Determine: (*a*) the impedance of the circuit; (*b*) the current flowing in the circuit; (*c*) the voltage across the resistance; (*d*) the voltage across the capacitance; (*e*) the angle between the voltage and the current; (*f*) the power; (*g*) the power factor of the circuit.

$$20\ \mu\text{f.} = 0.000020 \text{ farad.}$$

$$X_C = \frac{1}{2\pi 60 \times 0.000020} = 132.6 \text{ ohms.}$$

(*a*) $Z = \sqrt{(100)^2 + (132.6)^2} = \sqrt{27{,}600} = 166$ ohms. *Ans.*

(*b*) $I = {}^{120}\!/_{166} \doteq 0.723$ amp. *Ans.*

(*c*) $E_R = IR = 0.723 \times 100 = 72.3$ volts. *Ans.*

(*d*) $E_C = IX_C = 0.723 \times 133 = 96.2$ volts. *Ans.*

$$\sqrt{(72.3)^2 + (96.2)^2} = 120 \text{ volts } (\textit{check}).$$

(*e*) $\tan \theta = X_C/R = {}^{132.6}\!/_{100} = 1.326.$

$\theta = 53.0°.$ *Ans.*

(f) $P = I^2R = (0.723)^2 \times 100 = 52.2$ watts. *Ans.*
(g) $\cos\theta = R/Z = {}^{100}\!/_{166} = 0.602$. *Ans.*
Also,

$$\text{P. F.} = \frac{P}{EI} = \frac{52.2}{120 \times 0.723} = 0.602 \text{ (check).}$$

21. Circuit Containing Resistance, Inductance, and Capacitance in Series.—Figure 36 shows a resistance R, an inductive reactance X_L, and a condensive reactance X_C, all connected in series. The voltage across the circuit is E volts, the frequency is f cycles per second, and the current is I amp.

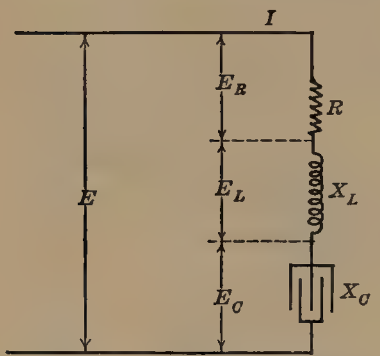


Fig. 36.—Circuit containing resistance, inductance, and capacitance in series.

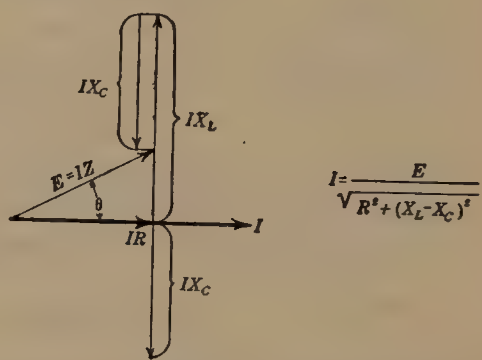


Fig. 37.—Vector diagram for circuit containing resistance, inductance, and capacitance, all in series.

As this is a series circuit, the current is the same in all parts of the circuit, and for convenience the current vector I is laid off horizontal in the circuit vector diagram (Fig. 37). The voltage E_R $(= IR)$ across the resistance is *in phase* with the current and is laid off to scale along the current vector. The voltage E_L $(= IX_L)$ across the inductance is laid off at right angles to the current and *leading*. The voltage E_C $(= IX_C)$ across the condenser is laid off at right angles to the current and *lagging*.

An examination of Fig. 37 shows that the voltage across the inductance and that across the capacitance are in opposition, so that the resultant voltage of these two is their arithmetical difference. In this particular case, IX_L is shown as being greater than IX_C. IX_C, therefore, is subtracted directly from IX_L. The line voltage must be the vector sum of the three

voltages and is the hypotenuse of a right triangle of which IR and $(IX_L - IX_C)$ are the other sides. Therefore,

$$E = \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$

$$E = I\sqrt{R^2 + (X_L - X_C)^2}. \tag{20}$$

Solving for I,

$$I = \frac{E}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{E}{Z} \tag{21}$$

which is the equation for the series alternating-current circuit in the steady state.

The values of X_L and X_C may be substituted in Eq. (21). It then becomes

$$I = \frac{E}{\sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}}. \tag{22}$$

The phase angle θ is found as follows:

$$\tan \theta = \frac{X_L - X_C}{R}. \tag{23}$$

If X_L is greater than X_C, the tangent is positive, and θ is positive, as shown in Fig. 37. This indicates a *lagging* current. If X_C is greater than X_L, the tangent becomes negative and the angle θ becomes negative. This indicates a *leading* current.

The power factor of the circuit

$$\text{P. F.} = \cos \theta = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{R}{Z}. \tag{24}$$

Example.—A series circuit consisting of a resistance of 50 ohms, a capacitance of 25 μf., and an inductance of 0.15 henry is connected across 120-volt, 60-cycle mains.

Find: (*a*) the impedance of the circuit; (*b*) the current in the circuit; (*c*) the voltage across the resistance; (*d*) the voltage across the inductance; (*e*) the voltage across the capacitance; (*f*) the power taken by the circuit; (*g*) the phase angle of the circuit; (*h*) the power factor of the circuit.

$$X_L = 2\pi 60 \times 0.15 = 377 \times 0.15 = 56.6 \text{ ohms.}$$

$$X_C = \frac{1}{2\pi 60 \times 0.000025} = 106 \text{ ohms.}$$

(*a*) $Z = \sqrt{(50)^2 + (56.6 - 106)^2} = \sqrt{(50)^2 + (-49.4)^2} = 70.2$ ohms. *Ans.*

(*b*) $I = 120/70.2 = 1.71$ amp. *Ans.*

(*c*) $E_R = IR = 1.71 \times 50 = 85.5$ volts. *Ans.*

(*d*) $E_L = IX_L = 1.71 \times 56.6 = 96.8$ volts. *Ans.*

(*e*) $E_C = IX_C = 1.71 \times 106 = 181.1$ volts. *Ans.*

(*f*) $P = I^2R = (1.71)^2 \times 50 = 146$ watts. *Ans.*

(*g*)
$$\tan\theta = \frac{X_L - X_C}{R} = \frac{56.6 - 106}{50} = \frac{-49.4}{50} = -0.988.$$

$\theta = -44.6°$. Therefore, the current leads. *Ans.*

(*h*)
$$\cos\theta = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{50}{70.2} = 0.712. \quad \textit{Ans.}$$

$$\cos\theta = \frac{P}{EI} = \frac{146}{120 \times 1.71} = 0.712 \ (\textit{check}).$$

Figure 38 gives the vector diagram for the circuit conditions represented by this problem.

It will be observed that the voltage across the condenser exceeds the line voltage by a considerable amount. This would be impossible under like conditions in a direct-current circuit, for the voltage across any part of the circuit cannot exceed the line voltage. This condition can exist in an alternating-current circuit, because the condenser voltage and the inductance voltage are in direct opposition. Both may be large, provided their difference is less than the line voltage.

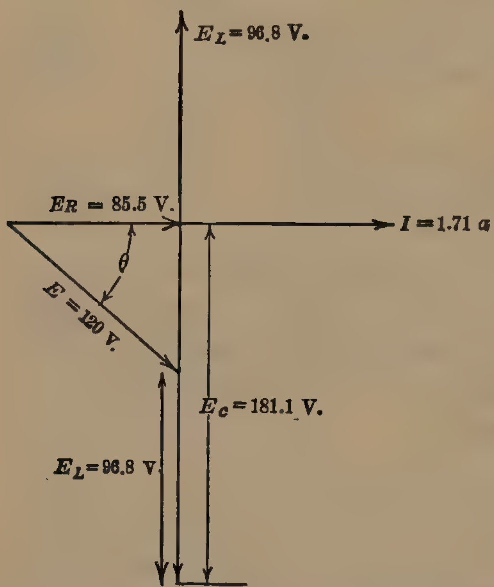


FIG. 38.—Vector diagram for series circuit, giving numerical values.

22. Resonance in a Series Circuit.—The general equation (22) for the current in a series circuit shows that for fixed values of resistance and impressed voltage, the current is a maximum when the expression in the parenthesis under the square-root sign is equal to zero.

That is, in the equation

$$I = \frac{E}{\sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}}$$

the current is a maximum when

$$\left(2\pi fL - \frac{1}{2\pi fC}\right) = 0$$

and then becomes

$$I = \frac{E}{\sqrt{R^2 + (0)}} = \frac{E}{R}$$

its Ohm's-law value.

Under these conditions,

$$2\pi fL = \frac{1}{2\pi fC} \tag{25}$$

and

$$2\pi fLI = \frac{I}{2\pi fC}.$$

That is, the voltage across the inductance is equal to the voltage across the capacitance. As these two voltages are in exact opposition, they balance each other, so that the IR drop is equal to the line voltage. This is illustrated in Fig. 39.

When the foregoing conditions exist, the circuit is said to be in *resonance*. The current is then in phase with the line voltage, and the power $P = EI$.

Solving Eq. (25) for the frequency,

$$2\pi fL - \frac{1}{2\pi fC} = 0$$

$$f = \frac{1}{2\pi\sqrt{LC}}. \tag{26}$$

This is the frequency for which a circuit having fixed values of L and C will be in resonance. It is sometimes called the *natural frequency* of the circuit, because it is the frequency at which the current in the circuit will oscillate, if no external frequency is impressed on the circuit, provided the resistance R is less than $\sqrt{4L/C}$. For example, in a radio circuit, a condenser C, charged to a high voltage, is discharged into an inductance L, of negligible resistance. The frequency of the resulting oscillations as determined by the values of L and C, is given in Eq. (26).

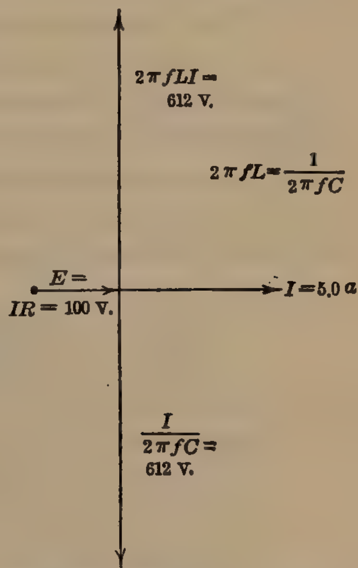


FIG. 39.—Vector diagram for series circuit in resonance.

As the voltage across the inductance equals the voltage across the capacitance, when the circuit is in resonance, and the two are

in opposition, each may reach a high value, even with moderate line voltage. This is illustrated by the following example:

Example.—A circuit has a resistance of 20 ohms, an inductance of 0.3 henry, and a capacitance of 20 μf. (*a*) For what value of the frequency will the circuit be in resonance? If the current is 5 amp., find: (*b*) the line voltage; (*c*) the voltage across the inductance; (*d*) the voltage across the capacitance; (*e*) the power consumed by the circuit. Draw a vector diagram for the circuit.

(*a*) $f = 1/2\pi\sqrt{0.3 \times 0.000020} = 65$ cycles. *Ans.*
(*b*) $E = IR = 5 \times 20 = 100$ volts. *Ans.*
(*c*) $E_L = 2\pi fLI = 6.28 \times 65 \times 0.3 \times 5 = 612$ volts. *Ans.*
(*d*) $E_C = I/(2\pi fC) = 612$ volts. *Ans.*
(*e*) $P = EI = 100 \times 5 = 500$ watts. *Ans.*
The vector diagram is shown in Fig. 39.

It will be observed that the voltage across the inductance and that across the capacitance are equal, each being 612 volts, or more than six times the line voltage.

It should be noted that the current is a *maximum* when a series circuit is in resonance.

Resonance Characteristics of Circuits.—In any circuit whose frequency is fixed, there is an infinite number of combina-

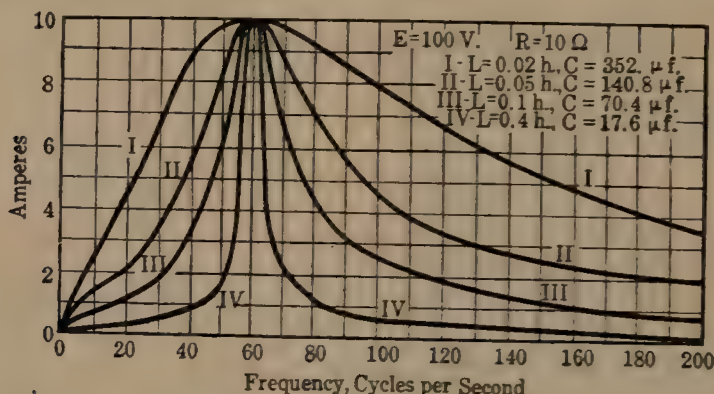


Fig. 40.—Resonance curves.

tions of inductance and capacitance which will give resonance. This may be seen from an examination of Eq. (26). It is only necessary that the product LC remain constant. For example, after the circuit has been adjusted to resonance, if the inductance be halved and the capacitance doubled, the resonant condition still exists. But the manner in which the current alters as the frequency changes depends on the relation of the inductance to

the capacitance. This is illustrated in Fig. 40. The voltage across a circuit having 10-ohms resistance is maintained constant at 100 volts. The circuit is first tuned to 60 cycles by adjusting the inductance and capacitance to 0.02 henry and 352 μf. The variation of current with frequency under these conditions is shown by curve I. The current is zero at zero frequency, since the condenser gives an open circuit for direct current. The current reaches its maximum when the frequency becomes 60 cycles per second. The current is zero at infinite frequency, since with inductance the reactance is infinite at infinite frequency. Curve II shows the variation of current with frequency when the inductance is 0.05 henry and the capacitance is 140.8 μf. The values of current, except at the resonant frequency, are now considerably less than those given by curve I. Curve III shows the variation of current with frequency when the inductance is 0.1 henry and the capacitance is 70.4 μf.; curve IV shows the variation of current with frequency when the inductance is 0.4 henry and the capacitance is 17.6 μf.

It will be noted that as the inductance is decreased and the capacitance is correspondingly increased, the tuning of the circuit becomes sharper; that is, a very small variation of frequency on either side of the resonant frequency causes a large decrease in current. The tuning with curve IV is extremely sharp.

This relationship is particularly useful in communication circuits, as, for example, radio receiving sets, where sharp tuning is often essential.

It will be observed that, for each of the curves I to IV, the product of L and C is constant and is equal to 7.04×10^{-6}.

23. Parallel Circuits.—In practice, parallel circuits are more common than series circuits, because of the extended use of the multiple system of transmission and distribution. The solution of problems with two or more loads in parallel involves the finding of the current in each branch of the circuit and the combining of these currents *vectorially* to give the resultant current.

This is illustrated by the following example:

A resistance of 10 ohms, an inductive reactance of 8 ohms, and a condensive reactance of 15 ohms are all connected in parallel across 120-volt, 60-cycle mains, as shown in Fig. 41 (a). Determine: (a) the total current; (b) the circuit power factor; (c) the power.

The current taken by the resistance

$$I_R = 120/10 = 12 \text{ amp. in phase with } E.$$
$$I_L = 120/8 = 15 \text{ amp. in quadrature with } E \text{ and lagging.}$$
$$I_C = 120/15 = 8 \text{ amp. in quadrature with } E \text{ and leading.}$$

These currents are shown vectorially in Fig. 41 (*b*).

The voltage is the same for all three branches of the circuit and is laid off as the horizontal vector. The resistance current I_R is in phase with the voltage E. The inductive current *lags* the voltage by 90°, and the condensive current *leads* the voltage by 90°. As the inductive current and condensive current are in exact opposition, they subtract from each other,

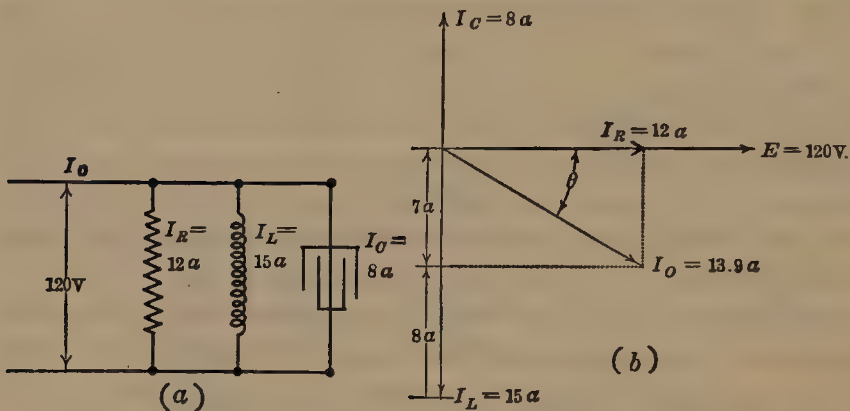


FIG. 41.—Alternating-current parallel circuit and vector diagram.

leaving 7 amp. lagging by 90°. The resultant current I_0 is the vector sum of the 7 amp. and the 12 amp.

(*a*) $I_0 = \sqrt{12^2 + 7^2} = 13.9$ amp. lagging. *Ans.*

(*b*) Obviously, the cosine of the angle θ between the voltage and the current is

$$\cos\theta = \frac{I_R}{I_0} = \frac{12}{13.9} = 0.864 = \text{P.F.} \quad \textit{Ans.}$$

(*c*) $P = EI_R = 120 \times 12 = 1{,}440$ watts. *Ans.*
Also $P = EI_0 \cos\theta = 120 \times 13.9 \times 0.864 = 1{,}440$ watts. *Ans.*

For convenience, the following equations are given for the parallel circuit:

R and *L* in parallel

$$Z = \frac{1}{\sqrt{(1/R)^2 + (1/X_L)^2}} = \frac{RX_L}{\sqrt{R^2 + X_L^2}}. \tag{27}$$

$$I_0 = \frac{E}{Z}.$$

R and C in parallel

$$Z = \frac{1}{\sqrt{(1/R)^2 + (1/X_C)^2}} = \frac{RX_C}{\sqrt{R^2 + X_C^2}}. \tag{28}$$

$$I_0 = \frac{E}{Z}.$$

R, L, and C in parallel

$$Z = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}} = \frac{RX_LX_C}{\sqrt{X_L^2X_C^2 + R^2(X_L - X_C)^2}}. \tag{29}$$

$$I_0 = \frac{E}{Z}$$

where I_0 is the total current and E is the circuit voltage.

Resonance in a Parallel Circuit.—Resonance (or anti-resonance) in a parallel circuit exists when the resultant current and the line voltage are in phase with each other. Under these

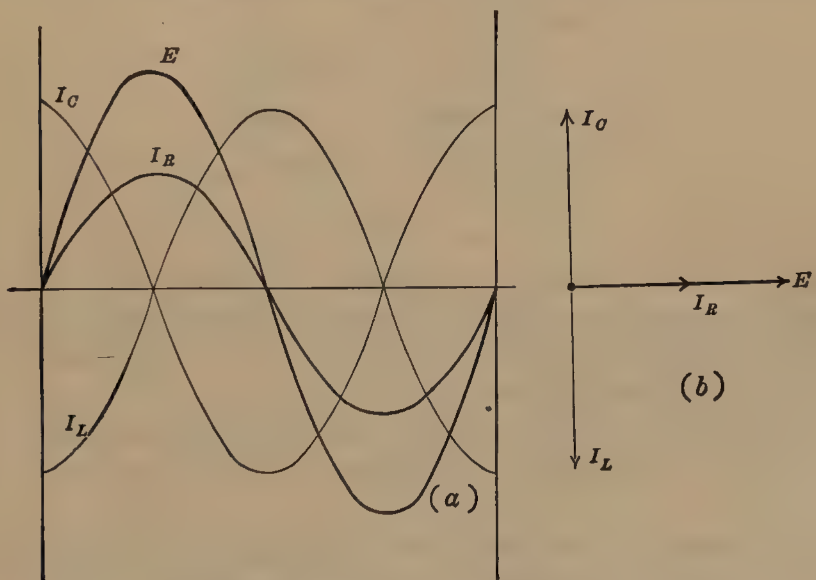


FIG. 42.—Resonance in parallel circuit.

conditions, the condensive current must be equal to the inductive current. These two, being opposite and equal, will balance each other, leaving only the resistance current. This is illustrated in Fig. 42 (*a*). E is the voltage wave; I_R is the current in the resistance; I_L is the current in the inductance; I_C is the current in the condenser and is equal to I_L. As the inductive current *lags* the voltage by 90° and the condensive current *leads*

the voltage by 90°, they are in direct opposition, and, being equal, they balance. This leaves only I_R.

Figure 42 (*b*) illustrates vectorially these circuit conditions. It will be observed that the total current is a *minimum* when the *parallel* circuit is in resonance,[1] whereas in the *series* circuit, the current is a *maximum* at resonance. In the *parallel* circuit, the inductive and condensive *currents* are opposite and equal; in the *series* circuit, the inductive and condensive *voltages* are opposite and equal. If a pure capacitance and a pure inductance were connected in parallel and adjusted for resonance, the line current would be zero, even though the inductance and condenser were each taking current.

Example.—A resistance of 12 ohms, an inductance of 0.2 henry, and a condenser are connected in parallel across 120-volt, 60-cycle mains. For what value of capacitance will the circuit be in resonance?

I_C must be equal to I_L

$$I_L = \frac{120}{2\pi 60 \times 0.2} = 1.59 \text{ amp.}$$

$$I_C = 120 \times 2\pi 60 \times C = 1.59 \text{ amp.}$$

$$C = \frac{1.59}{120 \times 2\pi 60} = 0.0000352 \text{ farad}$$

$$= 35.2\ \mu\text{f.} \quad \textit{Ans.}$$

In the parallel circuit, as well as in the series circuit, $LC\omega^2 = 1.0$ at resonance ($\omega = 2\pi f$), *when the inductive and capacitive branches contain only pure inductance and pure capacitance.*

Also, under these conditions, $f = 1/2\pi\sqrt{LC}$.

When there is resistance in either the inductive or the condensive branch, this relationship does not hold (see Par. 44, p. 68).

24. Polygon of Voltages; Three Voltages.—The inductances and condensers so far considered have been assumed as perfect, that is, as having no losses and with their currents exactly 90° from their respective voltages. In practice, this is impossible. The wire of which the inductance is made has a certain resistance, and if an iron core is used, the core losses are equivalent to an added resistance, since they involve a power loss. Condensers

[1] This is frequently called the *anti*-resonant condition, to distinguish it from the resonant condition, which, in a generalized network, occurs when the current is a maximum.

are made having very small losses and phase angles very nearly equal to 90°, but even such condensers are not ideal.

When an inductance coil is being considered, its resistance must be added to the other resistances in the circuit, in order to find the total circuit resistance.

Figure 43 (*a*) shows a series circuit connected across an alternating voltage E, having a frequency f. This circuit contains a resistance R and an impedance coil Z', having a resistance R' and an inductance L. The reactance X' of the impedance coil is equal to $2\pi fL$. Figure 43 (*b*) shows the vector diagram for this circuit. The voltage IR is in phase with the current I. The voltage E'_Z across the impedance coil is not 90° ahead of the

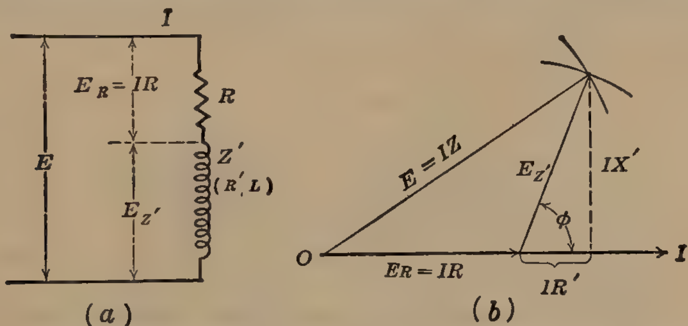


FIG. 43.—Circuit having resistance and impedance in series, and vector diagram.

current but leads the current by an angle ϕ which is less than 90°, due to the resistance of the impedance coil. The circuit voltage E is the vector sum of IR and E_Z'. The impedance voltage E_Z' consists of two components, IR' in phase with the current and IX' in quadrature with the current. The projection of E_Z', the voltage across the impedance, therefore, on the current I is the voltage drop due to the resistance of the impedance. Divide this projected voltage by the current, and the resistance of the impedance coil is obtained.

Figure 44 (*a*) shows a series circuit, containing a non-inductive resistance R and an impedance coil Z' in series across the voltage E. Let it be required to construct the vector diagram of this circuit. A voltmeter across the resistance R measures the voltage E_R; when across the impedance, it measures the voltage $E_{Z'}$; and when across the line, it measures the voltage E.

To construct the vector diagram of this circuit, the current vector I is laid off horizontally, as shown in Figs. 43 (*b*) and 44 (*b*).

The voltage $E_R = IR$ is laid off to scale in phase with the current I; from the outer end of E_R an arc is swung having $E_{Z'}$ for its radius. Then from O, the origin, another arc is swung having E for its radius. Lines drawn from the end of E_R and from O to the intersection of the arcs complete the vector diagram. By trigonometry, the angle θ, the circuit power-factor angle, and ϕ, the impedance-coil power-factor angle, can both be found. Knowing these, it is a simple matter to determine the power factor and the power of the circuit.

Example.—A resistance and an impedance coil are connected in series across a 60-cycle alternating-current circuit (Fig. 44 (*a*)), and the current is 4.0 amp. The voltage across the resistance is found to be 60 volts;

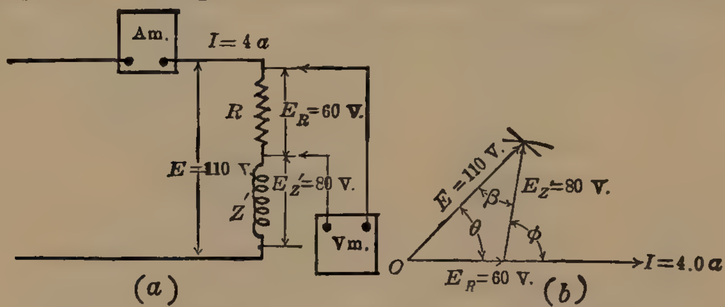


FIG. 44.—Triangle of voltages for circuit having resistance and inductive impedance in series.

that across the impedance coil, 80 volts; and the line voltage, 110 volts. Find: (*a*) the value of the resistance; (*b*) the circuit power-factor angle θ and the power factor; (*c*) the impedance-coil power-factor angle ϕ and the corresponding power factor; (*d*) the circuit power; (*e*) the impedance-coil power; (*f*) the impedance-coil resistance; (*g*) the impedance-coil reactance.

(*a*) $R = E_R/I = {}^{60}\!/_4 = 15$ ohms. *Ans.*

(*b*) Applying the law of cosines (p. 535) to Fig. 44 (*b*),

$\overline{80}^2 = \overline{110}^2 + \overline{60}^2 - 2 \times 110 \times 60 \cos \theta.$

$$\cos \theta = \frac{9{,}300}{13{,}200} = 0.704. \quad \textit{Ans.}$$

$$\theta = 45.2°. \quad \textit{Ans.}$$

(*c*) By the law of sines (p. 535),

$$\frac{\sin \beta}{\sin \theta} = \frac{60}{80}$$

$$\sin \beta = \sin 45.2° \left({}^{60}\!/_{80}\right) = 0.710 \left({}^{60}\!/_{80}\right) = 0.533$$

$$\beta = 32.2°$$

Since the exterior angle ϕ is equal to the sum of the interior angles θ and β, that is,

$$\phi = \theta + \beta$$

$$\phi = 45.2° + 32.2° = 77.4°. \quad \textit{Ans.}$$

$$\cos \phi = \cos 77.4° = 0.218. \quad \textit{Ans.}$$

(*d*) The circuit power

$$P = 110 \times 4 \times \cos \theta = 440 \times 0.704 = 310 \text{ watts.} \quad \textit{Ans.}$$

(*e*) The impedance-coil power

$$P' = E_{Z'} \times I \times \cos \phi$$
$$= 80 \times 4 \times 0.218 = 69.8 \text{ watts.} \quad \textit{Ans.}$$

(*f*) $I^2R' = 69.8$

$$R' = \frac{69.8}{16} = 4.36 \text{ ohms.} \quad \textit{Ans.}$$

(*g*) The reactance voltage in the impedance coil

$$E_{X'} = 80 \sin \phi = 80 \times 0.976 = 78.1 \text{ volts.}$$
$$\frac{78.1}{4} = 19.5\text{-ohms reactance.} \quad \textit{Ans.}$$

25. Polygon of Voltages, Four Voltages.—If three sides of a triangle are fixed, the triangle itself is fixed as regards both its

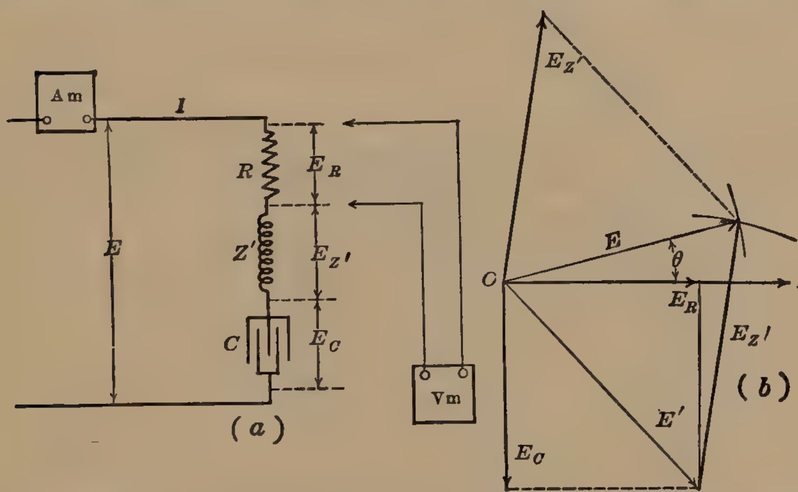


FIG. 45.—Polygon of voltages for series circuit containing resistance, inductive impedance, and capacitance.

area and its angles. If the four sides of a polygon are given, however, the polygon itself is not determined. In order to determine the polygon definitely, the angle included between two of its sides must be known. This is the condition which exists when there are resistance, inductance, and capacitance in a series circuit. These three voltages and the line voltage give four voltages which in themselves make an indeterminate polygon. If the angle between two of these voltages is known, the polygon and its angles are completely determined.

This is illustrated in Fig. 45, in which resistance, impedance, and capacitance are all connected in series, and the current I

flows in the circuit. Assume that the condenser power-factor angle is 90°, which is practically the case in most commercial condensers. This constitutes the angle which determines the polygon of voltages. Along I lay off E_R to scale (Fig. 45 (b)). Ninety degrees behind I lay off E_C to scale. Add these two vectorially, giving $\dot{E}' = \dot{E}_R + \dot{E}_C$. From the end of E' swing upward the vector $E_{Z'}$, and from O swing the line voltage E. Complete the polygon where these two arcs intersect. Then from O draw $E_{Z'}$ parallel to the $E_{Z'}$ swung from the end of E'.

It will now be seen that

$$\dot{E}_{Z'} + (\dot{E}_R + \dot{E}_C) = \dot{E}.$$

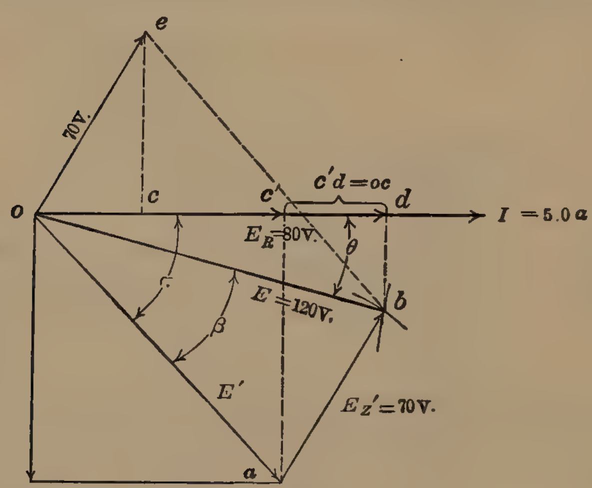


FIG. 46.—Polygon of voltages for alternating-current series circuit.

That is, the vector sum of the three voltages is equal to the line voltage, which condition exists in the circuit.

Example.—A resistance, an impedance coil, and a condenser are connected in series. The voltage across the resistance is 80 volts; that across the impedance coil is 70 volts; that across the condenser is 90 volts; and the line voltage is 120 volts. A current of 5 amp. flows in the circuit, and the condenser current leads its voltage by 90°. Determine: (a) the circuit power-factor angle θ; (b) the resistance and reactance of the impedance coil.

The voltage polygon is shown in Fig. 46.

(a) $E' = \sqrt{90^2 + 80^2} = \sqrt{14{,}500} = 120.5$ volts

$$\tan \alpha = {}^{90}\!/_{80} = 1.125 \quad \alpha = 48.4°.$$

Applying the law of cosines (p. 535) to triangle *oab*, $\overline{70}^2 = \overline{120.5}^2 + \overline{120}^2 - 2 \times 120.5 \times 120 \cos \beta$.

$$\cos \beta = \frac{24{,}000}{28{,}900} = 0.8305$$
$$\beta = 33.8°$$
$$\theta = \alpha - \beta = 48.4° - 33.8° = 14.6°. \quad \textit{Ans.}$$
$$\cos 14.6° = 0.968 \text{ (current leads).}$$

(*b*) The distance $od = 120 \cos \theta = 120 \times 0.968 = 116.3$ volts. $oc = c'd$, since *oc* is the projection of *oe* on *od*, and *c'd* is the projection of *ab* on *od*, and *ab* is equal and parallel to *oe*.

Therefore,

$$oc = od - 80 = 116.3 - 80 = 36.3 \text{ volts.}$$
$$\frac{36.3}{5} = 7.26\text{-ohms resistance in impedance coil.} \quad \textit{Ans.}$$
$$\overline{ce}^2 = \overline{70}^2 - \overline{36.3}^2$$
$$ce = \sqrt{3{,}590} = 59.9 \text{ volts.}$$
$$\frac{59.9}{5} = 11.98\text{-ohms reactance in impedance coil.} \quad \textit{Ans.}$$

26. Polygon of Currents.—Obviously, if the resistances, impedances, etc., are in parallel, the voltage is the same for each

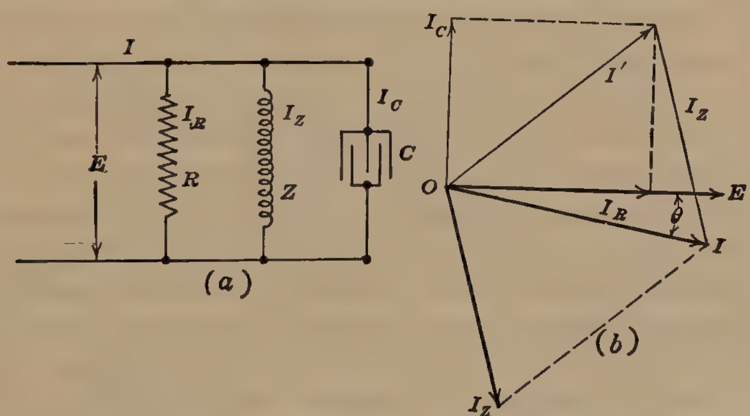


Fig. 47.—Parallel circuit, consisting of resistance, inductive impedance, and capacitance, all in parallel, with vector diagram.

branch of the circuit, but the respective currents may differ. The polygon is composed of currents, therefore, rather than of voltages. Figure 47 (*a*) shows a circuit consisting of resistance, inductive impedance, and capacitance all in parallel. Assume that the condenser current is in quadrature with its voltage. Figure 47 (*b*) then represents the polygon of currents. The voltage *E*, being common, is laid off horizontal. The current I_R is laid off in phase with *E*, and the current I_C leads *E* by 90°.

These two are combined to obtain I'. From the outer end of I', I_Z is swung to meet I, which is swung from O. This completes the polygon, which is similar to those shown in Figs. 45 and 46, except that the vectors are currents instead of voltages. With only three currents, the diagrams are analogous to those of Figs. 43 and 44, except that currents are substituted for voltages.

27. Effective Resistance.—A coil of copper wire with an air core is connected across a direct-current source, and its resistance is measured. The voltage across the coil is 22 volts when the current is 4.6 amp. This makes its resistance 4.78 ohms. This same coil is then connected across 110 volts, 60 cycles. It then takes 1.2 amp., and a wattmeter in circuit shows that the coil is taking 7.3 watts. If the direct-current resistance were used, the power should be only $(1.2)^2\,4.78 = 6.89$ watts. The greater loss with alternating current is due to the fact that the alternating current does not distribute itself uniformly over the cross-section of the wire (skin effect) and also that the resulting flux induces eddy currents in the conductor.

If an iron core be inserted in this coil, the voltage and frequency being maintained constant, the current drops to 0.20 amp., and the power becomes 0.26 watt. The power calculated on the basis of the direct-current resistance would be only $(0.20)^2$ $4.78 = 0.191$ watt. The greater loss is accounted for not only by the effects just mentioned but also by the eddy current and hysteresis losses in the iron caused by the alternating flux. It is thus seen that, with a given current, the losses with alternating current may be greater than with direct current. Since this is so, the resistance of the circuit with alternating current must be greater than it is with direct current. This resistance to alternating current is called *effective* resistance.

If R_e be the effective resistance of a circuit, the power loss P for a current I is

$$P = I^2R_e$$

and

$$R_e = \frac{P}{I^2}. \tag{30}$$

For example, in the illustration just given the *effective* resistance of the coil *without* iron is $7.3/(1.2)^2 = 5.07$ ohms, which is 6 per cent. greater than the direct-current resistance. *With*

iron, the effective resistance is $0.26/(0.20)^2 = 6.5$ ohms, or 36 per cent. greater than the direct-current resistance.

28. Energy and Quadrature Currents.—Figure 48 shows the vector diagram for a load connected across alternating-current mains. This load is typical of most commercial loads, except incandescent lamps. It takes a current I, lagging the voltage E by θ degrees. The current I may be resolved into two components, i_1 in phase with the voltage and i_2 in quadrature with the voltage. Obviously, I is the vector sum of i_1 and i_2.

The power taken by the load is

$$P = EI \cos \theta$$

but

$$I \cos \theta = i_1$$

Therefore,

$$P = Ei_1. \tag{31}$$

i_1 is called the *energy component* of the current, because this component multiplied by the voltage gives the circuit power.

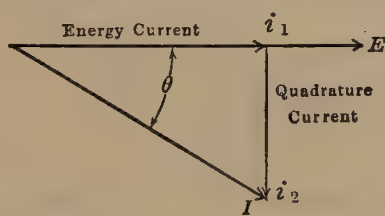


Fig. 48.—Energy and quadrature currents.

The component i_2 is in quadrature with the voltage and can contribute no power, therefore. i_2 is called the *quadrature* or wattless component of the current.

If this load is being supplied over a transmission line, the line loss is proportional to

$$I^2R = (i_1^2 + i_2^2)R = i_1^2R + i_2^2R$$

where R is the transmission-line resistance.

It will be observed that the quadrature component produces line loss yet contributes no power to the load. It is ordinarily desirable, therefore, to make i_2 as small as possible or, in other words, have the system operate at high power factor. For example, when $\theta = 45°$, P.F. $= 0.707$, the energy and quadrature currents are equal. The quadrature current contributes as much

to the line loss, therefore, as the energy current does, but it contributes nothing to the power supplied to the load.

Example.—A transmission line (Fig. 49 (*a*)), supplies 50 kw. at 220 volts, single-phase, to a load having a power factor of 0.60, lagging current. Each wire has a resistance of 0.02 ohm. Find: (*a*) the energy current; (*b*) the quadrature current; (*c*) the line loss due to the energy current; (*d*) the line loss due to the quadrature current; (*e*) the total line loss; (*f*) the line loss which would exist if the load power factor were unity.

The total current

$$I = \frac{50{,}000}{200 \times 0.6} = 379 \text{ amp.}$$

(*a*) $i_1 = 379 \cos \theta = 379 \times 0.6 = 227$ amp. *Ans.*

(*b*) $i_2 = 379 \sin \theta = 379 \times 0.8 = 303$ amp. *Ans.*

(*c*) $i_1^2 \times 0.04 = 2{,}070$ watts. *Ans.*

(*d*) $i_2^2 \times 0.04 = 3{,}680$ watts. *Ans.*

(*e*) $I^2 \times 0.04 = 5{,}750$ watts. *Ans.*

(*f*) If the power factor of the load were unity, the quadrature current i_2 would be zero, and the line current $I = i_1$.

Therefore, the loss would be

$$I^2 \times 0.04 = 2{,}070 \text{ watts. } \textit{Ans.}$$

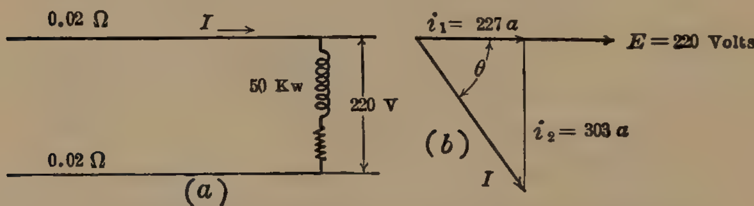


FIG. 49.—Energy and quadrature currents in transmission line.

In this particular case, the line loss due to the quadrature current is considerably in excess of that due to the energy current, yet the quadrature current contributes no power to the load.

From the foregoing, it must not be inferred that the energy and quadrature currents exist separately. Only one current actually flows, but this current is resolved into two components, each of which produces different effects in the circuit. The effect of each component can then be studied, resulting in a much better understanding of the circuit relations than if an attempt were made to consider the current as a whole.

29. Maximum Power in a Series Circuit.—If a series circuit across constant voltage have a variable resistance R and a fixed reactance X, the power taken by the circuit will be found to vary

as R is varied. When $R = 0$, the power is zero; when $R = \infty$, the power is zero. With a finite value of voltage, the power between these two values of R is not zero but must be finite. If the power be plotted as a function of R, it is zero when $R = 0$, increases to a maximum when $R = X$, and decreases to zero when $R = \infty$. The fact that the maximum power occurs when $R = X$ is shown by the following example:

Example.—A circuit having a fixed reactance of 12 ohms (either inductive or capacitive) in series with a variable resistance R is connected across 100-volt, 60-cycle mains. For what value of R is the power P a maximum?

The current

$$I = \frac{100}{\sqrt{R^2 + (12)^2}}$$

$$P = I^2R = \frac{(100)^2R}{R^2 + 144}$$

$$PR^2 - 10{,}000R = -144P. \qquad \text{(I)}$$

Dividing by P and completing the square,

$$R^2 - 10{,}000\frac{R}{P} + \left(\frac{5{,}000}{P}\right)^2 = \left(\frac{5{,}000}{P}\right)^2 - 144$$

$$R - \frac{5{,}000}{P} = \pm\sqrt{\left(\frac{5{,}000}{P}\right)^2 - 144}$$

$$R = \frac{5{,}000}{P} \pm \sqrt{\left(\frac{5{,}000}{P}\right)^2 - 144}. \qquad \text{(II)}$$

It is obvious that when $(5{,}000/P)^2$ is less than 144, the quantity under the radical sign becomes negative, and, the square root of a negative quantity being imaginary, the resistance becomes imaginary, which is impossible. Hence, the maximum value of the power occurs when $(5{,}000/P)^2 = 144$.

$$P = \frac{5{,}000}{12} = 417.$$

Substituting this value of power in (II),

$$R = \frac{5{,}000}{417} = 12 \text{ ohms.} \quad \textit{Ans.}$$

This value of resistance is equal to the reactance X.

If the resistance R is in series and the impressed voltage is constant, with an impedance Z' whose resistance is R' and reactance is X', it can be shown that R takes the maximum power when $R = Z'$. The demonstration is somewhat more complicated than the foregoing.

CHAPTER III

COMPLEX QUANTITIES

From the preceding two chapters, it is apparent that alternating-current problems cannot ordinarily be solved by the use of simple algebra. This is due to the fact that alternating currents and voltages are vector rather than scalar quantities, and, hence, simple algebraic operations do not give correct results. By means of *complex algebra*, however, it is possible to solve alternating-current problems by algebraic operations only. That is, it is not necessary to employ the usual trigonometric combinations of vector quantities. Furthermore, without complex algebra, many problems would be extremely difficult, if not impossible to solve.

30. Rectangular Notation of Complex Quantities.—In Fig. 50 are shown the usual coordinate axes XX and YY. Consider a vector $+A$ lying along the X-axis in the positive direction. If this vector is operated upon by the factor (-1), it becomes $-A$ and its position is now along the X-axis to the left of the origin (Fig. 50). That is, by operating on $+A$ by the factor (-1), it is caused to rotate through an angle of 180°. Since (-1) is also equal to $(\sqrt{-1}\ \sqrt{-1})$, this same result may be obtained by operating on $+A$ with the operator $(\sqrt{-1}\ \sqrt{-1})$. That is, by operating on $+A$ twice with the operator $\sqrt{-1}$, the vector $+A$ is caused to rotate through 180°. Hence, if the vector $+A$ is operated on but once by the factor $\sqrt{-1}$, it is caused to rotate through only 90°. It has been agreed, moreover, that $\sqrt{-1}$ causes rotation in a positive or counterclockwise direction. That is, the vector $+A$ when operated on once by $\sqrt{-1}$ takes a position along the Y-axis in a positive direction (Fig. 51).

In algebra, it is well known that the square root of a negative quantity cannot denote a physical entity. That is, no real quantity squared, whether positive or negative, can be equal

to a negative quantity. Hence, because intrinsically $\sqrt{-1}$ cannot represent a physical quantity, it is called an *imaginary quantity.* Since all vectors which lie along the Y-axis are designated by this operator $\sqrt{-1}$ (or $-\sqrt{-1}$), the Y-axis is called the *axis of imaginaries.* The X-axis is called the *axis of reals.* Quantities lying along the X-axis are called *real* quantities. Quantities lying along the Y-axis are called *imaginary* quantities. The terms *axis of imaginaries* and *imaginary quantities* are somewhat unfortunate, for they imply non-existent quantities. In complex algebra, quantities along the axis of imaginaries are just as much physical entities as quantities along the axis of reals.

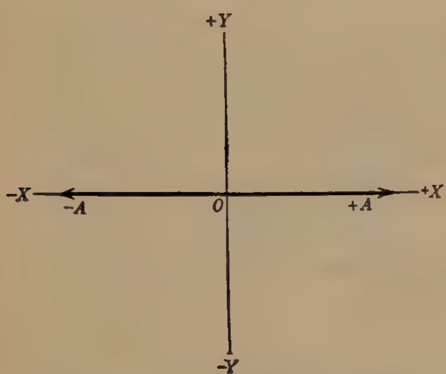


FIG. 50.—Operating on vector with (-1).

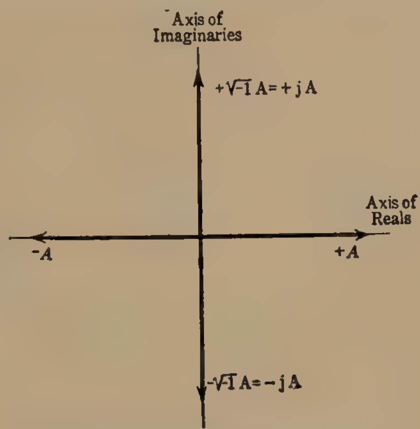


FIG. 51.—Operating on vector with $\sqrt{-1}$.

In electrical engineering, the operator $+\sqrt{-1}$ is represented by $+j$.[1] The factor $\sqrt{-1}$ (or j) is merely an operator which causes the vector on which it operates to be rotated through an angle of 90° in a counterclockwise direction.

The factor $[(-\sqrt{-1})\,(-\sqrt{-1})]$ also rotates the vector $+A$ through an angle of 180° in the same manner as does the factor $(\sqrt{-1}\sqrt{-1})$. If $+\sqrt{-1}$ causes positive rotation through 90°, $-\sqrt{-1}$ causes negative or clockwise rotation through 90°. A vector $+A$ operated on by $(-\sqrt{-1})$ or by $-j$, therefore, lies along the Y-axis in the downward or negative direction, as shown in Fig. 51.

[1] In pure mathematics, $\sqrt{-1}$ is represented by i. The fact that in electrical engineering i is the symbol for current has caused the adoption of the symbol j for $\sqrt{-1}$.

Since complex algebra concerns points in a plane rather than simple scalar quantities, the plane represented by the coordinate axes (Figs. 50 and 51) is called the *complex plane.*

Rectangular Vectors.—As is well known, a vector can be resolved into two or more components, and each component may be treated by itself. It is usually more convenient, if the components are at right angles to each other, to take the direction of one along the X-axis and the other along the Y-axis. In complex algebra, each vector is so resolved into components at right angles to each other. The component along the Y-axis is designated by $\pm j$. For example (in Fig. 52), the vector A lying in

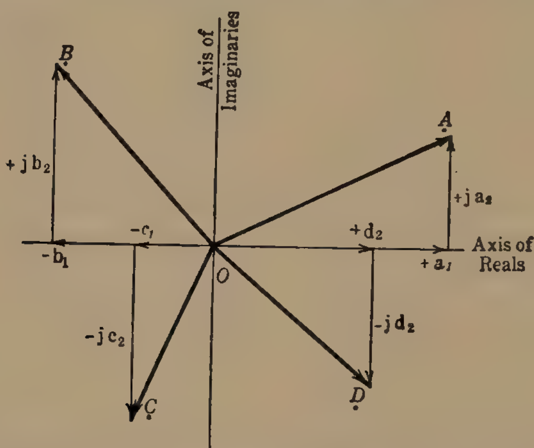


FIG. 52.—Rectangular, complex vectors.

the first quadrant is resolved into two components, a_1 along the axis of reals and $+ja_2$ along the axis of imaginaries. That is, $\underset{\cdot}{A} = a_1 + ja_2$. In a similar manner, vector B in the second quadrant $\underset{\cdot}{B} = -b_1 + jb_2$; vector C in the third quadrant, $\underset{\cdot}{C} = -c_1 - jc_2$; vector D in the fourth gradient, $\underset{\cdot}{D} = d_1 - jd_2$. Vectors defined by their components along the axis of reals and axis of imaginaries will be termed *rectangular vectors.*

In the algebra of complex quantities, ordinary algebraic operations are followed. The operator j is treated like any ordinary coefficient and in algebraic operations is given its algebraic value of $\sqrt{-1}$ (for example, $j^2 = -1$).

31. Addition and Subtraction of Rectangular Vectors.—Let it be required to add the vectors A and B (Fig. 53), where $\underset{\cdot}{A} = a_1 + ja_2$ and $\underset{\cdot}{B} = b_1 + jb_2$. The addition involves merely the

adding together of the real and of the imaginary components of these two vectors. For example,

$$\underset{\cdot}{C} = \underset{\cdot}{A} + \underset{\cdot}{B} = (a_1 + b_1) + j(a_2 + b_2) \tag{32}$$

If any of the quantities a_1, b_1, a_2, b_2 are negative, they are given the negative sign.

Example.—Add $8 - j10$ to $6 + j4$

$$(8 - j10) + (6 + j4) = 14 - j6. \quad \textit{Ans.}$$

This vector has a magnitude of $\sqrt{(14)^2 + (6)^2} = 15.3$ and lies in the fourth quadrant. It makes an angle $\tan^{-1}\dfrac{-6}{14} = -23.2°$ with the axis of reals.

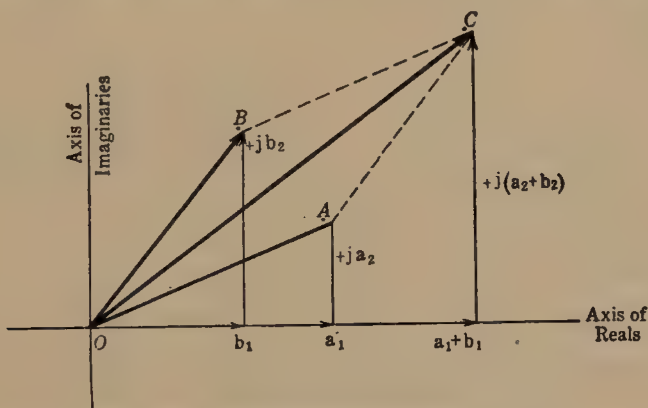


FIG. 53.—Addition of rectangular vectors.

Subtraction is accomplished in the same manner as addition.

Example.—Subtract $12 - j10$ from $7 + j4$.

$$(7 + j4) - (12 - j10) = 7 + j4 - 12 + j10 = -5 + j14. \quad \textit{Ans.}$$

This vector lies in the second quadrant and makes an angle with the axis of reals of $\tan^{-1}\dfrac{14}{-5} = \tan^{-1}(-2.80) = 109.7°$ with the axis of reals.

32. Multiplication of Rectangular Vectors.—Let it be required to multiply vector A by vector B. Ordinary algebraic procedure is followed. That is,

$$\begin{aligned} \underset{\cdot}{A}\underset{\cdot}{B} &= (a_1 + ja_2)(b_1 + jb_2) = a_1b_1 + ja_1b_2 + ja_2b_1 + j^2a_2b_2 \\ &= (a_1b_1 - a_2b_2) + j(a_1b_2 + a_2b_1). \end{aligned} \tag{33}$$

If

$$\begin{aligned} \underset{\cdot}{B} &= b_1 - jb_2 \\ \underset{\cdot}{A}\underset{\cdot}{B} &= (a_1b_1 + a_2b_2) - j(a_1b_2 - a_2b_1) \end{aligned} \tag{34}$$

(see Par. 37 for the physical significance of a vector product).

Example.—Determine the product of $8 - j10$ and $6 + j4$

$$(8 - j10)(6 + j4) = 48 + j32 - j60 - j^2 40 = 88 - j28. \quad \textit{Ans.}$$

This vector has a magnitude of $\sqrt{(88)^2 + (28)^2} = 92.3$ and lies in the fourth quadrant. It makes an angle

$$\tan^{-1} \frac{-28}{88} = -17.6° \text{ with the } X\text{-axis.}$$

33. Reciprocals of Rectangular Vectors.—Let it be required to determine

$$\frac{1}{\dot{A}} = \frac{1}{a_1 + ja_2}. \tag{I}$$

(I) is rationalized by multiplying numerator and denominator by $a_1 - ja_2$.

That is,

$$\frac{1}{\dot{A}} = \frac{1}{a_1 + ja_2} \cdot \frac{a_1 - ja_2}{a_1 - ja_2} = \frac{a_1 - ja_2}{a_1^2 - ja_1a_2 + ja_1a_2 - j^2a_2^2}$$

$$= \frac{a_1}{a_1^2 + a_2^2} - j\frac{a_2}{a_1^2 + a_2^2} \tag{35}$$

(see Par. 42).

Example.—Find $\dfrac{1}{8 - j10}$

$$\frac{1}{8 - j10} \cdot \frac{8 + j10}{8 + j10} = \frac{8}{64 + 100} + j\frac{10}{64 + 100} = 0.0488 + j0.0610. \quad \textit{Ans.}$$

34. Division of Rectangular Vectors.—Let it be required to determine

$$\frac{\dot{A}}{\dot{B}} = \frac{a_1 + ja_2}{b_1 + jb_2}. \tag{I}$$

The denominator of (I) is rationalized, as was done in Par. (33).

$$\frac{\dot{A}}{B} = \frac{a_1 + ja_2}{b_1 + jb_2} \cdot \frac{b_1 - jb_2}{b_1 - jb_2} = \frac{a_1b_1 - ja_1b_2 + ja_2b_1 + a_2b_2}{b_1^2 + b_2^2}$$

$$= \frac{a_1b_1 + a_2b_2}{b_1^2 + b_2^2} - j\frac{a_1b_2 - a_2b_1}{b_1^2 + b_2^2}. \tag{36}$$

Example.—Divide the quantity $8 - j10$ by $6 + j4$.

$$\frac{8 - j10}{6 + j4} \cdot \frac{6 - j4}{6 - j4} = \frac{48 - j32 - j60 - 40}{36 + 16} = \frac{8 - j92}{52} = 0.154 - j1.77. \quad \textit{Ans.}$$

This is a vector whose magnitude is $\sqrt{(0.154)^2 + (1.77)^2} = 1.78$ and lies in the fourth quadrant.

35. Polar Notation.—A vector in the complex plane may also be defined by its magnitude and its direction angle with respect

to the X-axis. For example, the vector A (Fig. 54) is defined as $A\underline{/\alpha}$; vector B is defined as $B\underline{\backslash\beta}$; vector C is defined as $C\overline{\backslash\gamma}$. Vector C may also be defined as $C\underline{/-\gamma}$. It follows that $\overline{\backslash\gamma} = \underline{/-\gamma}$, etc.

Vectors defined by the foregoing notation will be termed *polar vectors*. The magnitudes such as A or B are called the *modulus* or absolute value of the vector; the quantities $\underline{/\alpha}$ or $\underline{/\beta}$ are called the *argument* of the vector.

In the foregoing notation, the modulus or magnitude A and the argument or direction angle $\underline{/\alpha}$ are *not a product* and cannot be treated as such.

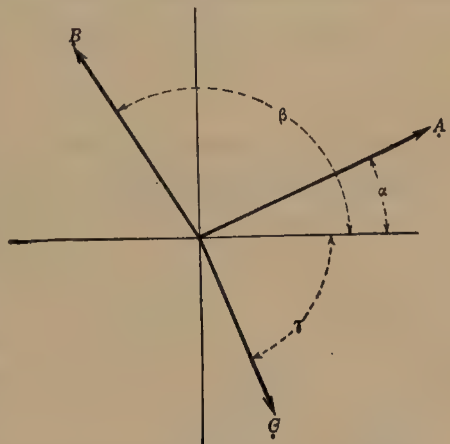


FIG. 54.—Polar notation of vectors.

36. Addition of Polar Vectors.—Polar vectors cannot be added or subtracted without first converting them into rectangular vectors. For example, in Fig. 54, let it be required to add vectors A and B. Referring to Fig. 55,

$$\underset{\cdot}{A} = a_1 + ja_2 = A(\cos\alpha + j\sin\alpha) \tag{37}$$

$$\underset{\cdot}{B} = -b_1 + jb_2 = B(\cos\beta + j\sin\beta) \tag{38}$$

$$\underset{\cdot}{A} + \underset{\cdot}{B} = (A\cos\alpha + B\cos\beta) + j(A\sin\alpha + B\sin\beta). \tag{39}$$

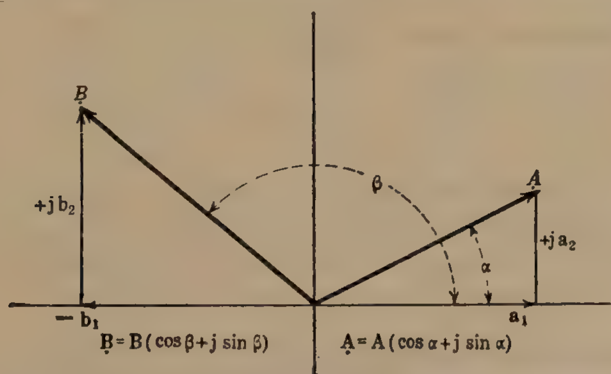


FIG. 55.—Polar vectors expressed as rectangular vectors.

Example.—Add $12\underline{/38^\circ}$ and $8\overline{\backslash 46^\circ}$

$$12\underline{/38^\circ} = 12(\cos 38^\circ + j\sin 38^\circ) = 9.46 + j7.39$$

$$8\overline{\backslash 46^\circ} = 8(\cos 46^\circ - j\sin 46^\circ) = 5.56 - j5.75$$

$$12\underline{/38^\circ} + 8\overline{\backslash 46^\circ} = 15.02 + j1.64 = 15.1\underline{/6.2^\circ}. \quad \textit{Ans.}$$

where

$$\tan 6.2^\circ = \frac{1.64}{15.02} = 0.109$$

37. Multiplication of Polar Vectors.—The product of two polar vectors is found by taking the product of their magnitudes and the sum of their angles. Thus, in Fig. 56,

$$\dot{C} = \dot{A}\dot{B} = AB\underline{/\alpha + \beta}. \tag{40}$$

Example.—Determine the product of $16\underline{/32^\circ}$ and $20\overline{\backslash 72^\circ}$.

$16\underline{/32^\circ} \cdot 20\overline{\backslash 72^\circ} = 320\underline{/32^\circ - 72^\circ} = 320\underline{/-40^\circ} = 320\overline{\backslash 40^\circ}$. *Ans.*

This vector obviously lies in the fourth quadrant.

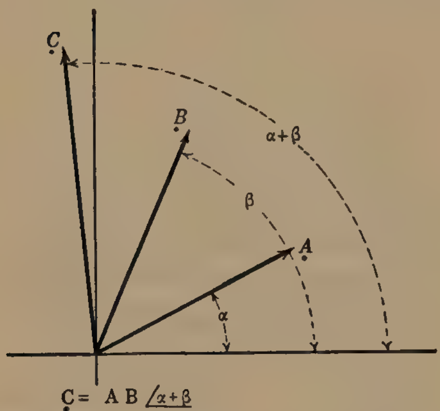


FIG. 56.—Multiplication of polar vectors.

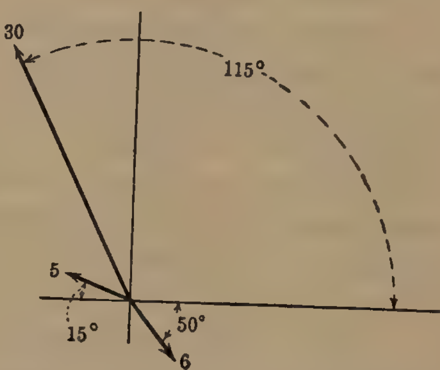


FIG. 57.—Division of polar vectors.

38. Reciprocals of Polar Vectors.—The reciprocal of $A\underline{/\alpha}$

$$= \frac{1}{A\underline{/\alpha}} = \frac{1}{A}\underline{/-\alpha} = \frac{1}{A}\overline{\backslash \alpha}. \tag{41}$$

Example.—Determine the reciprocal of $25\underline{/32^\circ} = 1/25\underline{/32^\circ} = 0.04\overline{\backslash 32^\circ}$. This vector lies in the fourth quadrant. It follows that an angle may be transferred from denominator to numerator and from numerator to denominator, if its sign is changed.

39. Division of Polar Vectors.—The quotient of two polar vectors is found by taking the quotient of their magnitudes and the difference of their angles, thus:

$$\frac{A\underline{/\alpha}}{B\underline{/\beta}} = \frac{A}{B}\underline{/\alpha - \beta} \tag{42}$$

Example.—Divide $30\backslash\underline{115^\circ}$ by $6\backslash\overline{50^\circ}$ (see Fig. 57).

$$\frac{30\backslash\underline{115^\circ}}{6\backslash\overline{50^\circ}} = 5/\underline{115^\circ + 50^\circ} = 5\backslash\underline{165^\circ} \quad \textit{Ans.}$$

40. Powers and Roots of Polar Vectors.—To find the nth power of a polar vector, take the nth power of its magnitude and n times its angle. For example,

$$(A/\underline{\alpha})^n = A^n/\underline{n\alpha}. \tag{43}$$

Example.—Find $(4\backslash\overline{64^\circ})^3$.

$$(4\backslash\overline{64^\circ})^3 = 4^3\backslash\overline{3 \times 64^\circ} = 64/\overline{192^\circ}$$
$$= 64\backslash\underline{168^\circ}. \quad \textit{Ans.}$$

To find the nth root of a polar vector, take the nth root of its magnitude and $1/n$th of its angle. That is,

$$\sqrt[n]{A/\underline{\alpha}} = \sqrt[n]{A}/\underline{\alpha/n}. \tag{44}$$

Example.—Determine $\sqrt{4\backslash\overline{64^\circ}}$.

$$\sqrt{4\backslash\overline{64^\circ}} = 2\backslash\overline{32^\circ}. \quad \textit{Ans.}$$

41. Rotation of Vectors.—To rotate a polar vector $A/\underline{\alpha}$ through an angle $\pm/\underline{\beta}$, the angle $\pm\beta$ is merely added to the angle α. Thus, $A/\underline{\alpha \pm \beta}$.

To rotate a rectangular vector through a positive angle β, without changing its magnitude, it is multiplied by $(\cos\beta + j\sin\beta)$. This may be proved as follows:

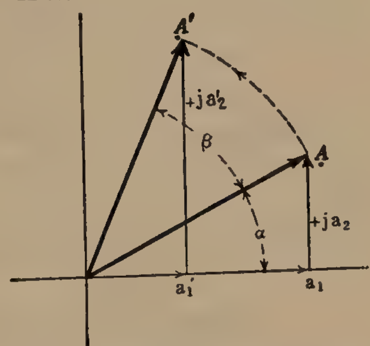


FIG. 58.—Rotation of rectangular vector through angle β.

In Fig. 58, the vector $\underset{\cdot}{A} = a_1 + ja_2$ is shown making an angle α with the axis of abscissas. The vector A' is of the same magnitude as vector A but rotated in a clockwise direction through the angle β from $\underset{\cdot}{A}$.

$$\underset{\cdot}{A}' = A'\,[\cos(\alpha + \beta) + j\sin(\alpha + \beta)]$$
$$= A'\,(\cos\alpha\cos\beta - \sin\alpha\sin\beta + j\sin\alpha\cos\beta + j\cos\alpha\sin\beta)$$

(see Eqs. 36 and 34, p. 535).

$$= A'\,[\cos\beta\,(\cos\alpha + j\sin\alpha) + j\sin\beta\,(\cos\alpha + j\sin\alpha)]$$
$$= A'\,(\cos\alpha + j\sin\alpha)(\cos\beta + j\sin\beta)$$

Since $A' = A$ numerically

$$\underset{\cdot}{A}' = \underset{\cdot}{A}\,(\cos\beta + j\sin\beta) \text{ Q.E.D.} \tag{45}$$

In a similar manner, $(\cos\beta - j\sin\beta)$ rotates any vector $\underset{\cdot}{A}$ through an angle β in a clockwise direction.

Example.—Rotate the vector $-6.0 - j8.5$ in a clockwise direction through an angle of 72°.

$$\text{Cos } 72° = 0.309 \quad \sin 72° = 0.951$$

$$(-6.0 - j8.5)(0.309 - j0.951) = -1.854 + j5.706 - j2.63 - 8.08 = -9.93 + j3.08 = 10.41\backslash\underline{162.8°}.$$

Since $-6.0 - j8.5 = 10.41/\overline{125.2°}$, it has been rotated from the third into the second quadrant through an angle of 72°, without changing its magnitude.

From the foregoing operations, it is clear that:

To add or subtract vectors, they must first be expressed in rectangular vectors, that is, vectors having their components along the axes of reals and imaginaries. This is the more convenient method of expressing vectors when addition or subtraction is to be performed.

It is possible with rectangular vectors to multiply, to take the reciprocal, to divide, to rotate, and to raise to a power. It is very much simpler to perform these operations with the vectors expressed in polar form. It is practically necessary to use the polar notation in finding roots of vector quantities.

Sometimes it is easier to multiply and divide quantities when they are already expressed in rectangular vectors than to convert them into polar quantities and back again, etc.

APPLICATIONS OF COMPLEX QUANTITIES TO ALTERNATING CURRENTS

42. Simple Series Circuits.—In Fig. 59 (a) is shown a simple series circuit consisting of a non-inductive resistance R and an inductive reactance X_L. This circuit is identical with that shown in Fig. 32 (p. 34). Let the direction of I be so chosen that it lies along the axis of reals. That is, $\dot{I} = I + jO$. Since the IR vector is in phase with I, its complex notation is $IR + jO$. The IX_L vector is at right angles to I and leads. Hence, its complex notation is $+jIX_L$. Hence, the line voltage

$$E = IR + jIX_L = I(R + jX_L). \tag{46}$$

It is seen from (46) that the impedance is expressed as a complex quantity. Although expressed as a complex quantity, impedance itself is *not* a vector quantity. It does, however, resolve the resistance and reactance voltage drops into two voltage vectors at right angles to each other. Because it is

expressed algebraically as a complex quantity and treated as a complex quantity, impedance is often erroneously considered a vector. It is actually a *complex operator*.

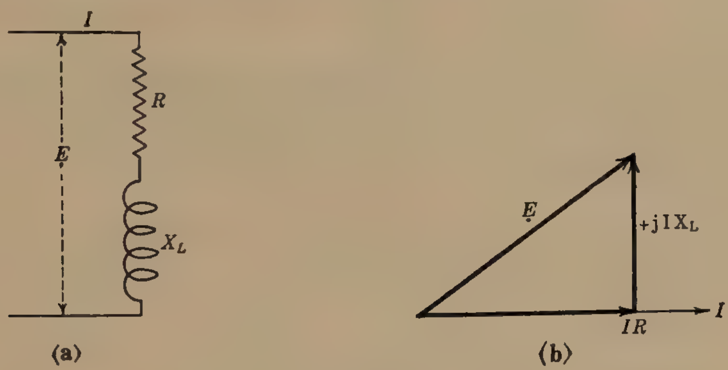


FIG. 59.—Simple series circuit and its complex vector diagram.

In the foregoing problem, the direction of the current was chosen along the axis of reals. Let the direction of the voltage now be taken along the axis of reals (Fig. 60).

The current may be found as follows:

$$I = \frac{E}{R + jX_L} = \frac{E}{R + jX_L} \cdot \frac{R - jX_L}{R - jX_L} = E\left[\frac{R}{R^2 + X_L^2} - j\frac{X_L}{R^2 + X_L^2}\right]$$
$$= E(g - jb) \qquad (47)$$

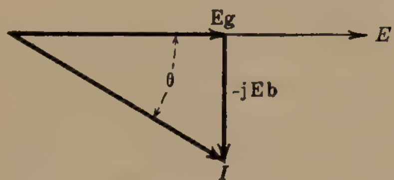


FIG. 60.—Complex diagram with voltage along axis of reals.

where
$$g = \frac{R}{R^2 + X_L^2} \text{ mhos} \qquad (48)$$

and is called the *conductance* of the circuit.

$$b = \frac{X_L}{R^2 + X_L^2} \text{ mhos} \qquad (49)$$

and is called the *susceptance* of the circuit. g and b are both expressed in reciprocal ohms or mhos.

$$\tan\theta = \frac{b}{g}.$$

The current I is shown in Fig. 60 lagging the voltage by the angle θ.

If the reactance is capacitive, it is denoted by $-jX_C$. The corresponding susceptance obviously will be $+jb$.

It is to be noted that inductive reactance is denoted by $+jX_L$ and susceptance by $-jb$. Capacitive reactance is denoted by $-jX_C$ and susceptance by $+jb$.

Impedance also may be expressed as a polar vector, a positive angle ($\angle$) being used for inductive reactance, and a negative angle ($\diagdown$) being used for capacitive reactance.

Example.—A capacitance of 20 μf. and a resistance of 100 ohms are connected in series across 120-volt, 60-cycle mains. Determine the current, choosing the position of the voltage vector so that its direction is along the axis of reals (see example, p. 37).

$$X_C = \frac{1}{20 \times 377 \times 10^{-6}} = 132.6\Omega$$

$$I = \frac{120}{100 - j132.6} = \frac{120}{100 - j132.6} \cdot \frac{100 + j132.6}{100 + j132.6}$$

$$= \frac{12{,}000}{10{,}000 + 17{,}600} + j\frac{15{,}910}{10{,}000 + 17{,}600}$$

$$= \frac{12{,}000}{27{,}600} + j\frac{15{,}910}{27{,}600}$$

$$= 0.435 + j0.577.$$

The absolute value of I,

$$|I| = \sqrt{(0.435)^2 + (0.577)^2} = \sqrt{0.523} = 0.723 \text{ amp.} \quad \textit{Ans.}$$

The phase angle, obviously, is $\tan^{-1} \frac{0.577}{0.435} = \tan^{-1} 1.326 = 53°$. *Ans.*

The impedance may be expressed in the polar form as follows:

$$100 - j132.6 = \sqrt{27{,}600}\,\diagdown 53° = 166\,\diagdown 53°$$

$$I = \frac{120}{166\,\diagdown 53°} = 0.723\,\underline{/53°} \text{ amp.} \quad \textit{Ans. (check).}$$

43. Power Determination.—If the voltage and current of a circuit are expressed as rectangular vectors, it is a simple matter to determine the power. For example, let the voltage $\dot{E} = e_1 + je_2$ (Fig. 61) be the voltage acting across a circuit, and let the corresponding current be $\dot{I} = i_1 - ji_2$. A study of Fig. 61 shows that the component voltage e_1 and the component current i_1 are in phase with each other. Since component voltages and currents may be treated as if they were acting alone, the power contributed by e_1 and i_1 obviously must be

$$p_1 = e_1 i_1.$$

Component e_2 is preceded by $+j$, and component i_2 is preceded by $-j$. Hence, these two components are in opposition, and their product gives negative power. That is,

$$p_2 = -e_2 i_2.$$

Components e_1 and i_2, and likewise components e_2 and i_1, contribute no power, since they are in quadrature. The total power under the foregoing conditions is, therefore,

$$p = p_1 + p_2 = e_1 i_1 - e_2 i_2 \quad (50)$$

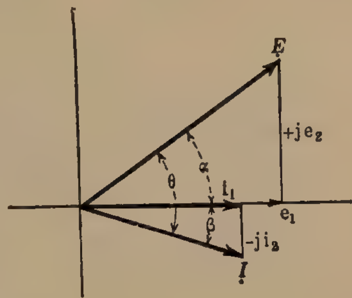


FIG. 61.—Power $= EI \cos \theta = e_1 i_1 - e_2 i_2$.

If the complex expressions for voltage and current be written as follows:

$$\dot{E} = e_1 + je_2$$
$$\dot{I} = i_1 + ji_2.$$

The total power is the sum of the product of the real quantities (e_1 and i_1) and the product of the imaginary quantities (e_2 and i_2), the signs being determined in the ordinary algebraic manner. The operator j, however, *must not be included* in the multiplication of the imaginary quantities. For example, in the expressions $\dot{E} = e_1 + je_2$ and $\dot{I} = i_1 - ji_2$, the plus and the minus signs preceding the components je_2 and ji_2 show that they are in opposition. Yet their algebraic product, including the operator, is plus. That is, $je_2\,(-ji_2) = +e_2 i_2$, which is incorrect.

A study of Fig. 61 shows that the phase angle between E and I

$$\theta = \alpha + \beta = \tan^{-1}\frac{e_2}{e_1} + \tan^{-1}\frac{i_2}{i_1}. \quad (51)$$

With polar vectors, the power is obtained in the usual manner, that is, by taking the product of the voltage and current magnitudes and multiplying this result by the cosine of the angle between them. Thus, if

$$\dot{E} = E\underline{/\alpha} \text{ and } \dot{I} = I\underline{/\beta}$$
$$P = EI \cos(\alpha - \beta).$$

Example.—When a voltage $60 + j80$ is acting on a circuit, the current is $-3 + j5$. Determine: (*a*) the complex expression for the impedance of the

circuit; (*b*) whether the impedance is capacitive or inductive in character; (*c*) the power; (*d*) the phase angle between current and voltage.

$$(a)\ \dot{Z} = \frac{60 + j80}{-3 + j5} = \frac{60 + j80}{-3 + j5} \cdot \frac{-3 - j5}{-3 - j5}$$

$$= \frac{-180 - j240 - j300 + 400}{9 + 25} = \frac{220 - j540}{34} = 6.47 - j15.88. \quad \textit{Ans.}$$

(*b*) Capacitive, since imaginary term is minus. *Ans.*

(*c*) $P = [60 \times (-3)] + [80 \times 5] = 220$ watts. *Ans.*

(*d*) $\tan \alpha = {}^{80}\!/_{60} = 1.333 \qquad \alpha = 53.1^\circ$

$\tan \beta = 5/-3 = -1.667 \qquad \beta = 121^\circ$

$\theta = 121^\circ - 53.1^\circ = 67.9^\circ$. *Ans.*

Example.—Express the voltage and current in the foregoing example as polar vectors, and repeat (*a*) and (*c*).

$$\dot{E} = \sqrt{(60)^2 + (80)^2}\ \underline{/\tan^{-1} {}^{80}\!/_{60}} = 100\underline{/53.1^\circ}$$

$$\dot{I} = \sqrt{(3)^2 + (5)^2}\ \underline{\backslash \tan^{-1} \frac{5}{-3}} = 5.83\underline{\backslash 121^\circ}$$

$$(a)\ Z = \frac{\dot{E}}{\dot{I}} = \frac{100\underline{/53.1^\circ}}{5.83\underline{\backslash 121^\circ}} = 17.15\underline{/53.1^\circ - 121^\circ} =$$

$$17.15\underline{/-67.9^\circ} \text{ ohms.} \quad \textit{Ans.}$$

(*c*) $P = 100 \times 5.83 \cos (53.1^\circ - 121^\circ) = 583 \cos (-67.9^\circ)$
$= 583 \times 0.376 = 220$ watts. *Ans.* (*check*).

44. The Parallel Circuit.—In Fig. 62 is shown a parallel circuit of two branches connected across voltage E. One branch consists of a resistance R_1 in series with an inductive reactance X_1; the second branch consists of a resistance R_2 in series with a capacitive reactance X_2. The respective currents are I_1 and I_2; the line current is I. Let the voltage E have its direction along the axis of reals. That is, $\dot{E} = E + jO$.

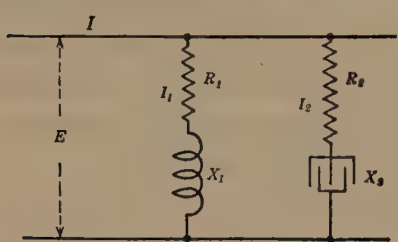


FIG. 62.—Series circuits in parallel.

$$\dot{I}_1 = \frac{E}{R_1 + jX_1} = \frac{E}{R_1 + jX_1} \cdot \frac{R_1 - jX_1}{R_1 - jX_1} =$$

$$E\left[\frac{R_1}{R_1^2 + X_1^2} - j\frac{X_1}{R_1^2 + X_1^2}\right] = E[g_1 - jb_1]$$

(see p. 65).

$$I_2 = \frac{E}{R_2 - jX_2} = \frac{E}{R_2 - jX_2} \cdot \frac{R_2 + jX_2}{R_2 + jX_2} =$$

$$E\left[\frac{R_2}{R_2^2 + X_2^2} + j\frac{X_2}{R_2^2 + X_2^2}\right] = E[g_2 + jb_2].$$

The total current

$$I = I_1 + I_2 = E[(g_1 + g_2) + j(-b_1 + b_2)] = E[G + jB].$$

G is the total *conductance* of the circuit.
B is the total *susceptance* of the circuit.
B is positive when the capacitive susceptance is greater than the inductive susceptance; it is negative when the inductive susceptance is greater than the capacitive susceptance.

Obviously, the energy current

$$I_e = EG;$$

the quadrature current

$$I_q = EB.$$

The power is equal to the product of the voltage and energy current (Par. 28, Eq. (31))

$$P = E(EG) = E^2G.$$

The power-factor angle,

$$\theta = \tan^{-1} \frac{B}{G}.$$

The absolute value of the current

$$|I| = E\sqrt{G^2 + B^2} = EY$$

where

$$Y = \sqrt{G^2 + B^2}.$$

Y is the *admittance* of the circuit and is equal to the reciprocal of the impedance.

That is,

$$Y = \frac{1}{Z}$$

Y is expressed in reciprocal ohms or mhos.

The power factor

$$\cos \theta = \frac{G}{\sqrt{G^2 + B^2}} = \frac{G}{Y}.$$

Also,

$$Z = \frac{1}{Y} = \frac{1}{G + jB} = \frac{1}{G + jB} \cdot \frac{G - jB}{G - jB} =$$

$$\frac{G}{G^2 + B^2} - j\frac{B}{G^2 + B^2} = R + jX.$$

That is,

$$R = \frac{G}{G^2 + B^2} \text{ and } X = \frac{B}{G^2 + B^2}$$

For unity power factor, the quadrature current must be zero. That is,

$$jEB = 0;$$

hence,

$$B = 0$$

or

$$b_1 + b_2 + b_3 + \cdots = 0.$$

Example.—In Fig. 62, let $R_1 = 8$ ohms, $X_1 = 12$ ohms, $R_2 = 15$ ohms, $X_2 = 20$ ohms, $E = 120$ volts, 60 cycles.

Determine: (*a*) the admittance of each branch; (*b*) the admittance of the entire circuit; (*c*) the impedance of the entire circuit; (*d*) the current in each branch; (*e*) the total current; (*f*) the power consumed in each branch; (*g*) the power factor of the entire circuit.

(*a*) $$g_1 = \frac{8}{64 + 144} = \frac{8}{208} = 0.0384 \text{ mho.}$$

$$b_1 = -\frac{12}{64 + 144} = \frac{-12}{208} = -0.0577 \text{ mho.}$$

$$Y_1 = 0.0384 - j0.0577 \text{ mho.} \quad \textit{Ans.}$$

$$g_2 = \frac{15}{225 + 400} = \frac{15}{625} = 0.0240 \text{ mho.}$$

$$b_2 = \frac{20}{225 + 400} = \frac{20}{625} = 0.0320 \text{ mho.}$$

$$Y_2 = 0.0240 + j0.0320 \text{ mho.} \quad \textit{Ans.}$$

(*b*) $$Y = G + jB = (0.0384 + 0.0240) + j(-0.0577 + j0.0320)$$
$$= 0.0624 - j0.0257 \text{ mho.} \quad \textit{Ans.}$$

$$|Y| = \sqrt{(0.0624)^2 + (0.0257)^2} = 0.0676 \text{ mho.} \quad \textit{Ans.}$$

(*c*) $$R = \frac{0.0624}{(0.0624)^2 + (0.0257)^2} = 13.68 \text{ ohms.}$$

$$X = \frac{0.0257}{(0.0624)^2 + (0.0257)^2} = 5.64 \text{ ohms.}$$

$$Z = 13.68 + j5.64 \text{ ohms.} \quad \textit{Ans.}$$

$$|Z| = \sqrt{(13.68)^2 + (5.64)^2} = 14.80 \text{ ohms.} \quad \textit{Ans.}$$

$$14.80 = 1/0.0676 \ (\textit{check}).$$

(*d*) $$I_1 = 120(0.0384 - j0.0577) = 4.61 - j6.92 \text{ amp.} \quad \textit{Ans.}$$

$$|I_1| = \sqrt{(4.61)^2 + (6.92)^2} = 8.32 \text{ amp.} \quad \textit{Ans.}$$

$$I_2 = 120(0.0240 + j0.0320) = 2.88 + j3.84 \text{ amp.} \quad \textit{Ans.}$$

$$|I_2| = \sqrt{(2.88)^2 + (3.84)^2} = 4.80 \text{ amp.} \quad \textit{Ans.}$$

(*e*) $$I = I_1 + I_2 = (4.61 + 2.88) + j(-6.92 + 3.84)$$
$$= 7.49 - j3.08 \text{ amp.} \quad \textit{Ans.}$$

$$|I| = \sqrt{(7.49)^2 + (3.08)^2} = 8.10 \text{ amp.} \quad \textit{Ans.}$$

Also,

$$I = EY = 120(0.0624 - j0.0257)$$
$$= 7.49 - j3.08 \text{ amp.} \quad (\textit{check}).$$

(*f*) $P_1 = E^2g_1 = (120)^2 0.0384 = 553$ watts. *Ans.*

$P_2 = E^2g_2 = (120)^2 0.0240 = 346$ watts. *Ans.*

Also,

$$P_1 = I_1^2R_1;\ P_2 = I_2^2R_2$$

(*g*) $\cos\theta = G/Y = 0.0624/0.0676 = 0.924$. *Ans.*

Also,

$$\cos\theta = \frac{P}{EI} = \frac{899}{120 \times 8.10} = 0.924 \text{ (check)}.$$

45. The Series-parallel Circuit.—The solution of series-parallel circuits is accomplished in the same manner as direct-current, series-parallel circuits (see Vol. I, p. 65), except that complex impedances rather than simple resistances are involved. The admittance of the parallel circuit is first found. The reciprocal of the admittance gives the impedance, just as with direct currents the reciprocal of the conductance gives the resistance. This impedance is then added to the series impedance in the ordinary manner. The procedure is best illustrated by the following example.

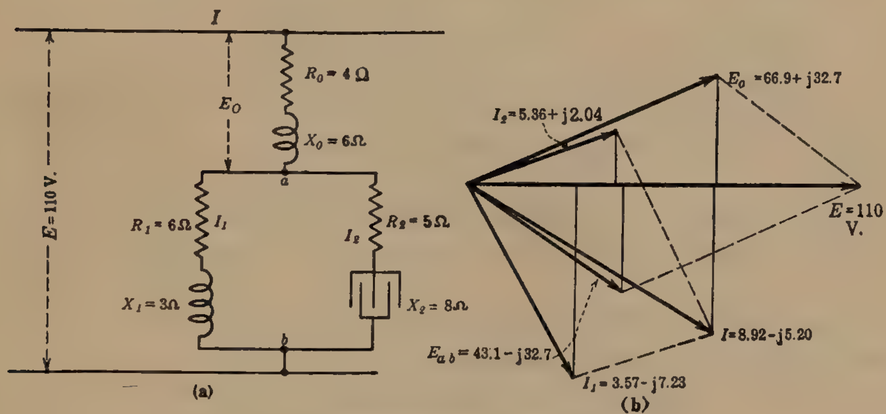


FIG. 63.—Series-parallel circuit.

Example.—Figure 63 (*a*) shows a series-parallel circuit consisting of an impedance $R_1 + jX_1 = 6 + j3$ and $R_2 + jX_2 = 5 - j8$ in parallel with each other and in series with the impedance $R_0 + jX_0 = 4 + j6$. The entire circuit is connected across 110-volt, 50-cycle mains. Determine: (*a*) the impedance of the parallel branch; (*b*) the equivalent impedance of the entire circuit; (*c*) the total current I; (*d*) the voltage E_{ab} across the parallel

branch; (e) the current in each impedance; (f) the power dissipated in each branch.

$$(a)\ g_1 - jb_1 = \frac{6}{36+9} - j\frac{3}{36+9} = 0.1333 - j0.0667.$$

$$g_2 + jb_2 = \frac{5}{25+64} + j\frac{8}{25+64} = 0.0562 + j0.0899.$$

$$Y' = G' + jB' = 0.1895 + j0.0232 \text{ mho}.$$

$$Z' = \frac{1}{Y'} = \frac{0.1895}{(0.1895)^2 + (0.0232)^2} - j\frac{0.0232}{(0.1895)^2 + (0.0232)^2} \text{ (see p. 69)}$$

$$= \frac{0.1895}{0.0364} - j\frac{0.0232}{0.0364} = 5.20 - j0.637 \text{ ohm.} \quad \textit{Ans.}$$

$$|Z'| = \sqrt{(5.20)^2 + (0.637)^2} = 5.23 \text{ ohms.} \quad \textit{Ans.}$$

$$(b)\ Z = (5.20 - j0.637) + (4 + j6) = (5.20 + 4) + j(-0.637 + 6)$$
$$= 9.20 + j5.36 \text{ ohms.} \quad \textit{Ans.}$$

$$|Z| = \sqrt{(9.20)^2 + (5.36)^2} = 10.65 \text{ ohms.} \quad \textit{Ans.}$$

(c) The line voltage vector is taken along the axis of reals.

$$I = \frac{110}{9.20 + j5.36} = \frac{110}{9.20 + j5.36} \cdot \frac{9.20 - j5.36}{9.20 - j5.36}$$

$$= \frac{110 \times 9.20}{(9.20)^2 + (5.36)^2} - j\frac{110 \times 5.36}{(9.20)^2 + (5.36)^2} = 8.92 - j5.20 \text{ amp.} \quad \textit{Ans.}$$

$$|I| = \sqrt{(8.92)^2 + (5.20)^2} = 10.33 \text{ amp.} \quad \textit{Ans.}$$

Also,

$$I = 110/Z = 110/10.65 = 10.33 \text{ amp. } (\textit{check}).$$

$$(d)\ E_{ab} = IZ' = (8.92 - j5.20)(5.20 - j0.637) = 43.1 - j32.7 \text{ volts.} \quad \textit{Ans.}$$

$$|E_{ab}| = \sqrt{(43.1)^2 + (32.7)^2} = 54.1 \text{ volts.} \quad \textit{Ans.}$$

Also, the voltage across $R_0 + jX_0$

$$E_0 = (8.92 - j5.20)(4 + j6) = 66.9 + j32.7. \quad \textit{Ans.}$$

$$E_0 + E_{ab} = 110 + j0 \ (\textit{check}).$$

$$(e)\ I_1 = E_{ab}Y_1 = (43.1 - j32.7)(0.1333 - j0.0667)$$
$$= 3.57 - j7.23 \text{ amp.} \quad \textit{Ans.}$$

$$|I_1| = \sqrt{(3.57)^2 + (7.23)^2} = 8.06 \text{ amp.} \quad \textit{Ans.}$$

$$I_2 = E_{ab}Y_2 = (43.1 - j32.7)(0.0562 + j0.0899)$$
$$= 5.36 + j2.04 \text{ amp.} \quad \textit{Ans.}$$

$$|I_2| = \sqrt{(5.36)^2 + (2.04)^2} = 5.73 \text{ amp.} \quad \textit{Ans.}$$

$$I_1 + I_2 = I \ (\textit{check}).$$

$$(f)\ P_0 = (66.9 \times 8.92) - (32.7 \times 5.20) \quad \text{(see Par. 43).}$$
$$= 597 - 170 = 427 \text{ watts.} \quad \textit{Ans.}$$

$$P_1 = (43.1 \times 3.57) + (32.7 \times 7.23)$$
$$= 153.7 + 236 = 389.7 \text{ watts.} \quad \textit{Ans.}$$

$$P_2 = (43.1 \times 5.36) - (32.7 \times 2.04) = 231 - 66.7 = 164.3 \text{ watts.} \quad \textit{Ans.}$$

Also,

$$P_0 = I_0^2R_0;\ P_1 = I_1^2R_1;\ P_2 = I_2^2R_2\ (check).$$

The total power

$$P = 427 + 389.7 + 164.3 = 981.0 \text{ watts}$$
$$= (110 \times 8.92) = (10.33)^2 \times 9.20\ (check).$$

The vector diagram for this circuit is shown in Fig. 63 (*b*). To avoid confusion, the individual impedance drops I_1R_1, I_1X_1, I_2R_2, I_2X_2 are omitted.

46. Equivalent Parallel Impedance.—The equivalent impedance, due to impedances in parallel, may also be determined without resort to conductance, admittance, etc. With resistances in parallel, it will be remembered that the reciprocal of the equivalent resistance

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \cdots$$

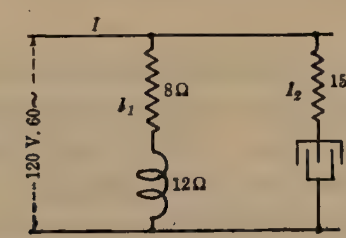


FIG. 64.—Series circuits in parallel.

With only two resistances,

$$R = \frac{R_1R_2}{R_1 + R_2} \text{ (see Vol. I, p. 62, Eq. (23)).}$$

Likewise, with impedances in parallel,

$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} \cdots$$

With two impedances, Z_1 and Z_2 in parallel, the equivalent impedance

$$Z = \frac{Z_1Z_2}{Z_1 + Z_2}$$

Each of the foregoing impedances must be expressed either in complex or in polar form.

Example.—Determine the equivalent impedance of the circuit given in the example, page 70 (see Fig. 64). Also, determine the current, the total power, and the power factor.

$$\dot{Z} = \frac{(8 + j12)(15 - j20)}{(8 + j12) + (15 - j20)}$$

$$= \frac{120 - j160 + j180 + 240}{23 - j8} = \frac{360 + j20}{23 - j8}$$

$$= \frac{360 + j20}{23 - j8} \cdot \frac{23 + j8}{23 + j8} = \frac{8{,}120}{593} + j\frac{3{,}340}{593}$$

$$= 13.68 + j5.64 \text{ ohms.} \quad \textit{Ans.}$$

$$|Z| = \sqrt{(13.68)^2 + (5.64)^2} = 14.80 \text{ ohms.} \quad \textit{Ans.}$$

$$\dot{I} = \frac{\dot{E}}{\dot{Z}} = \frac{120}{13.68 + j5.64} \cdot \frac{13.68 - j5.64}{13.68 - j5.64}$$

$$= \frac{1{,}641}{219} - j\frac{677}{219} = 7.49 - j3.09 \quad \textit{Ans.}$$

$$|I| = 8.10 \text{ amp.} \quad \textit{Ans.}$$

$$P = 120 \times 7.49 = 899 \text{ watts.} \quad \textit{Ans.}$$

$$\text{P.F.} = \frac{899}{120 \times 8.10} = 0.924. \quad \textit{Ans.}$$

This method of combining impedances in parallel has the advantage that decimal quantities having several ciphers after the decimal point are avoided. This is particularly the case with impedances whose values are in the hundreds of ohms.

47. Solution of Series-parallel Circuits with Polar Vectors.—The method given in Par. 46 is equally applicable to quantities expressed in polar vectors. For example, let it be required to solve the series-parallel circuit of Par. 45 (p. 71) by means of polar vectors.

(a) $\dot{Z}_1 = \sqrt{(6)^2 + (3)^2}/\underline{\tan^{-1} 3/6} = 6.71/\underline{26.6°}$

$\dot{Z}_2 = \sqrt{(5)^2 + (8)^2}\,\overline{\backslash\tan^{-1} 8/5} = 9.42\overline{\backslash 58.0°}$

$$\dot{Z}' = \frac{6.71/\underline{26.6°} \times 9.42\overline{\backslash 58.0°}}{(6 + 5) + j(3 - 8)} = \frac{63.2\overline{\backslash 31.4°}}{12.08\overline{\backslash 24.4°}}$$

$= 5.23\overline{\backslash 7°}$. *Ans.*

$= 5.23\ (\cos 7° - j \sin 7°) = 5.20 - j0.637$. *Ans.*

(b) $\dot{Z} = (4 + 5.20) + j(6.0 - 0.637) = 9.20 + j5.36$

$= 10.65/\underline{30.2°}$ *Ans*

(c) $\dot{I} = 110/\underline{0°}/10.65/\underline{30.2°} = 10.33\overline{\backslash 30.2°}$

$= 10.33\ (\cos 30.2° - j \sin 30.2°) = 8.92 - j5.20$. *Ans.*

(d) $\dot{E}_{ab} = \dot{I}\dot{Z}' = 10.33\overline{\backslash 30.2} \times 5.23\overline{\backslash 7°}$

$= 54.0\overline{\backslash 37.2°}$. *Ans.*

(e) $I_1 = 54.0\overline{\backslash 37.2^\circ}/6.71\underline{/26.6^\circ} = 8.06\overline{\backslash 63.8^\circ}$ amp. *Ans.*

$I_2 = 54.0\overline{\backslash 37.2^\circ}/9.42\overline{\backslash 58.0^\circ} = 5.73\underline{/20.8^\circ}$ amp. *Ans.*

(f) $P_0 = (10.33)^2\, 4 = 427$ watts. *Ans.*

$P_1 = 54.0 \times 8.06 \cos (63.8^\circ - 37.2^\circ) = 389$ watts. *Ans.*

$P_2 = 54.0 \times 5.73 \cos (37.2^\circ + 20.8^\circ) = 164$ watts. *Ans.*

Also,

$$P_1 = I_1^2R_1;\ P_2 = I_2^2R_2$$

It is thus seen that alternating-current networks in the steady state may be solved by means of complex quantities, expressed either in terms of real and imaginary components or as polar vectors. Problems are treated exactly as are similar direct-current problems, complex impedances being substituted for resistances.

CHAPTER IV

ALTERNATING-CURRENT INSTRUMENTS AND MEASUREMENTS

48. Siemens Dynamometer.—Several types of alternating-current instruments operate on the electrodynamometer principle. The Siemens dynamometer (Fig. 65) is an example of this type of instrument in simple form. It consists primarily of two sets of coils. The coil F is fixed and the coil M, whose axis is at right angles to the axis of F, is free to turn through a small angle. M is suspended by a silk thread, and its turning moment is opposed by a helical spring. Current s led into the moving coil through two mercury cups.

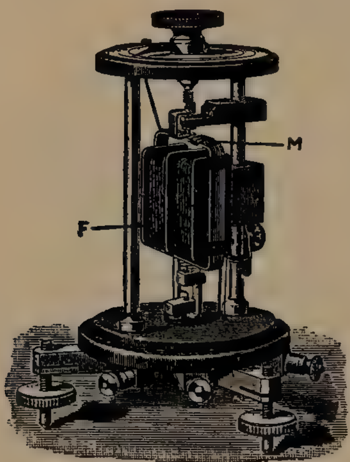


Fig. 65.—Siemens dynamometer.

When used as an ammeter, the two coils are wound with a few turns of coarse wire and are connected in series. When current flows through these coils, there is a tendency for the moving coil to swing into the plane of the fixed coil. When the current reverses, it reverses in the two coils simultaneously so that the torque is always in the same direction. The movable coil is not allowed to deflect, however, but is kept in its zero position by turning the knurled head at the top of the instrument which acts on the coil through the spring. The angle by which it is necessary to turn this head is proportional to the turning moment of the coil. The turning moment is proportional to the current *squared*, so that the deflection of the index

$$D = KI^2$$

where K is a constant.

The current

$$I = K'\sqrt{D} \tag{52}$$

where

$$K' = \frac{1}{\sqrt{K}}$$

As the deflections are proportional to the *square* of the current, the instrument gives effective values when Eq. (52) is applied. It therefore reads correctly for both alternating and direct currents. When direct current is used, it is advisable to reverse the direction of the current and average the set of readings. This eliminates the effect of stray fields.

This type of instrument is difficult to adjust and to manipulate, especially when the current fluctuates. It is not direct reading and, because of its construction, is adapted to laboratory work only.

If small wire be substituted for the coarse wire and an extension coil be connected in series, the instrument can be used as a voltmeter.

49. Indicating Electrodynamometer. As it is neither portable nor direct reading, the Siemens dynamometer itself is not adapted to portable and to switchboard instruments. Many types of portable and switchboard instruments operate on the Siemens dynamometer principle, however. The general construction of a portable type of electrodynamometer instrument is shown in Fig. 66.

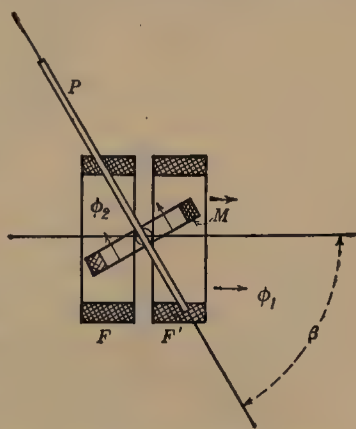


Fig. 66.—Principle of electrodynamometer.

Two fixed coils FF' are so connected that their magnetic fields act in conjunction. These coils may be considered as being two parts of a single coil, opened in the middle to allow the spindle of the moving coil to pass through.

M is a movable coil mounted on a vertical spindle. There is a hardened steel pivot at each end of the spindle, which turns in jewelled bearings. Two spiral springs similar to those used in direct-current instruments (see Vol. I, p. 144, Fig. 122) oppose the turning of coil *M* and at the same time carry the current into

the coil. As the springs can carry but a very small current, the movable coil is wound with fine wire.

Assume that at some instant the direction of the magnetic field ϕ_1, which is due to the fixed coils, is from left to right. At the same instant, the current in coil M produces a field ϕ_2 whose direction is along the axis of M. Coils tend to align themselves so that the number of magnetic linkages in the system is a maximum. The moving coil M, therefore, tends to turn in a clockwise direction so that its field will act in conjunction with ϕ_1. The turning of M is opposed by the control springs.

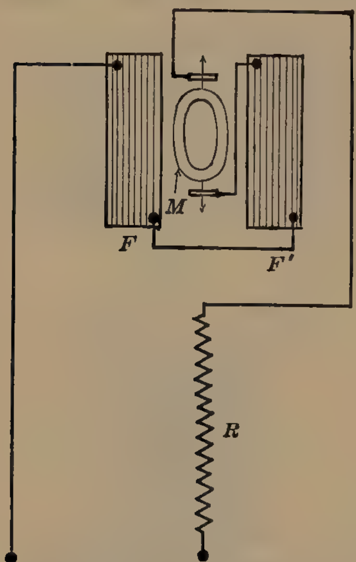


FIG. 67.—Diagram of dynamometer voltmeter.

Obviously, the torque developed is proportional to ϕ_1, ϕ_2, and $\sin \beta$, where β is the angle between the axis of coil M and the axis of coils FF'. As ϕ_1 and ϕ_2 are proportional to the currents in the coils FF' and M, respectively, the torque is proportional to the product of the two currents and $\sin \beta$.

50. The Electrodynamometer Voltmeter.—Some types of alternating-current voltmeter operate on the electrodynamometer principle. The fixed coils FF' (Fig. 67) are wound with fine wire and are connected in series with the moving coil M. A high resistance R is connected in series with the dynamometer to limit the current when the instrument is connected across the line. The current passing through the dynamometer is, therefore, proportional to the line voltage. The current passing through the instrument causes coil M to turn, and the pointer attached to it moves over a scale graduated in volts. The scale will not be divided uniformly like that of the direct-current voltmeter, for the deflections are very nearly proportional to the *square* of the voltage. The divisions at the lower part of the scale are so small that poor precision is obtained. The divisions at the middle and upper portions of the scale, however, are usually of such magnitude that they may be read with a high degree of precision.

This dynamometer type of voltmeter takes about five times as much current as a direct-current voltmeter of the same rating and consumes an appreciable amount of power. As the moving coil operates in a comparatively weak field, this type of instrument is very susceptible to stray fields. Unless the instrument is shielded, wires carrying currents, inductive apparatus, and even iron alone, if brought too near, may cause large errors in the indications of this type of voltmeter.

This instrument may be used for direct current as well as for alternating current. Reversed direct-current readings should

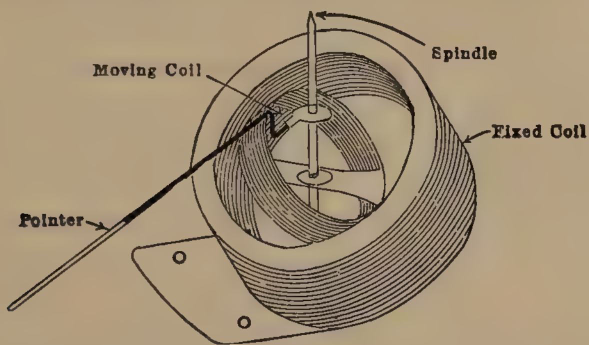


FIG. 68.—General Electric inclined-coil instrument.

be taken in order to eliminate the effect of the earth's field and of any other stray fields. As the deflections depend upon the *square* of the voltage, the instrument reads effective values.

51. Inclined-coil Voltmeters.—The inclined-coil type of voltmeter operates on the dynamometer principle. It differs from the previous types only in the geometrical relations of its fixed and moving coils. The axis of the fixed coil (Fig. 68) is set at a considerable angle with the vertical. The axis of the moving coil makes a considerable angle with the spindle. This moving coil is connected in series with the fixed coil, the current being carried to the moving coil through light springs. A resistance to limit the current is connected in series with the instrument.

When the pointer is at the zero position, there is a considerable angle between the axes of the fixed and the moving coils. When current flows through the instrument, the moving coil tends to take such a position that its axis coincides with the axis of the fixed coil, so that their magnetic fields act in conjunction. In

turning, the moving coil is opposed by flat spiral springs. The scale is calibrated in volts.

As this instrument is of the dynamometer type, its scale readings hold for both direct and alternating currents.

52. Dynamometer Ammeters.—Owing to the difficulty of leading a heavy current into the moving coil, dynamometer ammeters of the portable type and of the switchboard type are not common. It might appear that this type of instrument could be devised for use with a shunt, which would allow but a small portion of the total current to pass through the moving coil. This involves two difficulties. Alternating currents divide inversely as the circuit *impedances*. Impedances are determined by the frequency. Unless the ratio of inductance to resistance were the same in the shunt as in the moving coil, the instrument would be correct at only one frequency and might be in considerable error with an irregular wave shape, owing to the presence of higher-frequency harmonics. There would also be a considerable voltage drop across such a shunt.

Instruments of the dynamometer type, in which the above difficulties are, in part, overcome, are available, but the iron-vane type of instrument described in Par. 57 is so much simpler and less expensive that the shunted type is little used.

53. Wattmeter.—Alternating-current power is equal to the product of the effective current and the effective voltage only when the power factor is unity. The ammeter and voltmeter method, therefore, as used with direct currents, can seldom be used to measure alternating-current power. Consequently, a *wattmeter* is necessary for measuring alternating-current power.

The wattmeter shown in Fig. 69 operates on the electrodynamometer principle. M is a moving coil wound with fine wire and is practically identical with the moving coil of the dynamometer voltmeter (Fig. 67). It is connected across the line in series with a high resistance R. The current is led into this coil through springs. The two fixed coils FF are wound with a few turns of heavy wire, capable of carrying the load current. As there is no iron present, the field due to the current coils FF is proportional to the load current at every instant. The current in the moving coil M is proportional to the voltage at every instant. For any given position of the moving coil, therefore,

the torque is proportional at every instant to the product of the current and voltage or to the instantaneous power of the circuit. If the power factor is other than unity, there is negative torque for part of the cycle. That is, during the periods when there are negative loops in the power curve (Fig. 22, p. 25), the current in the fixed coil and the current in the moving coil reverse their directions with respect to each other and so produce a negative torque. The moving coil takes a position corresponding to the *average* torque. The torque is also a function of the angle between the fixed and moving coil axes, but this factor is taken into account by the scale calibration.

As the torque acting on the moving coil varies from instant to instant, having a frequency twice that of either the current or the voltage, the coil tends to change its position to correspond with these variations of torque. If the moving system had little inertia, the needle would vibrate so that it would be impossible to obtain a reading. Because of the relatively large moment of inertia of the moving system, the needle assumes a steady deflection for constant values of average power. The position taken by the coil corresponds to the *average* value of the power, which is the result desired.

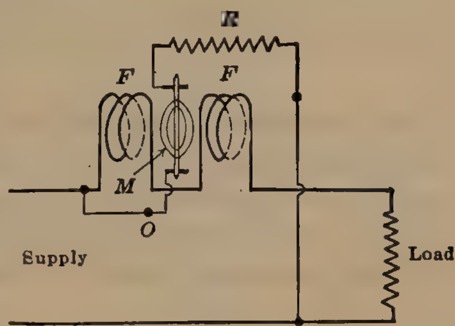


FIG. 69.—Connections for wattmeter.

It should be noted, in Fig. 69, that the voltage terminal marked *O* is connected directly to one end of the moving coil. This terminal always should be connected directly to that side of the line to which the current coil is connected. The fixed and moving coils are then at the same potential. If the moving coil is connected to the other side of the line, the potential difference between the fixed and moving coils is equal to the full-line potential, as shown in Fig. 70. In this diagram, the fixed coils are considered as being at zero or ground potential. The moving coil is then at the potential of the other side of the line, or 550 volts, and this is the difference of potential which exists between the fixed and the moving coils. This is dangerous from the insulation standpoint, and electrostatic forces existing between the

fixed and the moving coils may cause an error in the instrument reading. (The wattmeter is also briefly described in Vol. I, p. 183.)

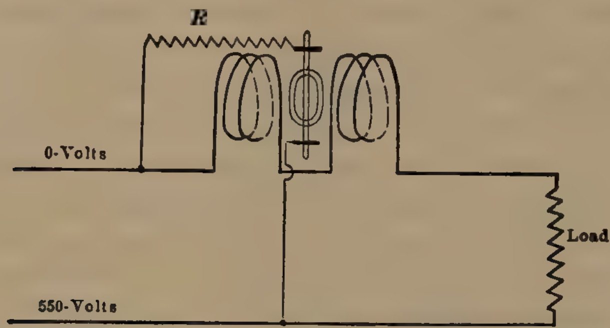
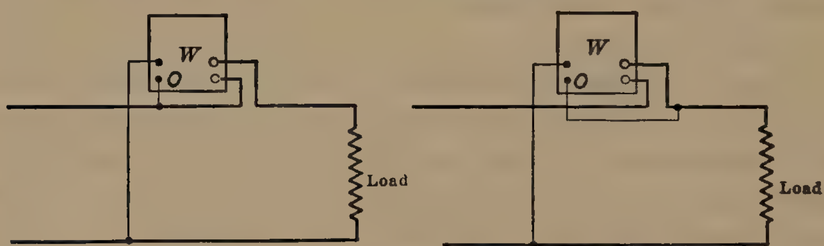


FIG. 70.—Incorrect method for connecting wattmeter.

54. Wattmeter Connections.—In Fig. 71 (*a*), wattmeter *W* is shown measuring the power taken by a certain load. In order to measure this power correctly, the wattmeter current coil should carry the *load* current, and the wattmeter voltage coil, in series with its resistance, should be connected directly across the *load.*



(*a*) Instrument measures power consumed by its own current coil.

(*b*) Instrument measures power consumed by its own potential circuit.

FIG. 71.—Methods for connecting wattmeter.

The current in the wattmeter current coil is the same as the load current; the wattmeter potential circuit is not connected directly across the load, however, but is measuring a potential in excess of the load potential by the amount of the impedance drop in the wattmeter current coil. The wattmeter reads too high, therefore, by the amount of power consumed in its own current coil. Under these conditions, the true power

$$P = P' - I^2R_c \tag{53}$$

where P' is the power indicated by the wattmeter, I is the current in the wattmeter current coil, and R_c is the resistance of this coil. This loss is ordinarily of the magnitude of 1 or 2 watts at the rated current of the instrument and may often be neglected.

If the wattmeter be connected as shown in Fig. 71 (*b*), the wattmeter potential circuit is connected directly across the load, but the wattmeter current coil carries the potential-coil current in addition to the load current. In fact, the wattmeter potential circuit may be considered as being a small load connected in parallel with the actual load whose power is to be measured. The power consumed by this potential circuit must be deducted, therefore, from the wattmeter reading. The true power taken by the load,

$$P = P' - \frac{E^2}{R_p} \tag{54}$$

where P' is the wattmeter reading, E the load voltage, and R_p the resistance of the wattmeter potential-coil circuit.

An idea of the magnitude of this correction may be obtained from the following example:

Example.—A certain wattmeter indicates 157 watts when it is connected in the manner shown in Fig. 71 (*b*). The line voltage is 120 volts, and the resistance of the wattmeter potential circuit is 2,000 ohms. How much power is taken by the load?

$$P = 157 - \frac{\overline{120}^2}{2{,}000} = 157 - 7.2 = 149.8 \text{ watts.}$$

It will be observed that a considerable percentage error would result in this case if the wattmeter loss were neglected.

The Weston Electrical Instrument Company manufactures an instrument which compensates for this loss. A small auxiliary coil, connected in series with the moving-coil system, is interwound with the fixed coils so that a small countertorque is exerted, this countertorque being proportional to the power consumed by the potential circuit.

The current and potential circuits of a wattmeter must each have a rating corresponding to the current and voltage of the circuit to which the wattmeter is connected. A wattmeter is rated in amperes and volts, rather than in watts, because the

indicated watts show neither the amperes in the current coil nor the voltage across the potential circuit.

If the current in an ammeter or the voltage across a voltmeter exceed the rating of the instrument, the pointer goes off scale and so warns the user. A wattmeter may be considerably overloaded and yet the load power factor be so low that the needle is well on the scale. For this reason, a voltmeter and an ammeter should ordinarily be used in conjunction with a wattmeter, so that it is possible to determine whether either the voltage or the current exceeds the wattmeter rating.

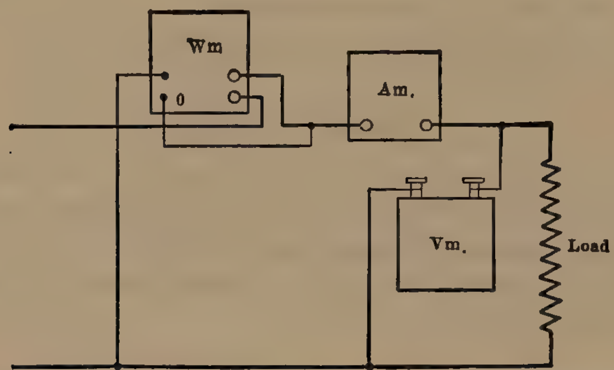


FIG. 72.—Wattmeter, ammeter, and voltmeter connections for measuring power.

Corrections for the power taken by ammeters and voltmeters are often necessary. For example, in Fig. 72, the I^2R loss of the ammeter and the E^2/R loss of the voltmeter must be deducted from the wattmeter reading, in addition to the wattmeter potential loss. The ammeter reads too high by the current taken by the voltmeter. This voltmeter current must be subtracted *vectorially* from the ammeter reading in order to obtain the true load current.

55. Polyphase Wattmeter.—Ordinarily, it requires two or more wattmeters to measure the total power of a two- or a three-phase circuit. If the load fluctuates, it is difficult to obtain accurate simultaneous readings of two wattmeters. At power factors less than 0.5, in a three-phase circuit, one of the wattmeters reverses its reading (see p. 116, Par. 72). This necessitates reversing the connections of one of the instruments, which is often inconvenient. If both wattmeters be combined in one, that is, if both moving coils be mounted on the same

spindle, the turning moments for each element add or subtract automatically, and the total power is read on a single scale.

Figure 73 shows the construction of a Weston polyphase wattmeter in which the two elements are clearly shown. Figure 74

FIG. 73.—Interior view, Weston polyphase wattmeter.

shows one method for connecting a polyphase wattmeter in a three-phase circuit. Note the symmetry of the connections. The two outer lines from the source connect to the two front current binding posts. Each of the two *o* potential binding posts connects to the source side of its respective current line.

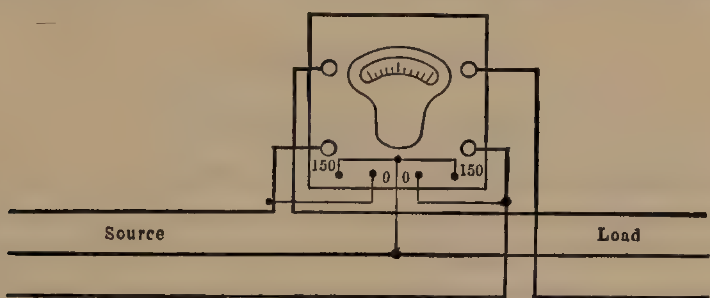


FIG. 74.—Connections for polyphase wattmeter on three-phase circuit.

Although it is often more convenient to use a polyphase wattmeter, two single instruments are better adapted to precision work, as there is no mutually inductive action between the elements of the instruments such as may occur between the elements of a polyphase instrument. With two individual instruments, it is a simple matter to apply scale corrections.

Wattmeter Calibration.—A dynamometer wattmeter is ordinarily calibrated with direct current, the connections for calibration being shown in Fig. 75. The voltage across the potential circuit is measured with a standard direct-current voltmeter. The current is accurately measured by means of a potentiometer, although a standardized direct-current ammeter is often sufficiently accurate. Both the current and the potential are reversed at each reading so as to eliminate the effect of the earth's field or of any stray field. The true power in watts is given by the product of the current and the voltage, as direct current is used.

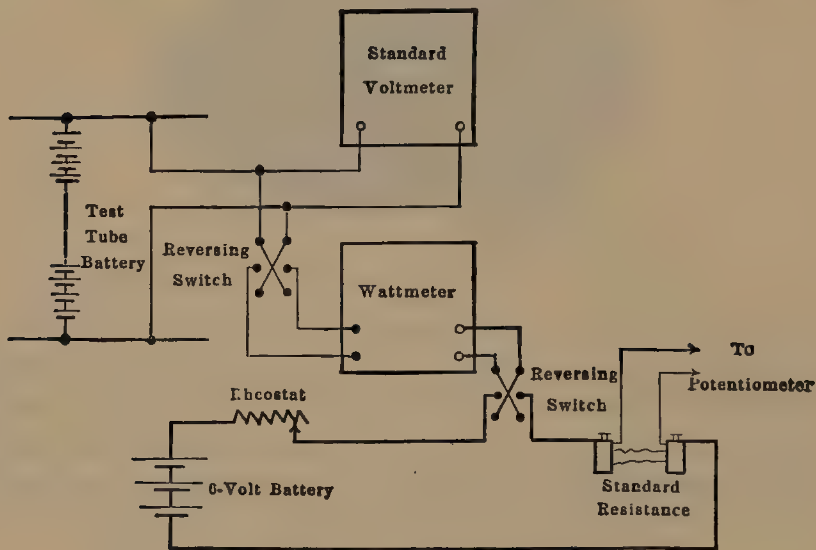


Fig. 75.—Connections for calibrating wattmeter.

IRON-VANE INSTRUMENTS

56. Voltmeters.—In Vol. I (Chap. VII), it was pointed out that instruments depending upon the solenoid action of an iron plunger were not satisfactory as ammeters. By the use of light iron vanes, jewelled bearings, etc., satisfactory types of commercial alternating-current instruments, based on the principle of magnetized iron, have been developed.

One such type of instrument, manufactured by the Weston Electrical Instrument Company, is shown in Fig. 76.

A small strip of soft iron M, bent into cylindrical form, is mounted axially on a spindle which is free to turn. Another similar strip F, which is more or less wedge shaped, and with a

larger radius than M, is fixed inside a cylindrical coil. The cylindrical coil is wound with fine wire and is connected in series with a high resistance. When connected across the line, the current through the instrument is substantially proportional to the circuit voltage. When current flows through this exciting coil, both iron vanes become magnetized. The upper edges of the two strips will always have the same magnetic polarity, and the lower edges will always have the same magnetic polarity, but when the upper edges are north poles, the lower edges are south poles. There will always be a repulsion, therefore, between the two upper edges and also between the two lower edges of the iron strips. This repulsion tends to move the spindle against the action of two springs. A pointer mounted on the spindle moves over a graduated scale and indicates the voltage.

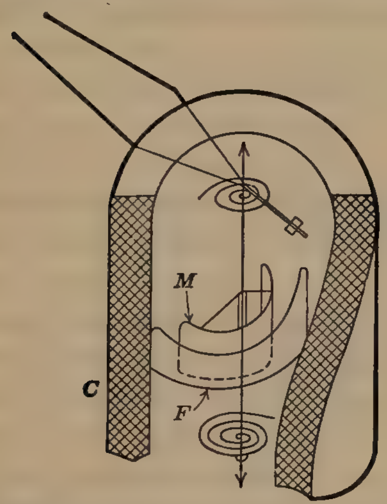


Fig. 76.—Weston iron-vane type of instrument.

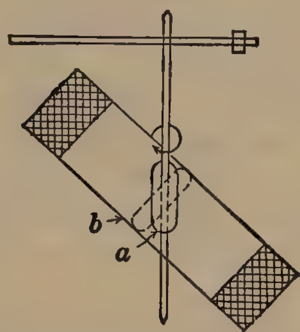


Fig. 77.—Inclined coil, iron-vane type of instrument.

This type of instrument can be used for direct current with a precision of 1 or 2 per cent. Its obvious advantages are its simplicity, its cheapness, and the fact that there is no current carried to the moving element. When carefully calibrated, a precision of 0.5 per cent., and better, can be obtained with alternating current. This type of instrument cannot be calibrated accurately with direct current on account of the effect of hysteresis on the vanes. It should be calibrated by comparison with an alternating-current standard. Air damping is obtained by the use of a light aluminum vane moving in a restricted space.

The iron-vane principle has been applied to the inclined-coil type of instrument. A small iron vane, mounted obliquely on the spindle (Fig. 77), replaces the inclined moving coil of Fig. 68 (p. 79). When the pointer is at zero, this vane lies at an angle to the coil axis, as at *a* (Fig. 77). When current flows in the coil, the vane attempts to take such a position that the direction of its axis shall coincide with that of the magnetic field, which acts along the coil axis. This position is shown at *b* (Fig. 77). The vane, in seeking this position, turns the spindle which carries the pointer. The turning moment is opposed by springs. In the later models, the coils of these instruments are surrounded by iron laminations which shield them from stray fields. In the cheaper models, air damping is used, being obtained by a light aluminum vane attached to the moving element. The more expensive models employ magnetic damping, such as is used with watthour meters, a light aluminum vane moving between the poles of permanent magnets.

57. Ammeters.—Owing to the difficulty of carrying any except the smallest currents into the moving system of dynamometer instruments, iron-vane ammeters are practically the only type used for commercial instruments. The Weston iron-vane ammeter operates on the same principle as the iron-vane voltmeter (Par. 56). The magnetizing coil in the ammeter is wound with a few turns of heavy wire instead of with the large number of turns of fine wire used with the voltmeter.

The General Electric Company's inclined-coil ammeter is of the same construction as the voltmeter, except that the coil is wound with coarse, instead of with fine, wire (see Fig. 77).

58. Hot-wire Instruments.—This type of instrument, described in Vol. I (Chap. VII, p. 151), reads equally well on both direct- and alternating-current circuits. As its deflection depends upon the square of the current (I^2R loss), the hot-wire instrument can be used as a transfer from alternating to direct current and *vice versa*. This type of instrument lacks high precision.

59. Alternating-current Watthour Meter.—The direct-current watthour meter can be used with alternating current, as the reversal of line voltage reverses both its armature and its field current simultaneously, and the direction of the torque remains unchanged. At low power factors, however, considerable error

may be introduced by the inductance of the armature circuit. This causes the armature current to lag the line voltage by a small angle, and although this has negligible effect at or near unity power factor, the error at low power factor is quite pronounced. This error may be compensated by shunting the current coils of the meter with a low non-inductive resistance.

The induction watthour meter is so much cheaper and so superior to the direct-current type that there is little necessity for using the direct-current type on alternating-current circuits.

A rear view of one type of induction meter is shown in Fig. 78. *P* is a potential coil which is highly inductive and is placed on one lug of the laminated magnetic circuit, this lug being over the disc *D*. *CC* are two series or current coils placed on two projecting lugs beneath the disc. These coils are so wound that if one tends to send flux upward, the other tends to send it downward. *cw* is a small auxiliary or compensating winding placed on the potential lug, and its ends are connected to the resistance *R*. In order that the meter may register correctly, the potential-coil flux must lag the line voltage by 90°. As it is impossible to make the resistance of the potential coil zero, its current will lag by an angle less than 90°. At low power factors, this introduces considerable error in the meter registration. By properly adjusting the resistance *R*, however, the potential-coil *flux* may be brought into the 90° relation, and the meter will register substantially correctly at all power factors. To adjust the compensation, the meter is made correct at unity power factor, and then the power factor is dropped to some low value, as 0.5. If the registration is now in error, it is due to improper compensation. The meter is again made to register correctly by changing the resistance

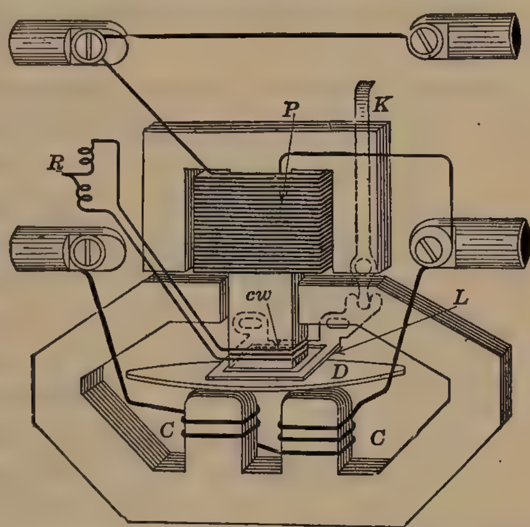


Fig. 78.—Diagram of induction watthour meter.

R, the two small wires of this resistance being either twisted or untwisted and then soldered. If the meter underregisters when the load current lags, the resistance R should be *decreased;* if the meter overregisters with lagging current, the resistance R should be *increased.* The reverse is true with leading current.

L is a small metallic stamping placed under the potential lug and can be moved laterally by means of the lever K. Its function is to provide the small torque just necessary to overcome the friction of the meter. The operation of this adjustment is as follows:

Figure 79 shows the stamping under the lug, set off center. When the flux starts to pass down through the lug, a current is immediately induced in the short-circuited stamping. This current, by Lenz's law, opposes the flux entering the stamping, so that the flux is crowded to the left-hand side of the lug, as shown. When the flux starts to decrease, the current in the short-circuited stamping tends to oppose the decrease in the flux. This retards the time phase of the flux in the right-hand side of the lug with respect to that in the left-hand side of the lug. The result is a sweeping of the flux from left to right across the lug. This sliding flux cuts the disc and sets up eddy currents in it. These currents, reacting with the flux, produce a torque tending to drive the disc in the direction in which the stamping is displaced from its position of symmetry. This is the "shaded-pole" principle, which is also used to start small single-phase induction motors (see Par. 149, p. 337).

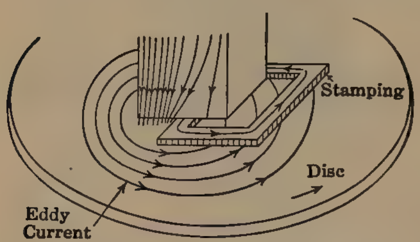


FIG. 79.—Shaded-pole principle of light-load adjustment.

The driving torque of the meter at unity power factor is produced as follows: Figure 80 (a) shows the current and the voltage wave in phase. If the meter is properly lagged, the potential flux ϕ_p is 90° behind E. The current flux ϕ_i is in phase with I. Figure 80 (b) shows the magnetic polarities of the meter poles for the various times indicated in (a). At 1, the current is zero so that no flux is produced by the current coils. The potential-coil flux is a negative maximum so that the potential pole is S.

The two current lugs, therefore, must be N poles. At 2, the potential-coil flux is zero, but the current is a maximum. The lower poles will be N and S, therefore, as shown, and the potential lug will have an S on one side and an N on the other. At 3, the upper lug is N, and the two lower ones S. Times 4 and 5 are also shown, 5 corresponding to 1.

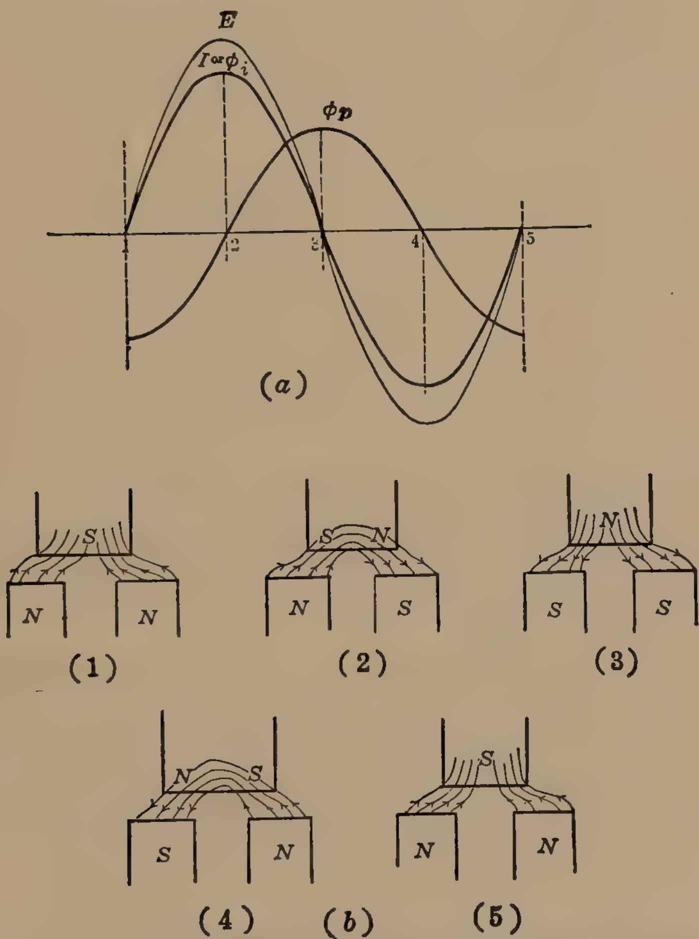


FIG. 80.—Gliding field in air-gap of induction watthour meter.

In (1), the entire upper lug is an S-pole. In (2), this S-pole has diminished in magnitude, has moved toward the left-hand side of the lug, and an N-pole appears on the right-hand side of this lug. In (3), an N-pole occupies the entire upper lug; and in (4), this has diminished and moved toward the left side of the lug.

A similar cycle takes place on the two lower lugs. In (1), both lugs are N-poles, making one large N-pole. In (2), this

large N-pole has diminished and moved toward the left, being followed by an S-pole appearing on the right. In (3), the N-pole has disappeared, both lugs becoming S-poles, etc. By following the cycle, it will be observed that an N-pole moves from right to left on both the upper and lower lugs. Similarly, an S-pole does likewise, following the N-pole. The field, therefore, "glides" laterally through the gap. In so doing, it cuts the disc and induces eddy currents. These eddy currents induced in the disc react with this gliding field, and by Lenz's law the disc tends to follow the field (see Induction Motor, Par. 128, p. 265).

If the power factor be zero, ϕ_i (Fig. 80 (a)) will be either in time phase with ϕ_p if the current lags or will be 180° out of phase with ϕ_p if the current leads. In either case, if instantaneous values of flux be taken, as in Fig. 80 (b), it will be found that there is no lateral displacement of the field in the gap but merely a sinusoidal pulsation of flux up and down in the gap. Under these conditions, the torque acting on the disc is zero.

The disc of the induction meter, like that of the direct-current meter, cuts a field of constant strength produced by *permanent* magnets. This causes a *retarding* torque which is proportional to the speed of the disc. Both the *driving* torque (motor action), therefore, and the *retarding* torque (generator action) are produced on the same disc.

60. Calibration and Adjustment of the Induction Watthour Meter.—The induction watthour meter is calibrated in much the same manner as the direct-current watthour meter. A standard indicating wattmeter is used to measure the average power over a stated interval, and the revolutions of the disc of the watthour meter are counted with the aid of a stop watch. The average meter watts are calculated by means of the equation

$$W = \frac{K \times N \times 3{,}600}{t} \tag{55}$$

where K is the meter constant, N the revolutions of the disc, and t the time in seconds.

As a rule, an ammeter and a voltmeter are used in connection with such a test (as shown in Fig. 81), in order to determine the power factor. Instrument losses should be carefully investigated, and corrections made if necessary.

After the meter is adjusted at full load and unity power factor by means of the retarding magnets, it is adjusted at light load by means of the light-load adjustment. The power factor is then lowered. Any error occurring now must be due to improper lagging. The registration is then made correct by adjusting the resistance *R* (Fig. 78), which is in series with the lagging coil. If the meter registers low with lagging current, the resistance *R* should be decreased; if it registers high, the resistance *R* should be increased. With leading current these operations should be reversed.

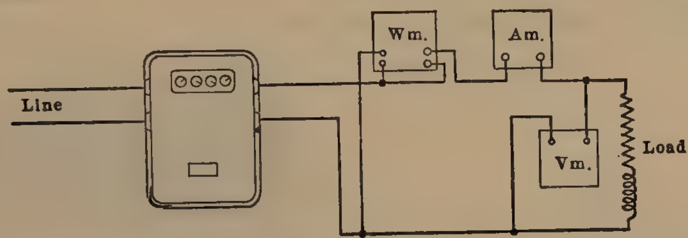


FIG. 81.—Connections for testing alternating-current watthour meter.

The induction watthour meter has certain advantages over the direct-current meter. As there is no coil-wound armature in addition to the disc, the rotating element of the induction meter is much lighter than that of the direct-current meter. It has, moreover, no commutator or delicate brushes, which are frequent sources of trouble with the direct-current meter.

The induction meter is also made in the polyphase type. Two single-phase elements act on a common spindle. There are two sets of damping magnets.[1]

61. Frequency Indicators.—Frequency indicators are based on two principles, that of electrical resonance and that of mechanical resonance. The latter type is the more common and is simpler in operation. A number of steel reeds, each having a white index on its end, are clamped between two metal strips. Each reed has its own mechanical frequency of vibration. Behind this bank of reeds there is an electromagnet, the coil of which is excited by the circuit whose frequency it is desired to measure. The reed whose frequency is that of the circuit will

[1] For a more detailed analysis, see F. A. LAWS, "Electrical Measurements."

vibrate with the greatest amplitude (Fig. 82). With the exception of one or two reeds near this one, none of the others will be affected. The frequency is determined, therefore, by noting the scale reading opposite this reed. Were the reeds unpolarized, they would be attracted equally well by either a north or a south pole. An adjacent permanent magnet keeps the reeds polarized, so that the reed of a particular mechanical frequency will respond to the same electrical frequency. The reeds are usually so arranged that there is a reed for every half-cycle. Figure 82 shows the Frahm type of indicator, as manufactured by Hartmann and Braun.

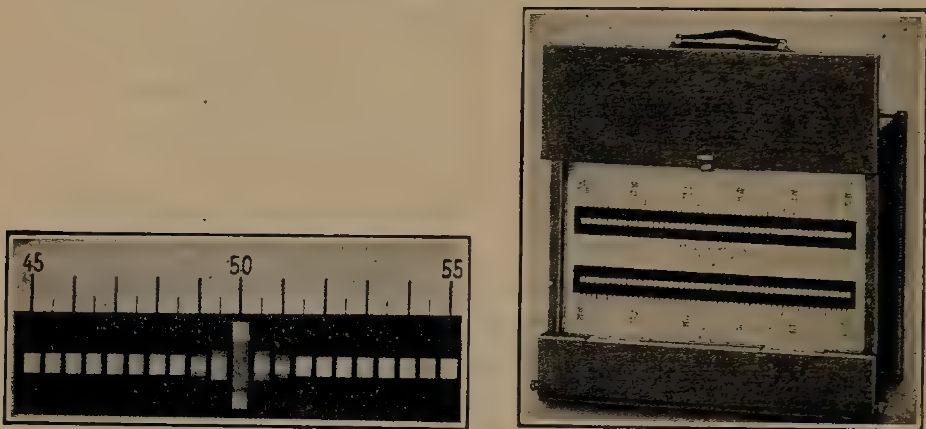


FIG. 82.—Frahm vibrating-reed frequency meter.

62. Power-factor Indicators.—Power-factor indicators and synchroscopes are based on the principle of the Tuma phase meter. In Fig. 83, F is a fixed coil carrying the circuit current. MM' are two flat coils wound with fine wire; they are fastened rigidly together and mounted on a spindle free to rotate. There is no mechanical control whatever of this moving element, such as springs, for example. The angle between the coils is 90°, or nearly so. The windings of the two coils MM' are connected together at the common point A, and A is connected to the same side of the circuit as F. A non-inductive resistance R is connected between M and the other side of the line. A high inductance L is connected between M' and the other side of the line. The currents in M and M' may be assumed to differ by 90° in time phase. Assume that the power factor of the load is unity.

The current in coil M' lags the line voltage by 90°, hence, lags the flux due to coil F by 90° and, therefore, exerts no torque. The current in coil M is in time phase with the line voltage and, hence, with the flux due to coil F and will, therefore, move into the plane of coil F as there is no restraining torque. Hence, at unity power factor, the entire moving element takes such a position that the coil M is in the plane of coil F.

If the power factor of the load is zero, the current and the voltage differ in phase by 90°. Hence, the current in coil M and the flux due to coil F have a time-phase difference of 90°, and coil M exerts no turning moment. The current in coil M',

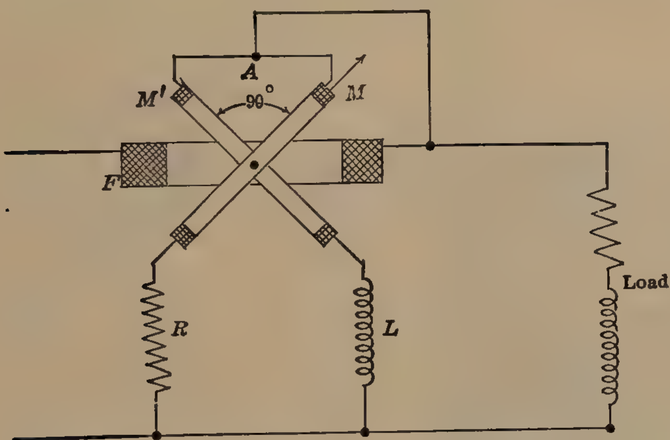


FIG. 83.—Principle of Tuma phase meter.

however, is now in time phase with the flux due to coil F, and, therefore, coil M' will move into the plane of coil F. The moving system will then have a position of 90° from its position at unity power factor. That is, when the current changes its time phase by 90°, the moving element of the indicator changes its space position by 90°. The direction in which the element turns depends on whether the current lags or leads the voltage. For intermediate power factors, it can be shown that the angle of the moving system corresponds to the circuit power-factor angle. If the scale is calibrated in degrees, the pointer can be made to indicate the power-factor *angle* of the circuit. To make the indicator read power factor, it is necessary merely to make the scale divisions proportional to the cosine of the power-factor angle. In practice, the current is led into the moving system

through strips of annealed silver foil which exert no appreciable control on the moving system.

As it is impossible to obtain either a pure resistance or a pure inductance, the currents in coils M and M' will not differ exactly by 90° in time phase. It can be shown that if the space angle between coils M and M' be made equal to the angle of phase difference of their currents, the instrument indicates correctly.

If the angle between the two coils MM' be made 120°, as shown in Fig. 84, the instrument can be made to indicate three-phase power factor, if the system is balanced. Non-inductive

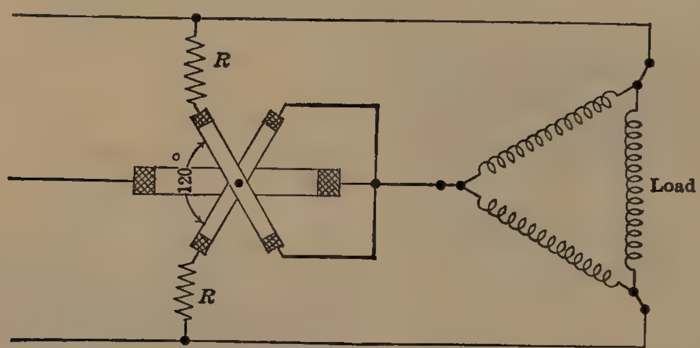


FIG. 84.—Three-phase power-factor indicator.

resistances R, R are now connected in series with each of the moving coils. The fixed coil is connected in one line of the three-phase system, and the common terminal of the two moving coils connects to this same line. The other terminal of each of the moving coils connects to one of the other two lines of the three-phase system, as shown in Fig. 84. This is the scheme of connections for the power-factor indicator of the General Electric Company, so often seen on switchboards. The instrument indicates the three-phase power factor if the system is very nearly balanced. If the system is unbalanced, the reading has little significance.

63. Synchroscope.—Before connecting an alternator to the bus-bars and in parallel with other alternators, it is necessary not only that its voltage be the same as that of the bus-bars but that it be in phase opposition as well. This corresponds to having direct-current generators of the same polarity before connecting them in parallel.

A synchroscope is an instrument for indicating when machines are in the proper phase relation for connecting in parallel and at the same time for showing whether the incoming machine is running fast or slow. This type of instrument is based on the principle of the power-factor indicator. A diagram of one type of synchroscope is shown in Fig. 85. A horseshoe magnetic

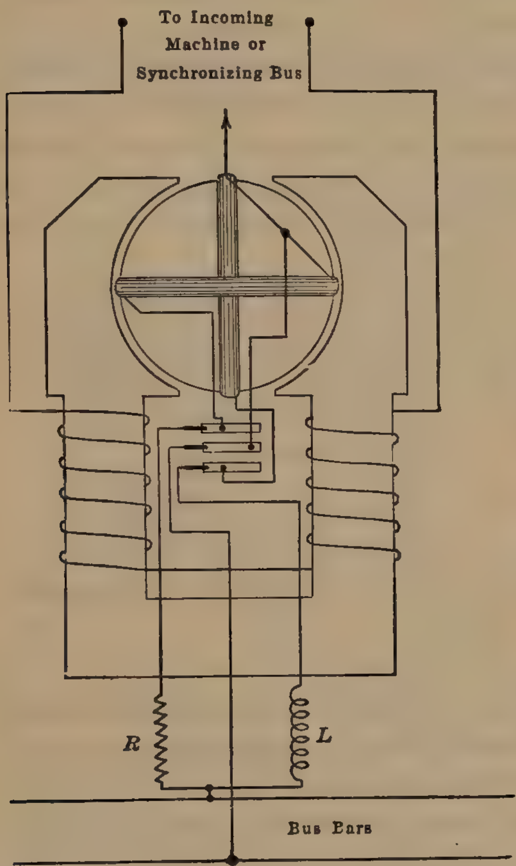


FIG. 85.—Typical connections for synchroscope.

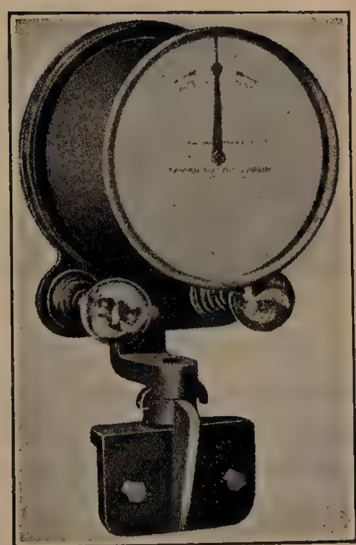
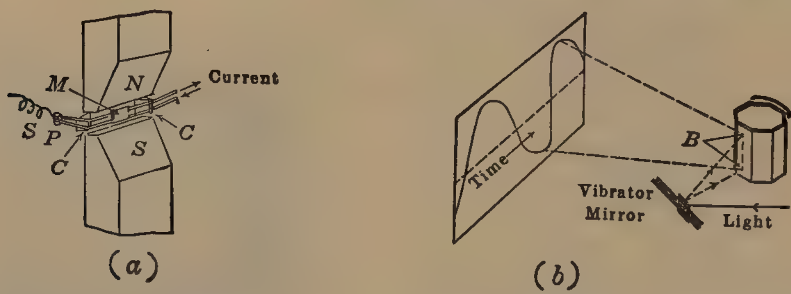


FIG. 86.—Synchronism indicator on swinging bracket.

circuit is excited by a winding which connects to the incoming machine, usually through a potential transformer. The moving coils are the same as those of the Tuma phase meter, except that the connections are made through slip-rings. This allows the coils to revolve freely. The moving element is connected across the bus-bars, usually through potential transformers. If the incoming machine has the same frequency as the bus-bars,

the pointer remains stationary. When the machines are in the proper phase relation for closing the switch, the pointer is over an index on the dial, this position being shown in Fig. 86. The direction of rotation of the pointer shows whether the incoming machine is fast or slow. The generator switch is usually thrown when the pointer is rotating slowly in the "fast" direction and is approaching the index. Figure 86 shows a General Electric synchroscope and its mounting.

64. Oscillograph.—It is often desired to investigate transient conditions in electrical circuits, such, for example, as the current and the voltage relations during the blowing of a fuse or during the short circuit of an alternator or in oscillations produced by switching, etc. Further, it is desirable to have apparatus which



Vibrating element of oscillograph.

Method of drawing out vibrating beam into wave.

FIG. 87.

will show the current and the voltage waves in alternating-current circuits during steady conditions. The oscillograph is an instrument which is capable of meeting these requirements.

Its principle is quite simple, being that of a D'Arsonval galvanometer (Vol. I, Chap. VII, p. 138), as shown in Fig. 87 (*a*). A small phosphor-bronze strip or filament is stretched over two clefts *CC*, around a small pulley *P* and back again. The spring *S* acting on the pulley keeps the two lengths of the strip in tension. This filament is placed between the poles of a strong electromagnet. When a current flows through the filament, one length of the filament moves outward and the other inward. A very small mirror *M* is cemented across the two lengths of the filament and is given a rocking motion by this movement of the filament. If a beam of light be reflected from this mirror, it

will be drawn out into a straight line by the mirror vibration. If this beam of light be made to strike a rotating mirror, in the manner shown in Fig. 87 (*b*), the rotation of the mirror introduces a time element, and the wave is drawn out so that its characteristics are shown.

The instrument is merely a galvanometer having a single turn and a very light moving element. This makes the moment of inertia very small. Also, the filament is under considerable tension, so that its natural frequency of vibration is very high, being from 3,000 to 10,000 cycles per second. These characteristics are necessary in order that the filament may respond accurately to the comparatively high-frequency variations which it is called upon to follow. The moving element is usually immersed in oil so that its movement is properly damped and the filament is kept cool.

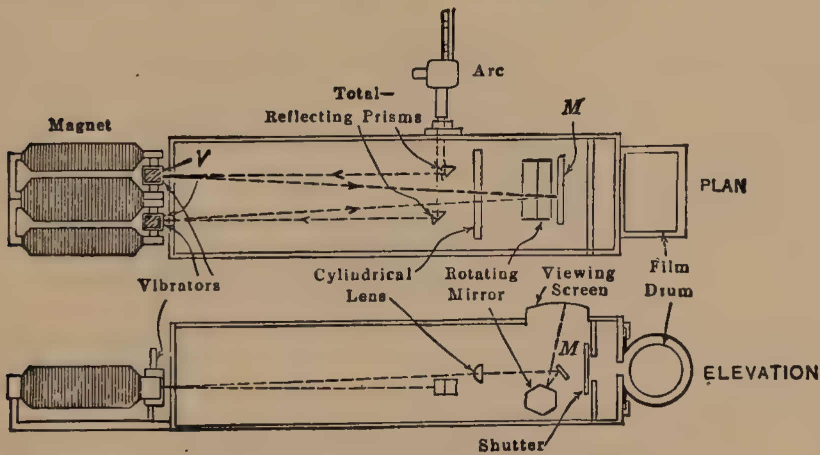


Fig. 88.—Typical oscillograph.

Figure 88 shows the general arrangement of a laboratory type of oscillograph.

The light from the arc lamp strikes the two total-reflecting prisms by means of which the beam is turned at right angles and directed upon the vibrator mirrors at V. These mirrors reflect the light back through the cylindrical lens, which concentrates the beam. A plane mirror M reflects the light down to a rotating mirror, which in turn reflects it, drawn out as a wave, on the viewing screen. It is often desired to obtain a photographic record of the phenomena which occur. For this purpose, a sensi-

tive photographic film is wound on the film drum, which is driven by a motor. The mirror M is then pulled up out of the way, and a mechanism causes the shutter to open and close during one revolution of the drum. In this case, the time axis is furnished by the movement of the film.

The oscillograph vibrators are connected into the circuit in the same manner as direct-current ammeter and voltmeter coils are connected (Fig. 89). As the current vibrator can carry but a small current—about 0.1 amp.—it is connected in parallel with a non-inductive shunt which is in series with the line. The

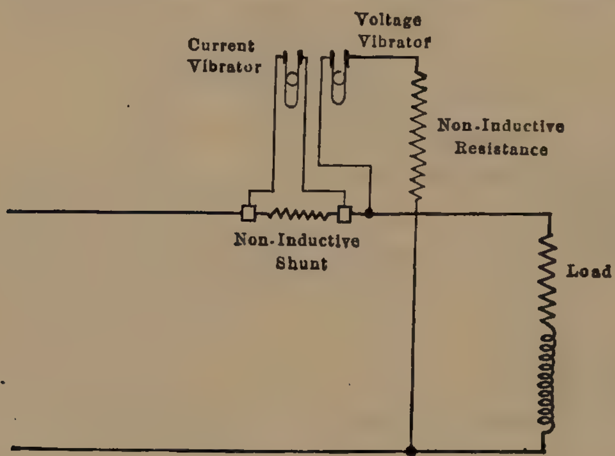


FIG. 89.—Method of connecting oscillograph vibrators in circuit.

voltage vibrator is connected across the line in series with a high non-inductive resistance. The current vibrator will then vibrate with an amplitude proportional to the circuit current and in phase with it. The current through the voltage vibrator will be proportional to the circuit voltage and in phase with it.

65. Impedance Bridge.—Impedances may be measured with a bridge in the same manner that resistances are measured by direct current with the Wheatstone bridge (Vol. I, p. 156). The usual connections are shown in Fig. 90. The unknown impedance Z_x, whose resistance is R_x and whose inductance is L_x, forms one arm of the bridge. Two of the arms M and N are non-inductive resistances. One arm, such as M, should be variable over a wide range. N may be adjustable to decimal values such as 1, 10, 100, etc., ohms. The fourth arm L of the bridge consists of a variable inductance standard or variometer

L, whose resistance is R_L. The variable resistance R may be connected into either bridge arm L or Z_x by moving the detector contact to either a or b. If the frequency is in the sensitive audio range, from 200 to 2,500 cycles, head phones T may be used as a detector. If low frequencies are used, such as from 20 to 100 cycles and more, a tuned vibration galvanometer may be used as a detector. If the impedances and resistances remain constant, the bridge balance is independent of frequency.

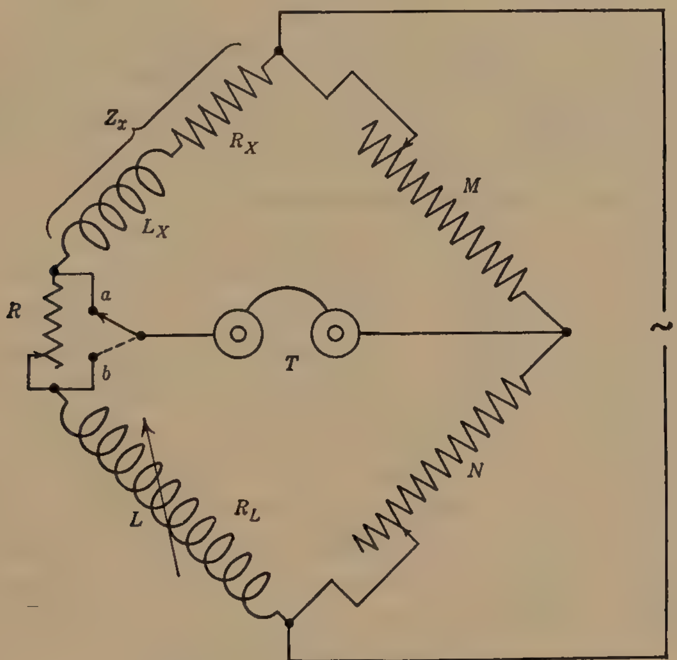


FIG. 90.—Impedance bridge.

If the bridge is in balance and the detector contact is at a,

$$\frac{L_x}{L} = \frac{M}{N} \text{ and } \frac{R_x}{R + R_L} = \frac{M}{N}; \tag{56}$$

if the detector contact is at b,

$$\frac{L_x}{L} = \frac{M}{N} \text{ as before, and } \frac{R_x + R}{R_L} = \frac{M}{N}. \tag{57}$$

These equations show that the inductance balance is independent of any resistances in the Z_x, L side of the bridge. With the values of M and N which are necessary to balance the inductances, it may be impossible at the same time to balance the

resistances. Hence, it is necessary to be able to connect R in either arm and adjust it for a balance.

It is not necessary that L be variable. A balance may be obtained with L a fixed standard, by adjusting M, N, and R. The impedance Z_x may be a capacitive impedance C_x, R_x. A capacitance C in the arm L is then necessary for a balance (see Vol. I, p. 258). When the bridge is in balance,

$$\frac{C_x}{C} = \frac{N}{M}.$$

Obviously, the positions of the alternating-current supply and the detector may be interchanged. There are many modifications of this bridge.[1]

[1] See F. A. Laws, "Electrical Measurements."

CHAPTER V

POLYPHASE SYSTEMS

66. Reasons for Use of Polyphase Systems.—In many industrial applications of alternating current, there are objections to the use of single-phase power.

In a single-phase circuit, the power delivered is pulsating. Even when the current and voltage are in phase, the power is zero twice in each cycle, as shown in Fig. 20 (p. 23). When the power factor is less than unity, the power is not only zero four times in each cycle, but it is also *negative* twice in each cycle. This means that the circuit returns power to the generator for a part of the time. This is analogous to a single-cylinder gasoline engine in which the flywheel returns energy to the cylinder during the compression part of the cycle. Over the complete cycle, both the single-phase circuit and the flywheel receive an excess of energy over that which they return to the source. The pulsating nature of the power in single-phase circuits makes such circuits objectionable in many instances.

A polyphase circuit is somewhat like a multicylinder gasoline engine. With the engine, the power delivered to the flywheel is practically steady, as one or more cylinders are firing when the others are compressing. This same condition exists in polyphase electrical systems. Although the power of any one phase may be negative at times, the *total power* is constant if the loads are balanced. This makes polyphase systems highly desirable for power purposes.

The rating of a given motor, or generator, increases with the number of phases—an important consideration. Below are the approximate capacities of a given machine for different numbers of phases, assuming the single-phase capacity as 100.

Single-phase	100
Two-phase	140
Three-phase	148
Six-phase	148
Direct-current	154

The same machine operating three-phase or six-phase has about 50 per cent. greater capacity than when operating single-phase. A machine has the same capacity whether connected three-phase or six-phase, because the same windings are used in the same manner for each. (The foregoing table does not apply to synchronous converters. The ratio of polyphase to single-phase capacity in converters is much greater than that shown in the above table. See p. 401.)

In single-phase synchronous machines, a pulsating armature reaction sets up eddy currents in the field structure, causing heating. Such reaction is negligible in polyphase machines with balanced loads.

A minor consideration in favor of three-phase power is the fact that with a fixed voltage between conductors, the three-phase system requires but three-fourths the weight of copper of a single-phase system, other conditions such as distance, power loss, etc., being fixed.

67. Symbolic Notation.—The solutions of problems involving circuits and systems containing a number of currents and volt-

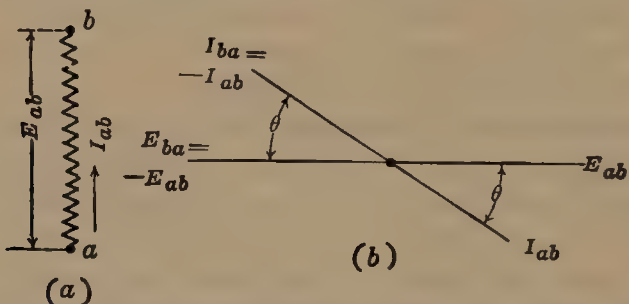


FIG. 91.—Symbolic notation applied to voltage and current vectors.

ages are simplified and are less susceptible to error if the current and voltage vectors are designated by some systematic notation, of which the following is one type: If a voltage is acting to send current from point a to point b (Fig. 91 (a)), it shall be denoted by E_{ab}. On the other hand, if the voltage tends to send current from b to a, it shall be denoted by E_{ba}. Obviously, $E_{ab} = -E_{ba}$. It may seem as if alternating currents cannot be considered as having direction, since they are undergoing continual reversal in direction. The assumed direction of a current, however, is determined by the actual direction of the flow of

energy. In an alternator, the energy comes *out* of the armature and the current is considered as flowing *out* of the armature, even although it is actually flowing *into* the armature for half the time.

Corresponding to the voltage E_{ab} (Fig. 91 (a)), the current I_{ab} flows from a to b in virtue of this voltage. The current flowing from b to a must be opposite in direction to that flowing from a to b. Therefore, $I_{ba} = -I_{ab}$. This relation is illustrated in Fig. 91 (b), in which I_{ab} differs in phase from I_{ba} by 180°. E_{ba} is 180° from E_{ab}.

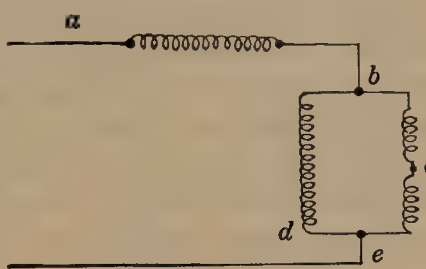


FIG. 92.—Circuit network.

Figure 92 represents a circuit network *abcde*. The parts of the network, *ab*, *bc*, etc., may be resistances, inductances, capacitances, or sources of e.m.f. such as alternator or transformer coils. It is obvious that the voltage from a to c is equal to the voltage from a to b plus the voltage from b to c. That is, $E_{ac} = E_{ab} + E_{bc}$. It is to be noted that when several voltages in series are being considered, the first letter of each subscript must be the same as the last letter of the preceding subscript. Figure 93 (a)

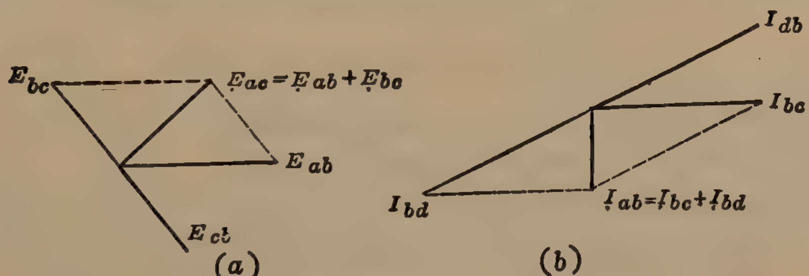


FIG. 93.—Examples of symbolic notation.

shows vectorially the voltage E_{ab} and the voltage E_{cb}. To obtain the voltage E_{ac}, E_{bc} is necessary. E_{cb} is reversed, therefore, giving E_{bc}. E_{bc} added vectorially to E_{ab} gives E_{ac}.

Currents may be treated in a similar manner, the principle involved being Kirchhoff's first law. For example, in Fig. 92, the current $I_{ab} = I_{bc} + I_{bd}$. Figure 93 (b) shows currents I_{bc} and I_{db}. I_{db} is reversed, giving I_{bd}, and this is combined vectorially with I_{bc} to obtain I_{ab}.

If all the currents at a junction are written on the same side of the equation, either the first letter or the last letter of every subscript is the same. For example, at any junction c,

$$I_{ac} + I_{bc} + I_{dc} + I_{ec} = 0$$

or

$$I_{ca} + I_{cb} + I_{cd} + I_{ce} = 0.$$

This notation not only distinguishes the various currents and voltages but the directions in which they act as well. By observing the foregoing rules as to the order of subscripts, a check on the accuracy of the equation is obtained. It is to be noted that the use of arrows is not necessary, the subscripts denoting the directions of the vectors.

68. Generation of a Three-phase Current.—As three-phase is now the most common of the polyphase systems, it will be

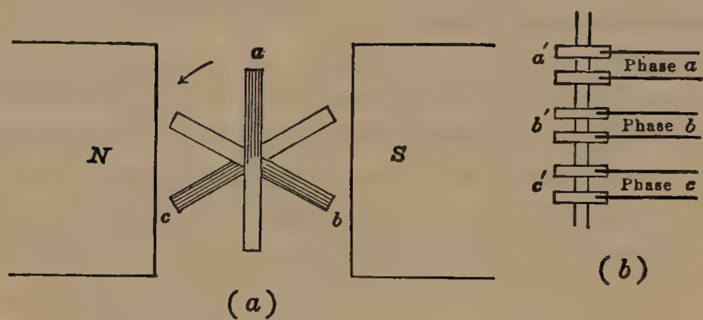


FIG. 94.—Generation of three-phase current.

considered first. Figure 94 (a) shows three simple coils, a, b, and c, 120° apart and fastened rigidly together.

The shaded sides of coils b and c correspond to the shaded side of a. That is, if one end of the winding of a is brought out of the shaded side of a toward the observer, the same is correspondingly true of b and c. It is important to keep this in mind.

These coils are mounted on an axis which can be rotated. The coils are shown rotating in a counterclockwise direction in a uniform magnetic field. The current can be conducted from each of these three coils by means of slip-rings, as shown in Fig. 94 (b). The terminals of coil a are connected to rings a', those of b to rings b', etc., making six slip-rings in all.

Figure 95 (a) shows as E_{oa} the voltage in coil a. E_{oa} is zero and is increasing in a positive direction when the time t is 0.

Obviously, the voltage induced in coil b will be 120 electrical time-degrees behind E_{oa}, and that induced in coil c will be 240 electrical time-degrees behind E_{oa}, as shown in Figs. 95 (a) and 95 (b). These three voltages constitute the elementary voltages generated in a three-phase system.

Their equations are as follows:

$$e_{oa} = \sqrt{2}E \sin \omega t$$
$$e_{ob} = \sqrt{2}E \sin (\omega t - 120°)$$
$$e_{oc} = \sqrt{2}E \sin (\omega t - 240°)$$

where E is the effective value of the three voltages.

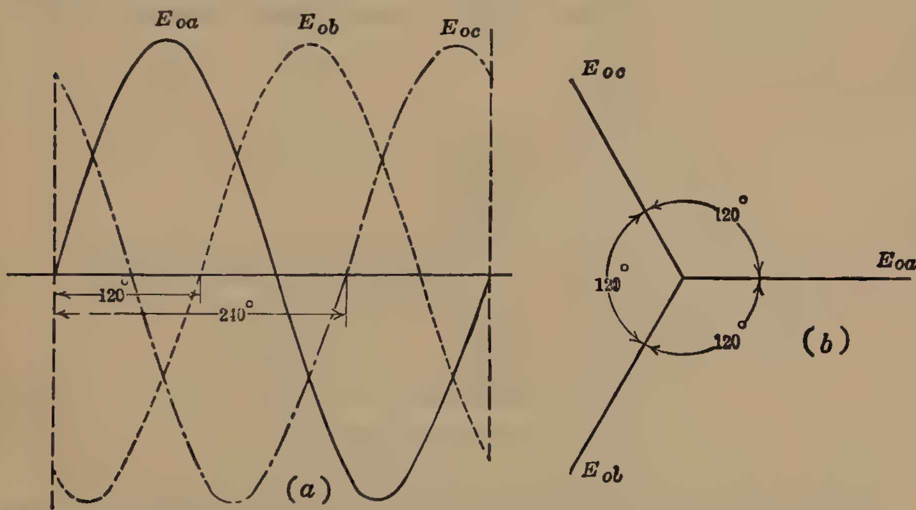


Fig. 95.—Three-phase voltage waves and vector diagram.

An examination of Fig. 95 (a) shows that for any particular instant of time, the algebraic sum of these three voltages is zero. When one voltage is zero, the other two are 86.6 per cent. of their maximum values and have opposite signs. When any one voltage wave is at its maximum, each of the others has the opposite sign to this maximum, and each is 50 per cent. of its maximum value.

Figure 95 (b) shows the vectors representing these three voltages, the vectors being 120° apart.

Each of the coils of Fig. 94 (a) can be connected through its two slip-rings to a single-phase circuit. This gives six slip-rings and three independent single-phase circuits. With a rotating

field and stationary armature type of generator, which is the most common type meet in practice, the six slip-rings would not be necessary, but six leads would be taken directly from the armature.

In practice, however, a machine seldom supplies three independent circuits by the use of six wires.

69. Y-connection.—The three coils of Fig. 94 are shown in simple diagrammatic form in Fig. 96. The three corresponding ends, one for each coil, are connected at the common point *o*. This is called the Y-connection of the coils. Ordinarily, only three wires, *aa′*, *bb′*, and *cc′*, lead to the external circuit, although the neutral wire *oo′* is sometimes carried along, making a three-phase, four-wire system.

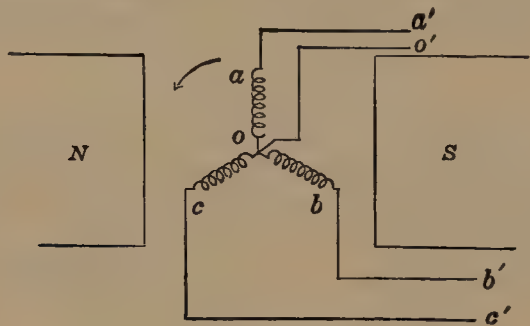


FIG. 96.—Y-connection of generator coils.

Figure 97 (*a*) again shows the three coils, and Fig. 97 (*b*) the three corresponding voltage vectors, E_{oa}, E_{ob}, and E_{oc}. These three voltages are called the *coil* or *Y-voltages*. Let it be required to find the three line voltages E_{ab}, E_{bc}, and E_{ca}. The line voltage $E_{ab} = E_{ao} + E_{ob}$ (Par. 67). E_{ao} is not on the original diagram but is obtained by reversing E_{oa}. E_{ao} is then added vectorially to E_{ob}, giving E_{ab}.

From geometry, E_{ab} lags the coil voltage E_{ob} by 30° and is 150° behind E_{oa}. Also, E_{ab} is numerically equal to $\sqrt{3}\ E_{ob} =$ 1.732 E_{ob}. In a similar manner, $E_{bc} = E_{bo} + E_{oc}$ and $E_{co} =$ $E_{co} + E_{oa}$. These three line voltages are shown in Fig. 98.

It is to be noted that in a balanced Y-system, the three line voltages are equal and differ in phase by 120°. *Each line voltage is* 30° *out of phase with one of its respective coil voltages. The three line voltages are each* $\sqrt{3}$, *or* 1.732 *times the coil voltage.*

It is obvious from Fig. 97 (a) that the three coil currents I_{oa}, I_{ob}, and I_{oc} are, respectively, equal to the three line currents $I_{aa'}$, $I_{bb'}$, and $I_{cc'}$, as the coil and line are in series.

Therefore, in a Y-system the line currents and the respective coil currents are equal. As the three coils meet at a common point, moreover, the vector sum of the three currents must be zero by Kirchhoff's first law, provided that there is no neutral conductor and current. That is,

$$\dot{I}_{oa} + \dot{I}_{ob} + \dot{I}_{oc} = 0.$$

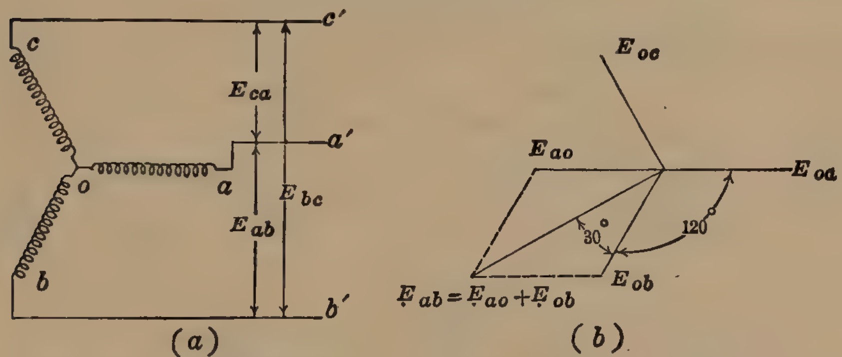


FIG. 97.—Y-connection and corresponding voltage vector diagram.

Power in Y-system.—Figure 98 shows the three currents I_{oa}, I_{ob}, and I_{oc} of coils oa, ob, and oc, respectively. Unity power factor is assumed, and the three currents are, therefore, in phase with their respective coil voltages. A balanced system is assumed, and the three currents are, therefore, equal in magnitude.

The coil current I_{oa} and the line current $I_{aa'}$ are the same current. The line current $I_{aa'}$, therefore, is 30° out of phase with the line voltage E_{ca}, when the power factor is unity. This is true for each phase.

The power delivered by each coil is

$$P' = E_{oa}I_{oa} \text{ (unity power factor)}$$

and the total power delivered by the generator is three times this.

$$P = 3E_{coil}I_{coil}$$

As the power in the line is the same as that delivered by the generator, substituting $E_{line}/\sqrt{3}$ for the value of E_{coil},

$$P = \frac{3}{\sqrt{3}} E_{line}I_{coil} = \sqrt{3}E_{line}I_{line}, \tag{58}$$

the coil current and the line current being equal.

In a balanced three-phase system, the line power at unity power factor is equal to $\sqrt{3}$ *times the line voltage times the line current.*

Figure 99 shows this same three-phase system when the power factor is no longer unity. Each coil current now lags its respective coil voltage by the angle θ.

The total coil power is now

$$P = 3E_{coil}I_{coil}\cos\theta_{coil}.$$

The system power is

$$P = \sqrt{3}E_{line}I_{line}\cos\theta_{coil} \tag{59}$$

and the system kilowatts are equal to

$$\frac{\sqrt{3}}{1{,}000}E_{line}I_{line}\cos\theta_{coil}. \tag{60}$$

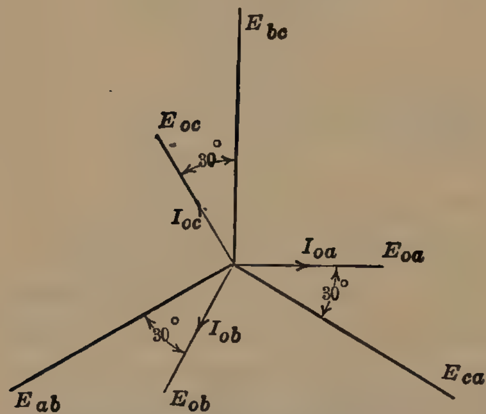


FIG. 98.—Relation of line to coil voltages and currents in a Y-system, unity power factor.

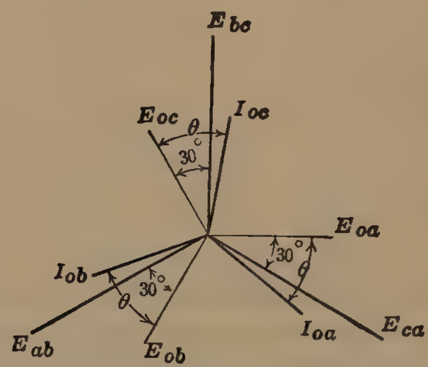


FIG. 99.—Relation of line to coil voltages and currents in a Y-system. Power factor = cos θ.

Therefore, in a balanced three-phase system, the system power factor is the cosine of the angle between the coil current and the coil voltage.

The angles between the line currents and the line voltages are not power-factor angles, for they involve the factors $(\theta - 30°)$ (Fig. 99) and, also, $(\theta + 30°)$, θ being the coil power-factor angle.

Obviously, the system power factor, which is the coil power factor, is

$$\text{P.F.} = \frac{P}{\sqrt{3}E_{line}I_{line}} \tag{61}$$

where P is the total system power.

If the system is unbalanced, that is, if the currents or voltages are not equal or do not differ in phase by 120°, the question arises as to just what the system power factor is under these conditions. Where such unbalancing is not very great, Eq. (61) is used, the line currents and voltages being averaged. The system power factor has little significance when the unbalancing is considerable.

Example.—A three-phase alternator has three coils each rated at 1,330 volts and 150 amp. What is the voltage, kilovolt-ampere, and current rating of this generator if the three coils are connected in Y?

$$E_{line} = \sqrt{3} \times 1{,}330 = 2{,}300 \text{ volts. } \textit{Ans.}$$
$$\text{Rating} = \sqrt{3} \times 2{,}300 \times 150 = 600 \text{ kv-a. } \textit{Ans.}$$
$$\text{Current rating} = 150 \text{ amp. } \textit{Ans.}$$

70. Delta-connection.—The three coils of Fig. 94 can be connected as shown in Fig. 100 (a), the diagram being simplified in

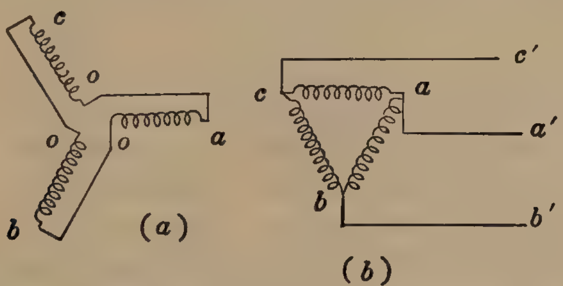


FIG. 100.—The delta-connection of alternator coils.

Fig. 100 (b). The end of each coil, which, in Fig. 96, was connected to the neutral, is now connected to the outer end of the next coil, as shown in Fig. 100 (a). As points o and a are now connected directly together, $E_{co} = E_{ca}$, etc., the o's are now superfluous and are dropped.

Figure 101 (a) shows vectorially the three voltages E_{ab}, E_{bc}, and E_{ca}, acting from a to b, b to c, and c to a, respectively.

At first sight, Fig. 100 looks like a short circuit, the three coils each containing a source of voltage, being short circuited on themselves. The actual conditions existing in this closed circuit may be demonstrated by the use of the subscript notation. Assume that the coil bc is broken at c' (Fig. 102 (a)). The voltage $\dot{E}_{bc} = \dot{E}_{ba} + \dot{E}_{ac}$. The vector sum of these two voltages, shown in Fig. 102 (b), lies along voltage $E_{bc'}$ and is equal to it.

The voltage $E_{c'c} = 0$, therefore, and points c and c' can be connected without any resulting flow of current. This is the same condition which exists when two direct-current generators having equal voltages are connected in parallel. No current flows between the two if the proper polarity is observed.

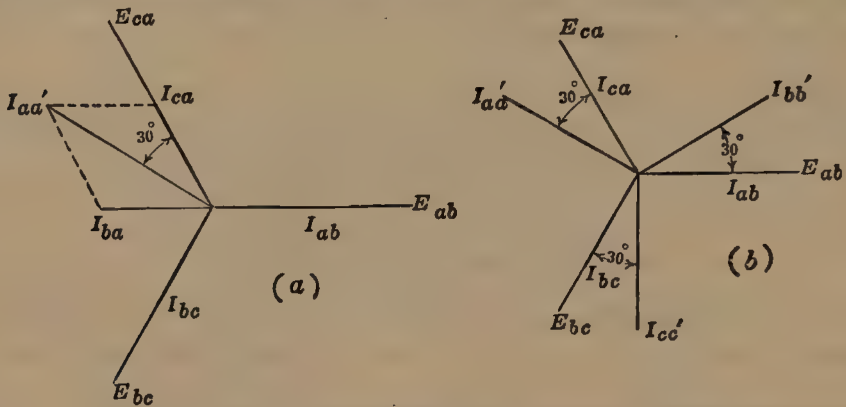


FIG. 101.—Relation of line to coil voltages and currents in delta-system, unity power factor.

Also, since the three voltages E_{ba}, E_{ac}, and E_{cb} ($= E_{oa}$, E_{oc}, and E_{ob}, Fig. 100 (a)) are in series, it follows, from Fig. 95 (a), p. 107 that their sum at every instant is zero.

The coil currents of Fig. 100 are shown in Fig. 101 as being in phase with their respective voltages, balanced conditions being assumed. The line current

$$I_{aa'} = I_{ba} + I_{ca}.$$

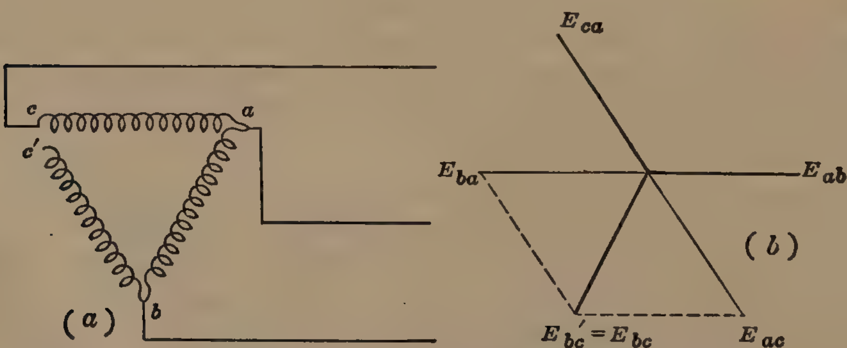
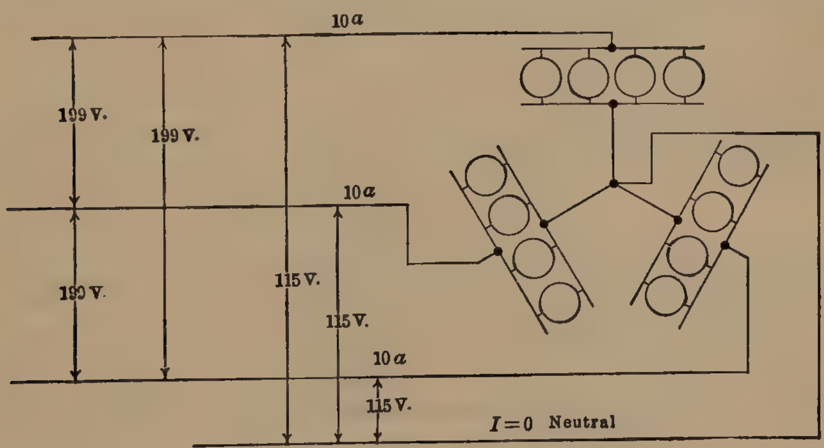


FIG. 102.—Showing that sum of three delta voltages is zero.

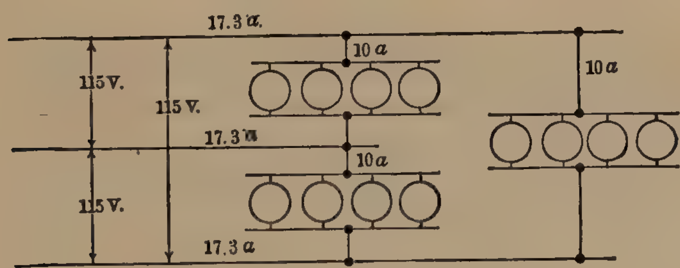
This addition is made vectorially in Fig. 101 (a), giving $I_{aa'}$, differing in phase from E_{ca} by 30°. It will be observed that $I_{aa'}$ is $\sqrt{3}$ times the coil current. Line currents $I_{bb'}$ and $I_{cc'}$ may be

found in a similar manner, with the result shown in Fig. 101 (*b*). In the delta-system, therefore, there is a phase difference of 30° between the line currents and the line voltages at unity power factor, just as in the Y-system.

It is obvious that the line voltage is equal to the coil voltage in a delta-system. The sum, moreover, of the three *voltages* acting around the delta must be zero by Kirchhoff's second law.



115-Volt Lamp Banks Connected in Y



115-Volt Lamp Banks Connected in Delta

FIG. 103.—Lamp loads in Y and in delta.

In a balanced delta-system, the line voltage is equal to the coil voltage, but the line current is $\sqrt{3}$ *times the coil current.*

Figure 103 shows three lamp loads, each requiring 10 amp. at 115 volts. They are first connected in Y and then in delta. In order to supply the proper voltage in each case, there are 199 volts across lines in the Y-system and 115 volts in the delta-system. There are 10 amp. per line in the Y-system and 17.3

amp. per line in the delta-system. The power supplied is the same in each system.

Power in Delta-system.—The total power in a delta-system is

$$P = 3E_{coil}I_{coil} \cos \theta_{coil}. \tag{62}$$

This power is equal to that in the line, as there is no intervening loss. Also, the line current

$$I_{line} = \sqrt{3}I_{coil}$$

and

$$E_{line} = E_{coil}.$$

Hence, substituting in Eq. (62),

$$P = \sqrt{3}\, E_{line}I_{line} \cos \theta_{coil}. \tag{63}$$

This equation is the same as Eq. (59) (p. 110) for the Y-system. This should be so, for the relations in a three-phase line are the same whether the power originates in a delta- or in a Y-connected generator.

The power factor of the delta-system is the same as that for a Y-system.

$$\text{P.F.} = \frac{P}{\sqrt{3}EI} = \cos \theta_{coil} \tag{64}$$

where P is the total power of the system, and E and I are the line voltage and line current, respectively.

The denominator $\sqrt{3}EI$ (Eq. (64)) gives the *volt-amperes* of the three-phase system. The *kilovolt-amperes* of a three-phase system are given by $\sqrt{3}EI/1{,}000$.

METHODS OF MEASURING POWER IN THREE-PHASE SYSTEM

71. Three-wattmeter Method.—Let (1), (2), and (3) (Fig. 104 (*a*)) be the three coils of either a Y-connected alternator or a Y-connected load. If the neutral of the Y is accessible, it is possible to measure the power of each phase by connecting the current coil of a wattmeter in series with the phase and by connecting the wattmeter potential coil across the phase, as shown in Fig. 104 (*a*). W_1, W_2, and W_3, therefore, measure the power in loads 1, 2, and 3, respectively, regardless of power factor, degree of balance, etc.

The total power

$$P = W_1 + W_2 + W_3$$

If the loads are balanced,

$$W_1 = W_2 = W_3.$$

If the potential circuits of the three wattmeters have equal resistances, these three potential circuits constitute a balanced Y-load, having a neutral O'. As coils 1, 2, and 3 and these three wattmeter potential circuits are both symmetrical systems, O' must be at the same potential as O. No current flows, therefore, between O and O', and the line can be cut at X without changing existing conditions. Figure 104 (*b*) shows the three-wattmeter connection for a three-phase system. It can be shown that the

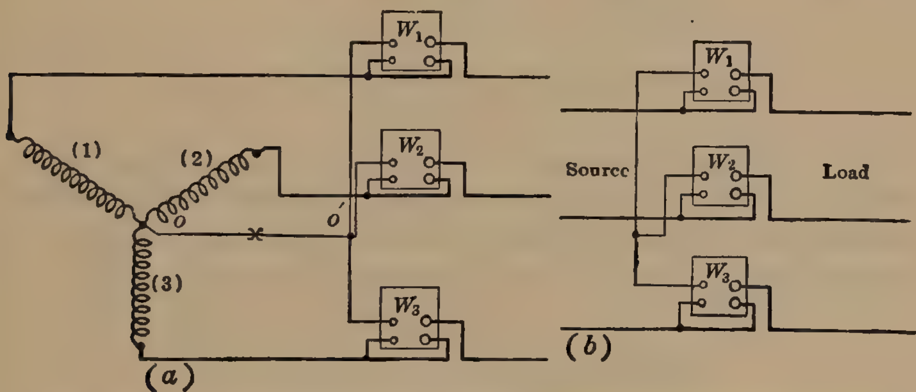


FIG. 104.—The three-wattmeter method of measuring three-phase power.

total power is the sum of the wattmeter readings even though the wattmeter potential circuits have different resistances. Under these conditions, however, the wattmeters may not all have the same reading, even with balanced loads.

The three-wattmeter method is well adapted to measuring power in a system where the power factor is continually changing, as in obtaining the phase characteristics of a synchronous motor. If the three instruments have equal potential-circuit resistances, they read alike, regardless of power factor, if the loads are balanced. The three-wattmeter method is necessary in a three-phase, four-wire system, as a system of n wires ordinarily requires $n - 1$ wattmeters in order to measure the power correctly.

The Y-box.—The use of the Y-box is based on the principle that each of the three wattmeters of Fig. 104 reads the same, if the

loads are balanced. Under these conditions, the total power $P = 3W_1$. If two resistances, each equal to the resistance of the potential coil of W_1, be used in conjunction with this potential

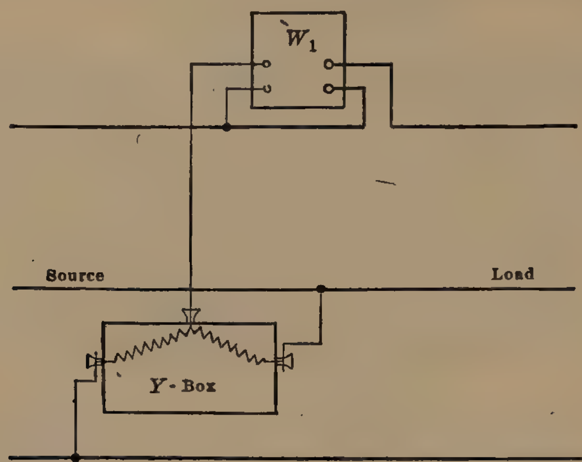


FIG. 105.—Use of the Y-box for measuring three-phase power.

coil, the wattmeters W_2 and W_3 are not necessary. As a rule, these two equal resistances are mounted in the same box and are connected as shown in Fig. 105. Accurate results can be obtained with this method only when the loads are balanced.

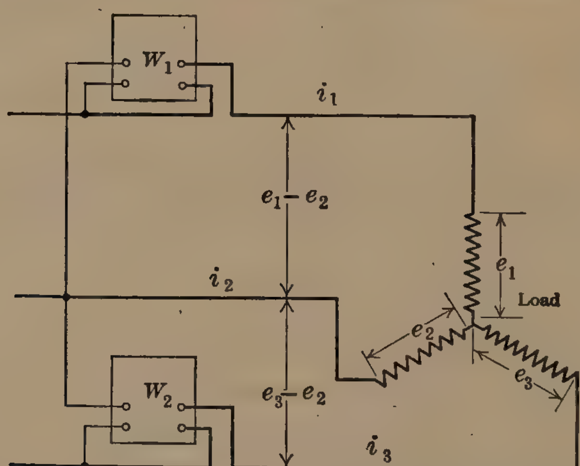


FIG. 106.—Two-wattmeter method of measuring three-phase power.

72. Two-wattmeter Method.—The power in a three-wire, three-phase system can be measured by two wattmeters connected as shown in Fig. 106. The current coils of the two instruments are connected in two of the lines, and the potential coil

of each instrument is connected from its own current coil to the line in which there is no current coil. Under these conditions, the total power passing through the system

$$P = W_1 \pm W_2$$

regardless of power factor, balance, etc. The choice of the plus or the minus sign will be explained later.

One method of proving that these instruments give the correct power is as follows: Let e_1, e_2, e_3, and i_1, i_2, i_3 be the voltages and currents of the three loads at any particular *instant*. These being *instantaneous* values, the power at the instant under consideration is equal to the sum of their products regardless of power factor. That is, the instantaneous power

$$p = e_1i_1 + e_2i_2 + e_3i_3$$

But

$$i_1 + i_2 + i_3 = 0 \text{ (Kirchhoff's first law)}$$
$$i_2 = -(i_1 + i_3)$$

Substituting,

$$\therefore p = e_1i_1 - e_2(i_1 + i_3) + e_3i_3$$
$$= (e_1 - e_2)i_1 + (e_3 - e_2)i_3.$$

As the line voltages in a Y-system are the *differences* of the proper coil voltages (p. 108, Par. 69),

$$W_1 \text{ reads } (e_1 - e_2)i_1$$

and

$$W_2 \text{ reads } (e_3 - e_2)i_3.$$

The same proof may be used for a delta-load, except that, in this case,

$$e_1 + e_2 + e_3 = 0.$$

It is shown, in Pars. 69 and 70, that a phase difference of 30° exists between the line voltage and line current at unity power factor. For power factors other than unity, this phase difference becomes (30° $\pm$ θ), where θ is the power-factor angle of the coil.

Figure 107 (a) shows two wattmeters, W_1 and W_2, measuring the power taken by a balanced three-phase, Y-connected load. The wattmeter W_1 is so connected that the current I_{bo} flows in its current coil and the voltage E_{ba} is across its potential circuit. The reading of W_1, therefore, is equal to the product of I_{bo},

E_{ba}, and the cosine of the angle between this current and this voltage. Figure 107 (*b*) gives the vector diagram of the load. The three coil voltages E_{ao}, E_{bo}, and E_{co} are all equal and 120° apart. The coil currents I_{ao}, I_{bo}, and I_{co} are equal and lag their respective coil voltages by the angle θ. The voltage E_{ba} is found by reversing E_{ao}, giving E_{oa}, and then adding E_{bo} and E_{oa} vectorially ($\dot{E}_{ba} = \dot{E}_{bo} + \dot{E}_{oa}$). The current I_{bo} is given. The angle between E_{ba} and I_{bo} is $30° - \theta$. The reading of this wattmeter is, therefore,

$$\begin{aligned} W_1 &= E_{ba} I_{bo} \cos (30° - \theta) \\ &= E_{line}\, I_{line} \cos (30° - \theta). \end{aligned}$$

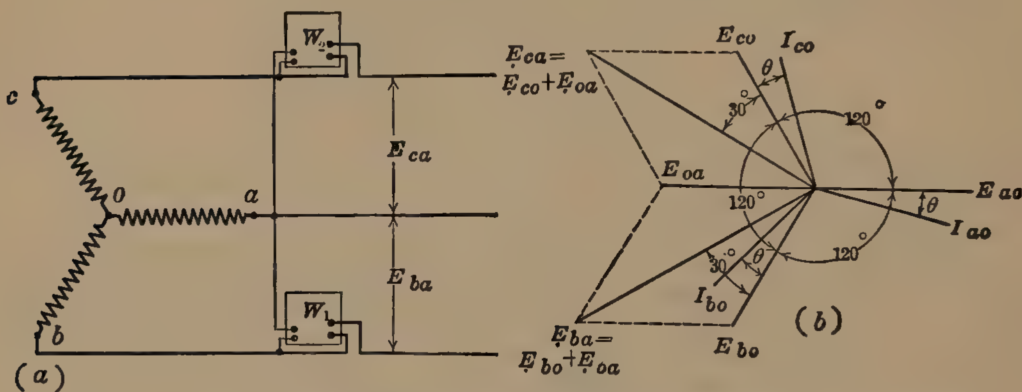


FIG. 107.—Vector diagram illustrating two-wattmeter method for measuring three-phase power, balanced load.

Likewise, the wattmeter W_2 reads the product of E_{ca}, I_{co}, and the cosine of the angle between them. From the vector diagram (Fig. 107 (*b*)), E_{ca} is found by adding vectorially E_{co} and E_{oa} ($\dot{E}_{ca} = \dot{E}_{co} + \dot{E}_{oa}$). The current I_{co} is given. The angle between E_{ca} and I_{co} is $30° + \theta$.

The reading of this wattmeter is, therefore,

$$\begin{aligned} W_2 &= E_{ca} I_{co} \cos (30° + \theta) \\ &= E_{line}\, I_{line} \cos (30° + \theta) \end{aligned}$$

Summarizing,

$$\begin{aligned} W_1 &= EI \cos (30° - \theta) \\ W_2 &= EI \cos (30° + \theta) \end{aligned}$$

where E and I are the line voltage and line current, respectively, the system being balanced.

W_1 and W_2 will read alike when $\theta = 0$ and $\theta = 180°$. Both conditions correspond to unity power factor. When θ equals 180°, however, the power has reversed. The two instruments also read alike at zero power factor ($\theta = 90°$), although this condition is seldom realized.

When $\theta = 60°$, corresponding to a power factor of 0.5 lag, W_2 reads zero, as $\cos(30° + 60°) = \cos 90° = 0$. In this case, the reading of W_1 gives the total power. For angles greater than 60°, corresponding to power factors less than 0.5, $\cos(30° + \theta)$ becomes negative, W_2 reads negative and the total power becomes

$$P = W_1 - W_2.$$

Discretion must be used, therefore, when two single instruments are employed, as the total power may be either the *sum* or the *difference* of the readings.

It may also be shown that

$$\tan\theta = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2} \tag{65}$$

where θ is the coil power-factor angle. It is possible, therefore, to obtain the power factor in a balanced three-phase system by means of the wattmeter readings alone.

Another convenient method for determining the power factor from the wattmeter readings for a balanced load is to divide the smaller wattmeter reading by the larger,

$$\frac{W_2}{W_1} = \frac{\cos(30° + \theta)}{\cos(30° - \theta)}. \tag{66}$$

The power factor corresponding to this ratio is obtained by substituting different values of θ and solving for $\cos\theta$. The power factors corresponding to different values of W_2/W_1 are plotted as abscissas, and $\cos\theta$ as ordinates (Fig. 108). For example, when $\theta = 30°$, $\cos 30° = 0.866$, and

$$\frac{W_2}{W_1} = \frac{\cos 60°}{\cos 0°} = 0.500.$$

By the use of Fig. 108, the power factor is read directly, the ratio W_2/W_1 being known. It is seen that when $W_2/W_1 = 1.0$, the power factor is 1.0; when $W_2/W_1 = 0$, the power factor is 0.5; when W_2/W_1 is negative, that is, when it becomes necessary to

reverse W_2, the power factor is less than 0.5. By means of a curve like that of Fig. 108, the power factor may be read directly from the ratio of the two wattmeter readings.

Example.—In a test of a three-phase induction motor, two wattmeters are used to measure the input. Their readings are 1,900 and 800 watts, respectively. Both instruments are known to be reading positive. What is the power factor of the motor at this load?

Using Eq. (65),

$$\tan \theta = \sqrt{3}\,\frac{1{,}900 - 800}{1{,}900 + 800} = \sqrt{3}\,\frac{1{,}100}{2{,}700} = 0.705$$

$$\theta = 35.3^\circ$$

$$\cos \theta = \cos 35.3^\circ = 0.815. \quad \textit{Ans.}$$

his result may be checked by Fig. 108.

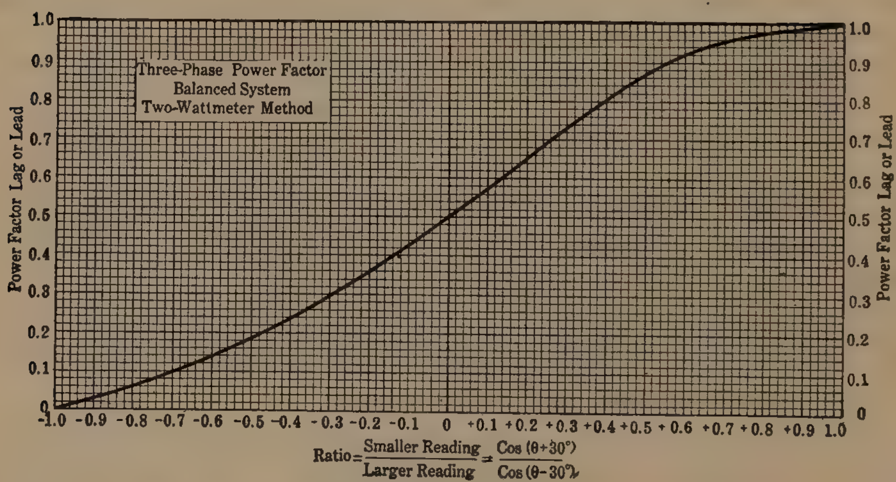


FIG. 108.—Power-factor diagram, two-wattmeter method.

If a polyphase wattmeter is used (p. 85, Fig. 73), the adding or subtracting is done automatically, as both elements of the instrument act on the same spindle. The polyphase instrument, therefore, if properly connected, reads the total power at all times.

The two-wattmeter method cannot be used to measure power in a three-phase, four-wire system unless the current in the neutral wire is zero. When the current in the neutral wire of Fig. 109 is zero, the power is correctly indicated by $W_1 \pm W_2$. Now apply load $B'O$ between line B and the neutral. The current to this load will complete its circuit from wire B through

the neutral without going through the current coil of either wattmeter. As neither wattmeter, therefore, indicates this additional load, the two wattmeters are not sufficient to measure the power in such a four-wire system under all conditions of load (also, see p. 129).

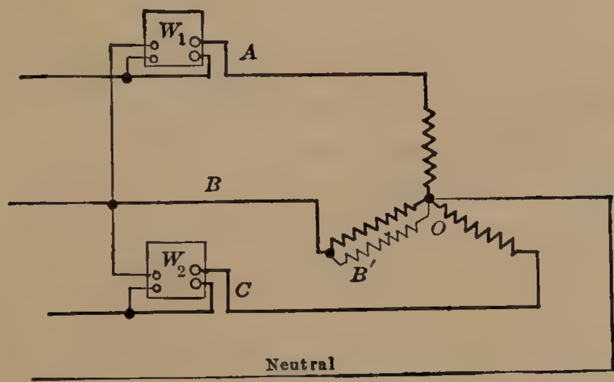


FIG. 109.—Two-wattmeter method generally not applicable to four-wire system.

TWO-PHASE SYSTEMS

73. Two-phase and Four-phase (Sometimes Called Quarter-phase) Systems.—Although three-phase systems are superseding other systems, there are still many two-phase and four-phase systems in existence. The two-phase system is rarely used for transmission but is used for distribution, and in some instances it is specially advantageous to use two-phase machines.

Two-phase current is generated in the elementary generator (Fig. 110 (*a*)) by two coils *A* and *B*, 90° apart. Figure 110 (*b*) shows the e.m.f. waves generated by these coils. The voltage of *A* leads that of *B* by 90°. When one voltage is a maximum, the other is zero. Figure 110 (*c*) shows these two-phase voltages represented vectorially (also, see Figs. 244 and 249, pp. 268 and 272).

The two phases may be carried along, insulated from each other, to supply two separate single-phase circuits, or they may supply a common load such as an induction motor (Fig. 111). The two phases are *entirely insulated* from each other in Fig. 111, and no single load can be applied between the two phases. Only one value of voltage is obtainable, moreover, as the voltages of the two phases are equal.

If, however, the generator coils be connected at their neutral points and a neutral conductor carried along with the other

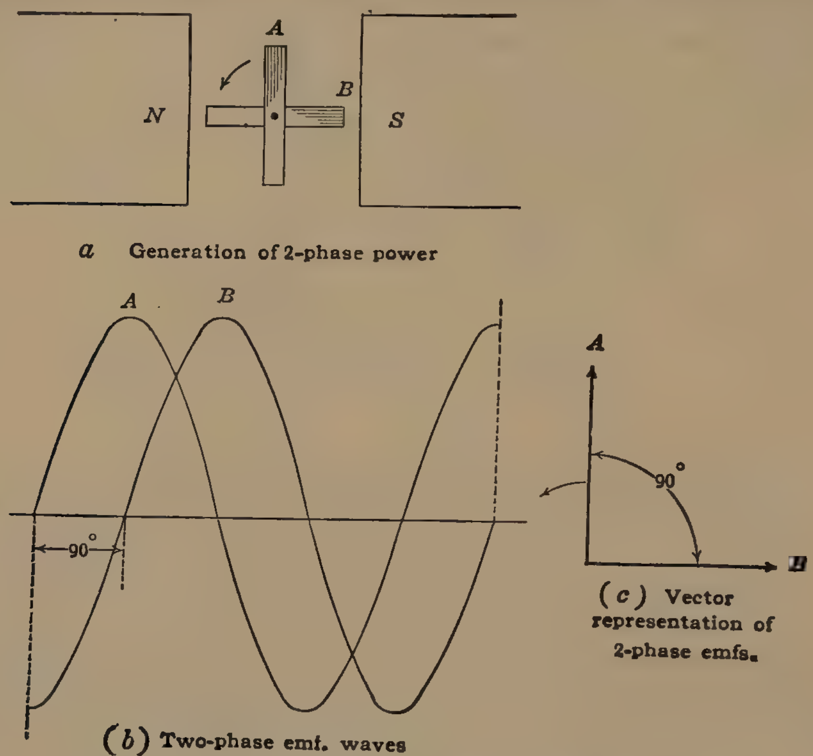


FIG. 110.—Phase relations of two-phase electromotive forces.

conductors, a four-phase or quarter-phase, five-wire system results, as shown in Fig. 112 (*a*). Three different voltages,

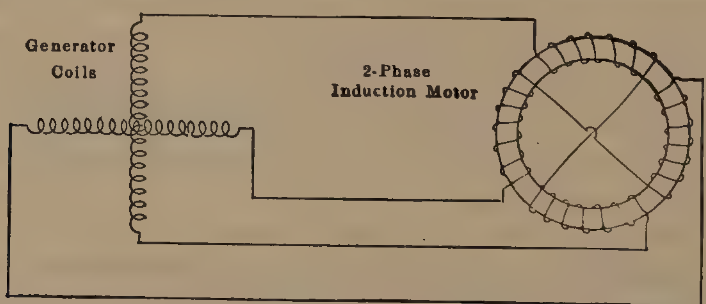


FIG. 111.—Two-phase circuit in which two phases are insulated.

moreover, are available. If the voltages between the outer wires of each phase be 200 volts, then 200, 100, and 141 volts are available, as shown in Fig. 112 (*b*). This system is more readily

unbalanced than the three-phase system, a fact which is an objection to its use. Another objection is the greater number of wires.

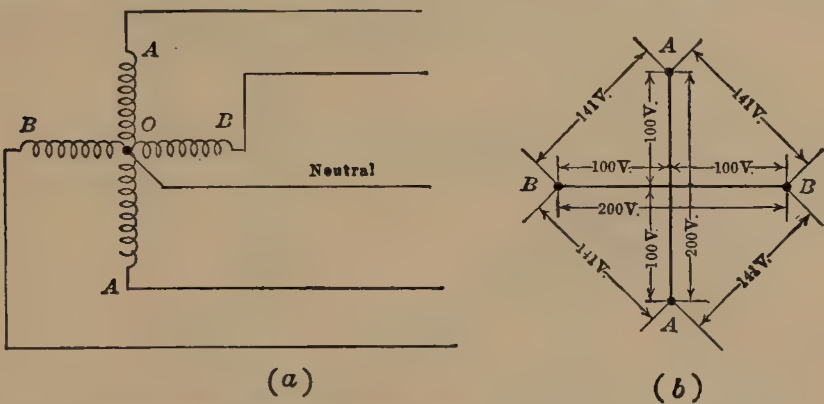


FIG. 112.—Two-phase interconnected system giving four-phase, five-wire system.

If one end of the coil A be connected to one end of the coil B, a three-wire, two-phase system results, as shown in Fig. 113. This gives two different values of voltage, 200 and 283 ($= 200 \sqrt{2}$) volts. This system is little used because of the considerable

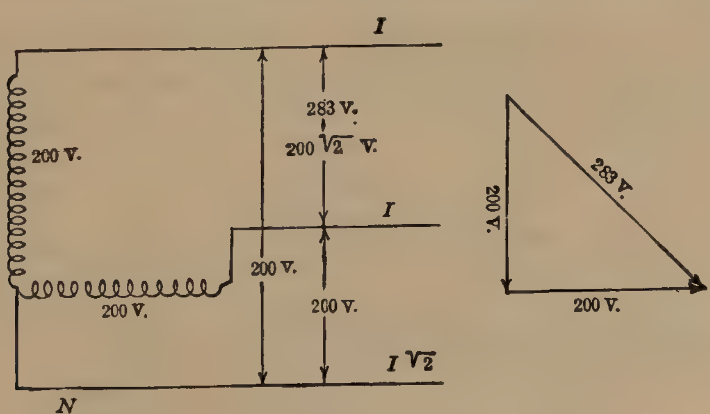


FIG. 113.—Two-phase, three-wire system.

amount of voltage unbalancing which results, even when moderate loads are applied. It should be noted that the common wire N carries a current $I\sqrt{2}$, where I is the current in each of the two outer wires. The wire N is not a true neutral conductor since its potential is not the center of gravity of the potentials of the system.

A two-phase or four-phase alternator may have a winding which consists of four coils. These coils may be connected in mesh, as shown in Fig. 114. This corresponds to the delta-

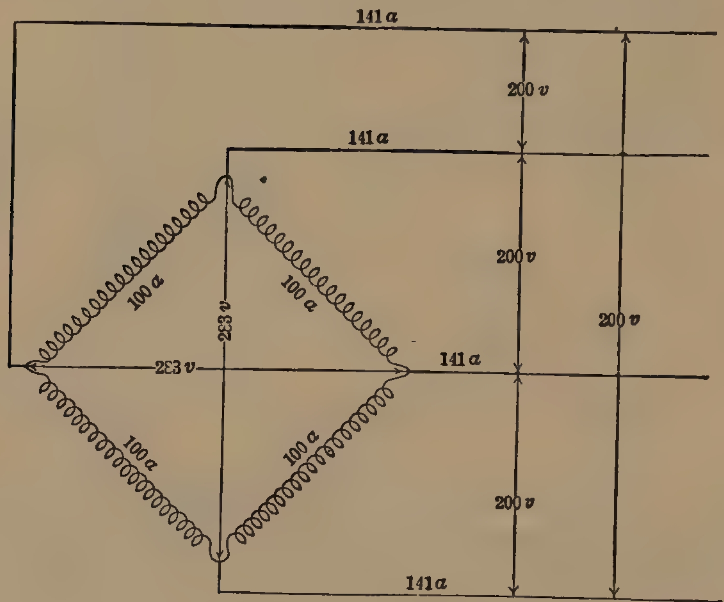


FIG. 114.—Mesh-connected, two-phase winding.

connection in a three-phase system. As in the case of the delta, if these coils are properly connected, the winding is not short circuited on itself.

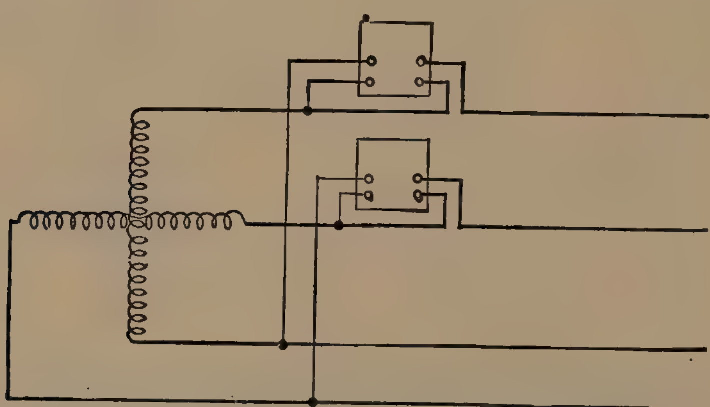


FIG. 115.—Measurement of power in insulated two-phase, four-wire system.

The line voltages are each equal to the coil voltage. The diametrical voltages are equal to $\sqrt{2}$ times the coil voltage. The line currents are equal to the $\sqrt{2}$ times the coil current,

because the line currents are the resultant of two equal currents having 90° phase difference.

In Fig. 114, the coil voltage is 200 volts, and the diametrical voltage is $200\sqrt{2} = 283$ volts. The coil current is 100 amp.

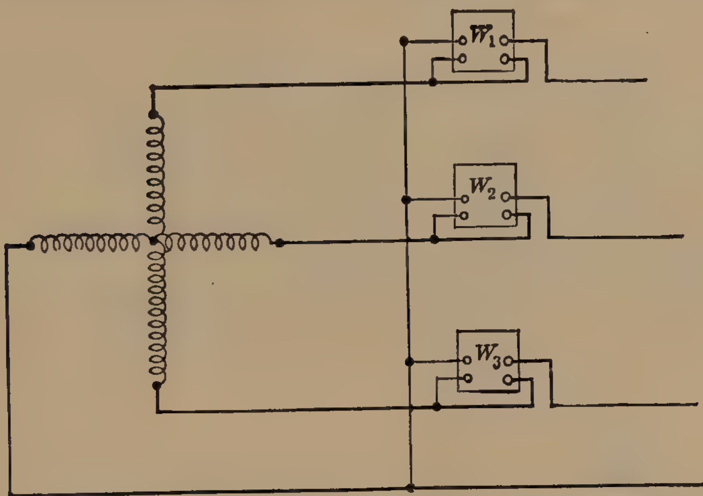


FIG. 116.—Measurement of power in four-phase or two-phase interconnected system.

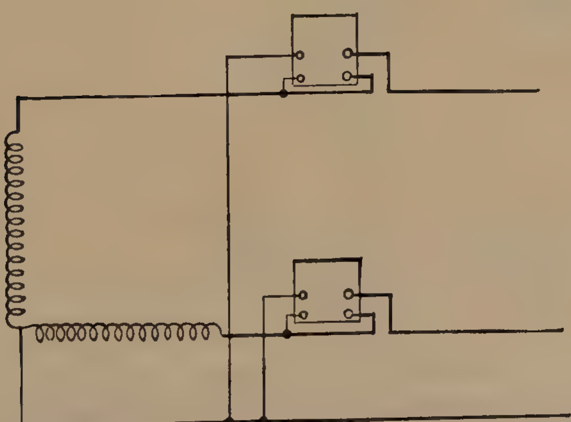


FIG. 117.—Measurement of power in two-phase, three-wire system.

and the line current is 141 amp. The total kilovolt-ampere capacity of this system is

$$\frac{4 \times 200 \times 100}{1{,}000} = 80 \text{ kv-a.}$$

Measurement of Power in Two-phase Systems.—In a two-phase, four-wire system, connected as shown in Fig. 115, the total power may be measured by two wattmeters. If the system is interconnected, the loads must be balanced or this method is incorrect. If, however, the two phases are insulated from each other, two wattmeters measure the power correctly regardless of unbalance, power factor, etc.

If the loads of a four-wire interconnected system are not balanced, three wattmeters must be used, as shown in Fig. 116. The power is the algebraic sum of their readings. The power in a two-phase, three-wire system may be measured by two wattmeters connected as shown in Fig. 117.

74. Applications of Complex Algebra to Polyphase Circuits.—The solution of polyphase networks is frequently not difficult if

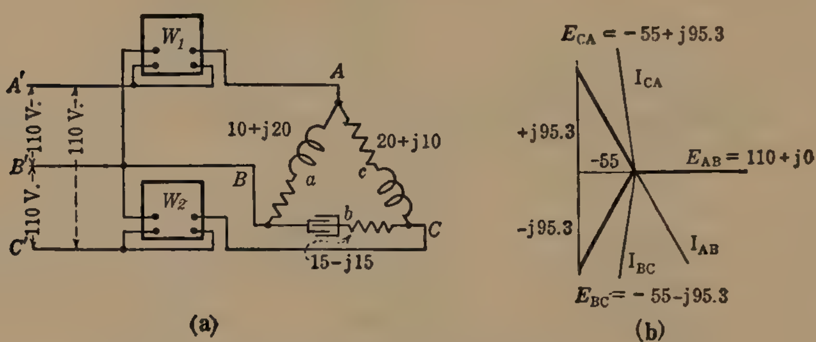


FIG. 118.—Unbalanced delta loads and vector diagram.

complex algebra is used. The methods of solving such problems is best illustrated by actual numerical examples.

Example 1.—Three impedances a, b, and c (Fig. 118 (a)), having resistances of, 10, 15, and 20 ohms and reactances of $+20$, -15, and $+10$ ohms, are connected in delta across the three lines $A'A$, $B'B$, and $C'C$ of a 110-volt, three-phase, 60-cycle system. Impedance a is connected between conductors A-B, impedance b between conductors B-C, and impedance c between conductors C-A. The sequence of phase rotation is AB, BC, CA. A wattmeter W_1 is connected with its current coil in line $A'A$ and its potential coil between lines A and B; wattmeter W_2 is connected with its current coil in line $C'C$ and its potential coil between lines C and B. Determine: (a) the current in each impedance; (b) the current in each line conductor; (c) the reading of each wattmeter.

(a) Assume that E_{AB} lies along the axis of reals (Fig. 118 (b)). Hence,

$$\dot{E}_{AB} = 110 + j0 = 110\backslash\underline{0^\circ}$$
$$\dot{E}_{BC} = -55 - j95.3 = 110/\overline{120^\circ}$$
$$\dot{E}_{CA} = -55 + j95.3 = 110\backslash\underline{120^\circ}$$
$$\dot{I}_a = \dot{I}_{AB} = \frac{100 + j0}{10 + j20} = \frac{110 + j0}{10 + j20} \cdot \frac{10 - j20}{10 - j20} = \frac{1{,}100}{500} - j\frac{2{,}200}{500}$$
$$= 2.20 - j4.40 \text{ amp.}$$
$$|I_a| = \sqrt{(2.20)^2 + (4.40)^2} = 4.92 \text{ amp.} \quad \textit{Ans.}$$
$$\dot{I}_b = \dot{I}_{BC} = \frac{-55 - j95.3}{15 - j15} = \frac{-55 - j95.3}{15 - j15} \cdot \frac{15 + j15}{15 + j15} = \frac{+602}{450} - j\frac{2{,}254}{450}$$
$$= +1.338 - j5.01 \text{ amp.}$$
$$|I_b| = \sqrt{(1.338)^2 + (5.01)^2} = 5.18 \text{ amp.} \quad \textit{Ans.}$$
$$\dot{I}_c = \dot{I}_{CA} = \frac{-55 + j95.3}{20 + j10} = \frac{-55 + j95.3}{20 + j10} \cdot \frac{20 - j10}{20 - j10} = \frac{-147 + j2{,}456}{500}$$
$$= -0.29 + j4.91$$
$$|I_c| = \sqrt{(0.29)^2 + (4.91)^2} = 4.92 \text{ amp.} \quad \textit{Ans.}$$

(b) $\dot{I}_{A'A} = \dot{I}_{AB} + \dot{I}_{AC} = (2.20 - j4.40) + (0.29 - j4.91) =$
$2.49 - j9.31$ amp.

$$|\dot{I}_{A'A}| = \sqrt{(2.49)^2 + (9.31)^2} = 9.64 \text{ amp.} \quad \textit{Ans.}$$

$\dot{I}_{B'B} = \dot{I}_{BC} + \dot{I}_{BA} = (1.338 - j5.01) + (-2.20 + j4.40) =$
$-0.88 - j0.61$ amp.

$$|\dot{I}_{B'B}| = \sqrt{(0.88)^2 + (0.61)^2} = 1.07 \text{ amp.} \quad \textit{Ans.}$$

$\dot{I}_{C'C} = \dot{I}_{CA} + \dot{I}_{CB} = (-0.29 + j4.91) + (-1.338 + j5.01) =$
$-1.628 + j9.92$ amp.

$$|\dot{I}_{C'C}| = \sqrt{(1.628)^2 + (9.92)^2} = 10.05 \text{ amp.} \quad \textit{Ans.}$$

(c) The current in wattmeter W_1 is $\dot{I}_{A'A}$, and its potential circuit is connected across $\dot{E}_{AB}$. $\dot{I}_{A'A} = 2.49 - j9.31$ and $\dot{E}_{AB} = 110 + j0$. Hence, by Par. 43

$$W_1 = 110 \times 2.49 = 274 \text{ watts} \quad \textit{Ans.}$$

The current in W_2 is $\dot{I}_{C'C}$, and its potential circuit is connected across E_{CB}. $\dot{I}_{C'C} = -1.628 + j9.92$ and $E_{CB} = +55 + j95.3$.
Hence,

$$W_2 = +55 \times (-1.628) + (95.3 \times 9.92) = 855.5 \text{ watts.} \quad \textit{Ans.}$$

The total power,

$$P = W_1 + W_2 = 1{,}130 \text{ watts.} \quad \textit{Ans.}$$

Since the wattmeters are connected to measure the power by the two-wattmeter method, 1,130 watts is the power taken by the entire system. This may be checked by determining the I^2R loss in each impedance. That is,

$$P_a + P_b + P_c = (4.92)^2 10 + (5.18)^2 15 + (4.92)^2 20 = 1{,}130 \text{ watts} \quad (\textit{check})$$

The foregoing example may also be solved using the polar-vector method, although a transformation to rectangular vectors is necessary in order to find the line currents.

For example,

$$\dot{E}_{AB} = 110\underline{/0^\circ};\ \dot{E}_{BC} = 110\overline{/120^\circ};\ \dot{E}_{CA} = 110\backslash\underline{120^\circ}$$

$$\dot{Z}_{AB} = 22.37\underline{/63.5^\circ};\ Z_{BC} = 21.2\overline{\backslash 45^\circ};\ Z_{CA} = 22.37\underline{/26.5^\circ}$$

$$\dot{I}_{AB} = \frac{110\underline{/0^\circ}}{22.37\underline{/63.5^\circ}} = 4.92\overline{\backslash 63.5^\circ} = 2.20 - j4.40, \text{ etc.}$$

Example 2.—Three loads having resistances of 20, 40, and 50 ohms and reactances of +30, −50, and 0 ohms are connected from conductors $A'A$, $B'B$, and $C'C$ to the neutral $O'O$ of the balanced 600-volt, 60-cycle, three-phase, four-wire system shown in Fig. 119. Three wattmeters W_1, W_2, and W_3 are connected with their current coils in conductors $A'A$, $B'B$, and $O'O$ and their potential circuits between conductors A, B, O, and conductor C. The sequence of phase rotation is AO, BO, CO. Determine: (*a*) the current in each line conductor; (*b*) the current in the neutral; (*c*) the reading

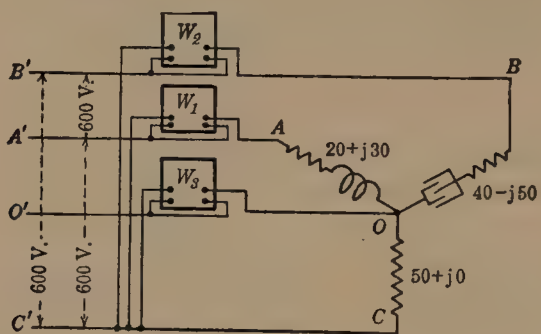


FIG. 119.—Unbalanced loads in three-phase, four-wire system.

of each wattmeter. (*d*) Show that the sum of the wattmeter readings is equal to the total power of the system, and compare with Fig. 116 (p. 125).

The voltage to neutral is $600/\sqrt{3} = 346.4$ volts.

$$\dot{E}_{AO} = 346.4 + j0;\ \dot{E}_{BO} = -173.2 - j300;\ \dot{E}_{CO} = -173.2 + j300.$$

$$(a)\quad \dot{I}_{AO} = \frac{346.4}{20 + j30} = \frac{346.4 \times 20}{1{,}300} - j\frac{346.4 \times 30}{1{,}300} = 5.33 - j8.00 \text{ amp.}$$

$$|I_{A'A}| = |I_{AO}| = \sqrt{(5.33)^2 + (8.00)^2} = 9.61 \text{ amp.}\quad \textit{Ans.}$$

$$\dot{I}_{BO} = \frac{-173.2 - j300}{40 - j50} = \frac{(-173.2 - j300)(40 + j50)}{4{,}100} = 1.968 - j5.04$$

$$|I_{B'B}| = |I_{BO}| = \sqrt{(1.968)^2 + (5.04)^2} = 5.42 \text{ amp.}\quad \textit{Ans.}$$

$$\dot{I}_{CO} = \frac{-173.2 + j300}{50 + j0} = -3.464 + j6.0 \text{ amp.}$$

$$|I_{C'C}| = |I_{CO}| = \sqrt{(3.464)^2 + (6.0)^2} = 6.93 \text{ amp.}\quad \textit{Ans.}$$

$$(b)\quad \dot{I}_{OO'} = \dot{I}_{AO} + \dot{I}_{BO} + \dot{I}_{CO}$$

$$= (5.33 - j8.00) + (1.968 - j5.04) + (-3.464 + j6.0) = 3.834 - j7.04$$

$$|I_{OO'}| = \sqrt{(3.834)^2 + (7.04)^2} = 8.02 \text{ amp.}\quad \textit{Ans.}$$

(*c*) The current in wattmeter W_1 is $\dot{I}_{AO} = 5.33 - j8.00$, and its potential circuit is connected across $\dot{E}_{AC} = \dot{E}_{AO} + \dot{E}_{OC} = 519.6 - j300$. Hence, by Par. 43 (p. 66), $W_1 = 519.6 \times 5.33 + 300 \times 8.00 = 5{,}170$ watts. *Ans.*

The current in W_2 is $\dot{I}_{BO} = 1.968 - j5.04$, and the potential across W_2 is $\dot{E}_{BC} = \dot{E}_{BO} + \dot{E}_{OC} = -j600$.
Hence, $W_2 = 600 \times 5.04 = 3{,}024$ watts. *Ans.*

The current in W_3 is $\dot{I}_{O'O} = -3.834 + j7.04$, and the potential across W_3 is $E_{OC} = 173.2 - j300$. Hence, $W_3 = 173.2 \times (-3.834) + (-300 \times 7.04) = -2{,}776$ watts. *Ans.*
The total power

$$P = W_1 + W_2 + W_3 = 5{,}170 + 3{,}024 - 2{,}776 = 5{,}418 \text{ watts. } \textit{Ans.}$$

(*d*) The total power is also equal to the sum of the I^2R losses in the impedances. That is,

$$P = I^2_{AO} \times 20 + I^2_{BO} \times 40 + I^2_{CO} \times 50$$

$$= (9.61)^2 \times 20 + (5.42)^2 \times 40 + (6.93)^2 \times 50 = 5{,}420 \text{ watts. } \textit{Ans. (check)}$$

Hence, the three wattmeters as connected in Fig. 119 measure the total power of the system, although W_3 reads backward. The current connection of W_3 must be reversed, therefore, and its reading subtracted from the sum of the other two readings. To measure the power in any polyphase system of n conductors, the $(n - 1)$ wattmeters may have their current coils connected in *any* $(n - 1)$ of the conductors, and the potential circuits to the remaining conductor.

CHAPTER VI

THE ALTERNATOR

75. Rotating-field Type.—The generation of an electromotive force in a conductor may take place with the magnetic field stationary and the conductor moving through this field, as in a direct-current generator, or with the conductor stationary and the field moving past the conductor. It is merely necessary that there be *relative* motion between the conductor and the field. In direct-current machines, the commutator makes it necessary either that the armature be the rotating member or that the brushes revolve with the field.

As alternators have no commutator, it is not necessary that the armature be the rotating member. Most commercial alternators have stationary armatures, inside which the field poles rotate, as shown in Figs. 122, 123, etc. This construction has two distinct advantages. A rotating armature requires two or more slip-rings for carrying the current from the armature to the external circuit. Such rings must be more or less exposed and are difficult to insulate, particularly for the higher voltages of 6,600 and 13,200 volts at which alternators are commonly operated. These rings may become a frequent source of trouble, due to arc-overs, short circuits, etc. A stationary armature requires no slip-rings, and the armature leads can be continuously insulated conductors from the armature coils to the bus-bars. It is more difficult to insulate the conductors in a rotating armature than in a stationary one, because of centrifugal force and the vibration resulting from rotation.

When the field is the rotating member, the field current must be conducted to the field winding through slip-rings. As the field voltage seldom exceeds 250 volts and the amount of power is small, no particular difficulty is encountered in the operation of such slip-rings.

Usually, it is difficult to find sufficient space for the copper on the surface of an armature. This is particularly true with high-speed, high-voltage machines having armatures of small diameter. Increased space for copper may be obtained by deepening the slots. If the armature be the rotating member, the deepening of the slots is limited by the contraction of the tooth necks, as shown in Fig. 120 (*a*). No such difficulty is encountered if the armature be stationary, since the tooth necks increase in width with the deepening of the slots (Fig. 120 (*b*)).

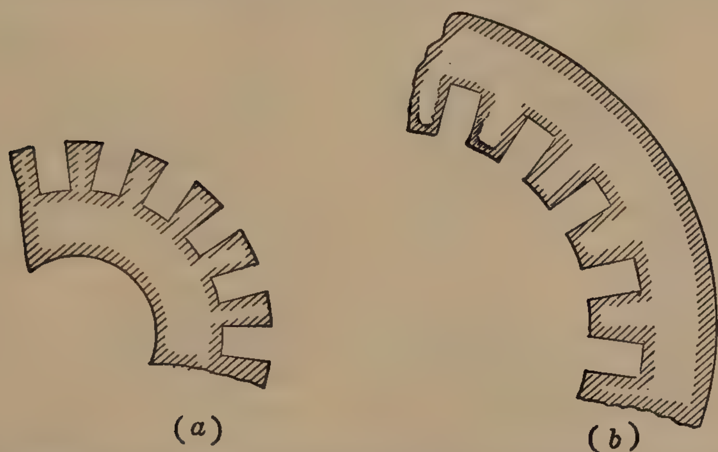


FIG. 120.—Effect of slot depth on the width of tooth necks in rotor and in stator.

ALTERNATOR WINDINGS

76. General Principles.—The usual direct-current armature generates alternating current, and, if provided with properly connected slip-rings, alternating current may be obtained from it. On the other hand, only certain types of alternator windings can be used for direct-current armatures. The ordinary direct-current winding is a *closed winding* (see Vol. I, p. 269), but alternator windings may be either open or closed.

The general principles which govern direct-current windings hold, also, for windings of alternators. The span of each coil must be approximately one pole pitch; that is, the two sides of any coil must lie under adjacent poles. The coils must be so connected that their electromotive forces add.

Alternator windings are divided into several general classes. There are single-layer and two-layer windings, usually made up of former-wound coils. Windings may be either of the lap-

winding type, shown in Figs. 121 (*a*), 124, 128, 132, and 134, or of the wave-winding type, shown in Fig. 121 (*b*). In the direct-current machine, the wave winding gives a higher voltage than the lap winding, if the number of series-connected conductors and other conditions are the same. In the alternator, the wave and lap windings give the *same* voltage, if the number of series-connected conductors and other conditions are the same. An inspection of Fig. 121 (*a*) and (*b*) shows that each winding has the same number of series conductors between terminals.

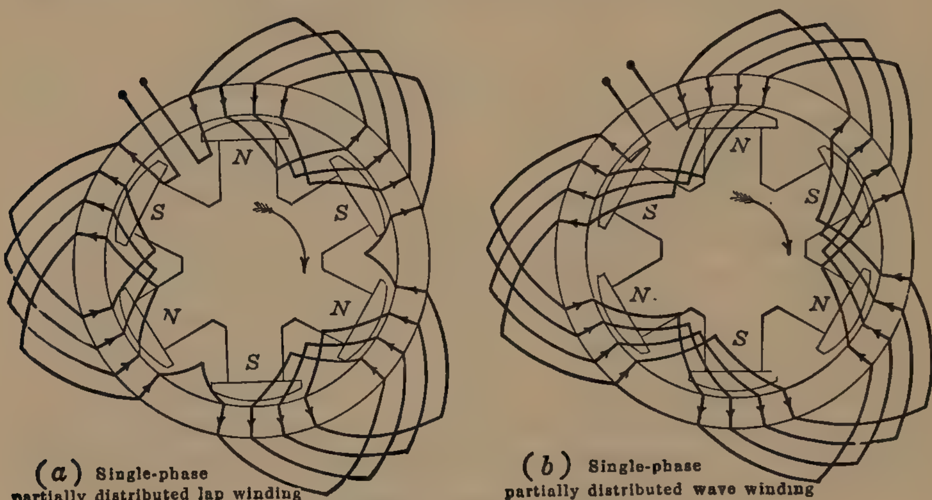


Fig. 121.—Single-phase lap and wave windings.

The type of winding shown in Figs. 121, 124, 131, etc., is called the *barrel* winding. The type shown in Fig. 125 (*a*) is called the *spiral* winding. A development of the spiral into the *chain* winding is shown in Figs. 127 and 130.

77. Single-phase Windings.—Figure 122 shows a single-phase, single-layer, half-coil winding for a four-pole machine. This machine has four slots and four poles, making one slot per pole. This winding is called a *half-coil* winding, because there is but one-half coil or coil group per pole. The two coils are shown connected in series, and T_1 and T_2 are the terminals of the winding.

Figure 123 shows the same type of winding as Fig. 122, except that four coils are now used. There are two coil sides, therefore, per slot. This is called a single-phase, two-layer, *whole-coil* winding. It is called a whole-coil winding because there is one

coil or coil group per pole. The winding of Fig. 122 may be obtained from Fig. 123 by swinging coil B into the plane of coil A and coil D into the plane of coil C.

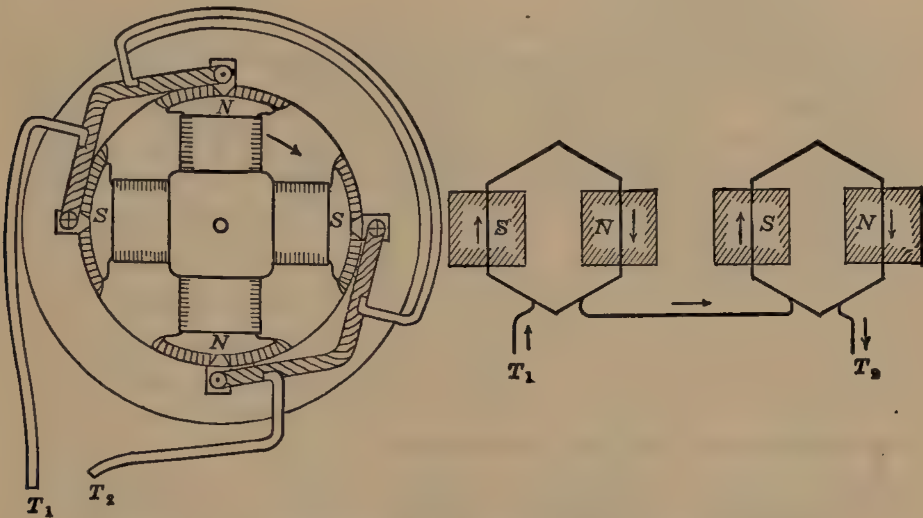


FIG. 122.—Single-layer, half-coil winding, one slot per pole.

One slot per pole is seldom found in practice, as the surface of the armature is not used economically, and, in addition, a poor voltage wave results. Figure 124 shows the winding of Fig. 123,

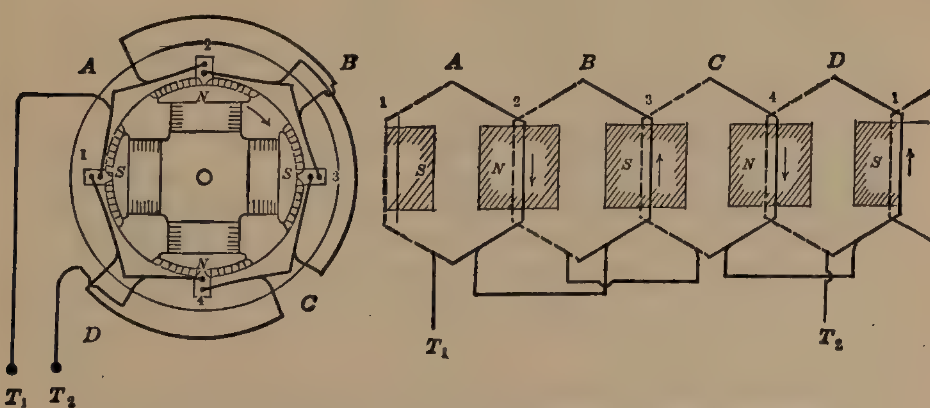


FIG. 123.—Two-layer, whole-coil winding, one slot per pole.

except that there are now two slots per pole instead of one. This is also a two-layer winding, as there are two coil sides per slot, placed one above the other.

Instead of making the coils lap one another, as is done in Fig. 124, the winding may be placed on the armature in the manner

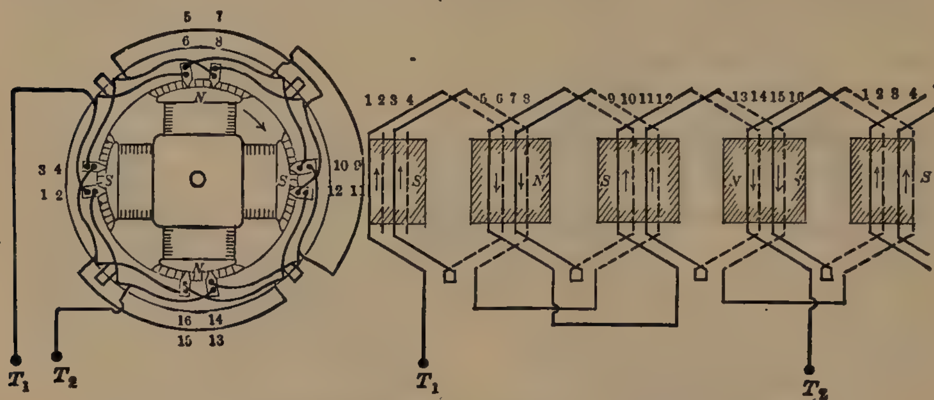
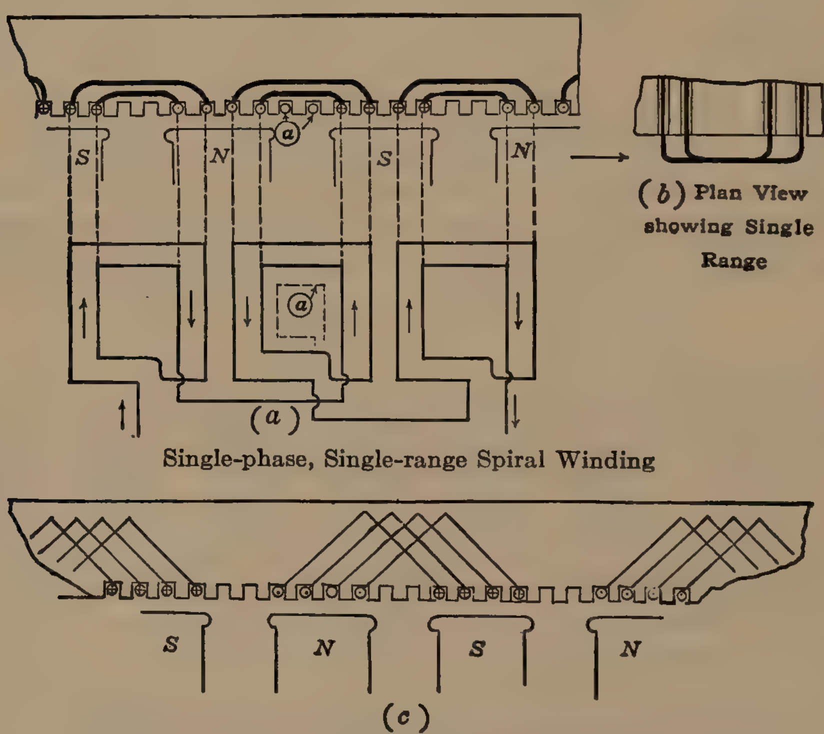


FIG. 124.—Whole-coil, two-layer winding, two slots per pole.



(b) Plan View showing Single Range

(a) Single-phase, Single-range Spiral Winding

(c) Barrel-type Winding which can replace Spiral Winding of (a)

FIG. 125.

shown in Fig. 125. This is called a *spiral* winding. It will be observed that in this particular winding the coils themselves

have a pitch less than 180 electrical space-degrees. Notwithstanding this lesser pitch, the winding is not considered as having the properties peculiar to a fractional-pitch winding. The slot conductors may be reconnected by barrel-type end connections, as shown in Fig. 125 (*c*), without changing the electrical characteristics of the winding. This gives a full-pitch, half-coil, *barrel* winding. The spiral winding for which this can be substituted is considered, therefore, as a full-pitch winding. The differential action of the coil sides of Fig. 125, due to their not having a full pitch, is taken into consideration by the belt-factor constant (see p. 153, Par. 83).

The inside coil shown dotted at *a* may be added to the winding, but it contributes so little to the generated e.m.f., because of its small pitch, that to use it is wasteful. This winding has but one coil side per slot, so that it is also a single-layer winding. As the ends of the coils may be bent so that they all lie in a single vertical plane, as shown at (*b*) (Fig. 125), it is called a *single-range* winding. Two- and three-range windings are also used in practice. In this country, the lap winding has practically replaced the spiral winding.

At the present time, single-phase machines are somewhat limited in their field of application. They are used more or less extensively for single-phase railway electrification and for some electric furnace work. Instead of building a single-phase machine for these purposes, however, Y-connected, three-phase machines are commonly used, as such machines are standard. Two phases of the Y are used in series. A spare phase is also available.

78. Two-phase Windings.—Two-phase windings are merely two single-phase windings displaced 90 electrical space-degrees from each other on the armature. If another winding be added to Fig. 123, the coil sides of this new winding being midway between those shown in the figure, a two-phase winding results, as shown in Fig. 126. These two windings are 90 electrical space-degrees apart, so that their voltages differ in time phase by 90°.

Figure 127 shows a two-phase spiral or chain winding. This is merely adapting the winding of Fig. 125 to two phases. There are now eight slots per pole rather than six. As the coil ends in this winding must necessarily lie in two different vertical

planes, in order to pass one another, the winding is called a *two-range* winding.

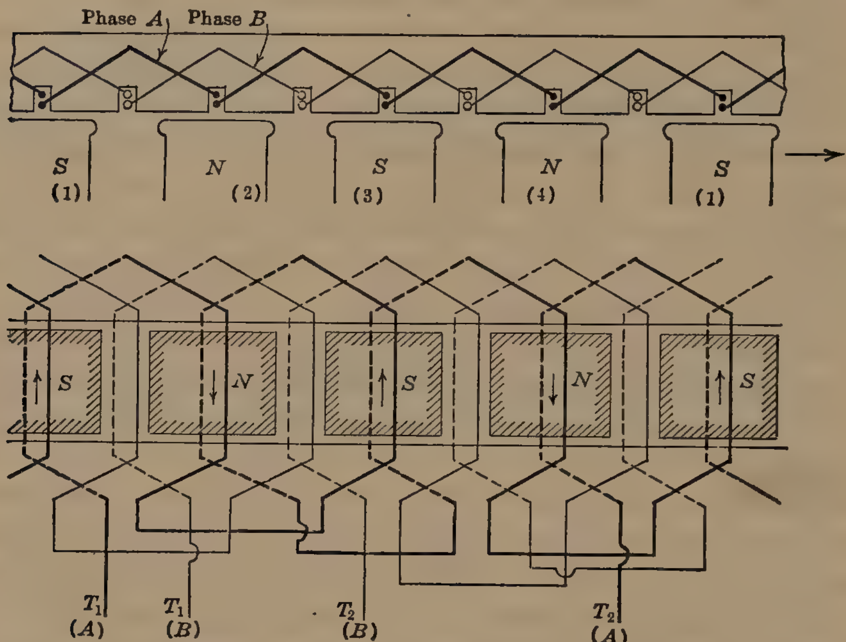


FIG. 126.—Two-phase, two-layer winding, one slot per pole per phase.

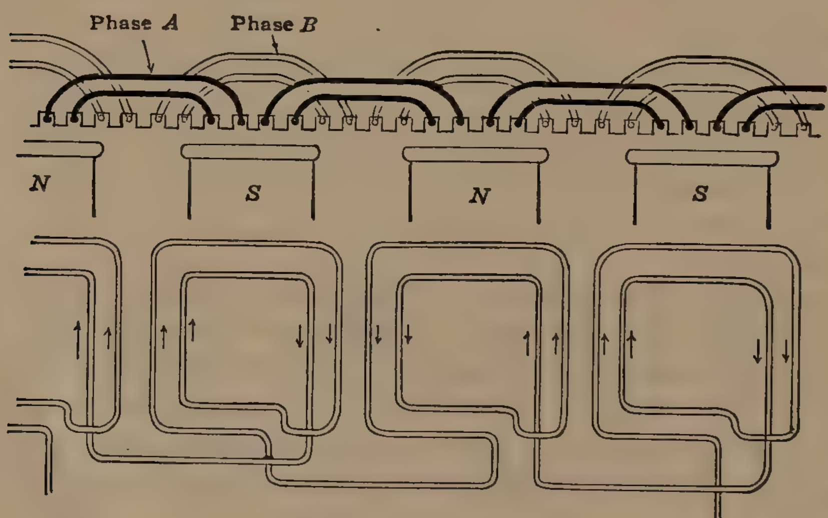


FIG. 127.—Two-phase chain winding, two-range.

The chief advantage of a chain winding is the considerable space between the coil ends, so that there is little opportunity for electrical breakdown at these points. They are, therefore,

admirably adapted to high-voltage machines. Although coils of several different sizes must be kept in stock as spares, a coil may be replaced more easily than it can be in the lap winding, where a large number of coils must often be removed in order to replace a single coil. In this country, the lap winding is rapidly replacing the chain winding.

79. Two-phase Lap Winding.—The lap winding is the most common type of alternator winding. With it, there are very few limitations in the choice of number of slots, pitch, etc. The coils are all alike, requiring the minimum number of spares, and the winding is very flexible in the matter of connections. For example, with a lap winding it is a simple matter to change a 440-volt winding to one of 220 volts by paralleling.

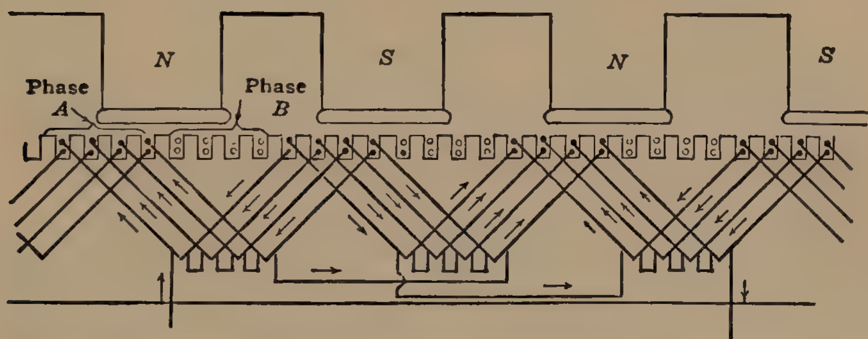


FIG. 128.—Two-phase, full-pitch lap winding, four slots per pole per phase

To obtain a lap winding, more coils are added to the winding shown in Fig. 123. The connections of the coils of any one phase are almost identical with those in the direct-current windings described in Vol. I (Chap. X). Direct-current lap windings may be used for single-phase and polyphase voltages by taps at suitable points, with connections to slip rings, as is done in the synchronous converter (see p. 387, Par. 171).

Figure 128 shows a two-phase lap winding, in which there are eight slots per pole, making four slots per pole per phase. This is a full-pitch winding, the coil pitch being eight slots, which is the number of armature slots per pole. The connections of phase *B* are omitted for the sake of clearness, as they are identical with those of phase *A*. It will be observed that the coil sides in any one slot are both of the same phase. This is not the case with fractional-pitch windings.

80. Three-phase Windings.—The difference between two-phase and three-phase windings is merely in the number of phase belts per pole. Figure 129 shows the simple winding of Fig. 123, adapted to three-phase. For clearness, the end connections of phase *A* alone are shown. It is necessary merely to add two more windings, equally spaced, between those of Fig. 123 in order to obtain a three-phase winding having one slot per pole

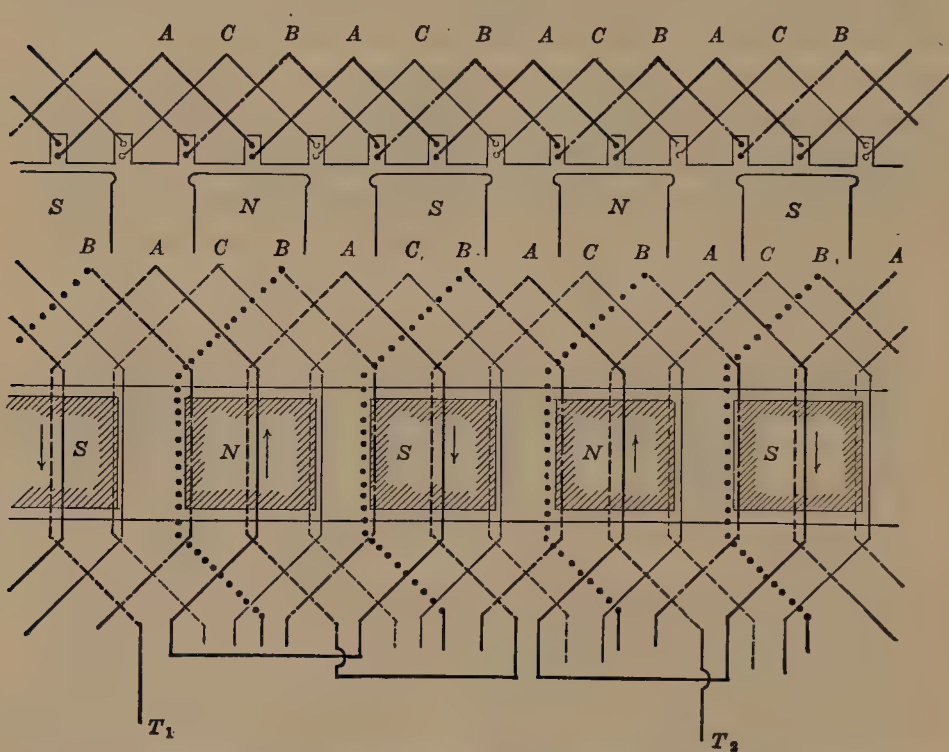


FIG. 129.—Three-phase, two-layer winding, one slot per pole per phase.

per phase. The three phases of Fig. 129 may be connected either in Y or in delta.

Figure 130 shows a three-phase chain winding, in which there are six slots per pole, making two slots per pole per phase. This is a two-range winding, for the coil ends in order to pass one another must lie in two different planes perpendicular to the machine shaft. If the number of coil groups per phase is odd, which occurs if the number of poles is not a multiple of four, coils having one long side and one short side must be used to complete the winding. This occurs in the six-pole winding of Fig.

130, in which two coils d and d' must be of trapezoidal shape in order to pass the coil ends of phase A and so complete the winding. A plan view of such coils is shown at (a) (Fig. 130). Six different sizes of coils are required in this winding, making it neces-

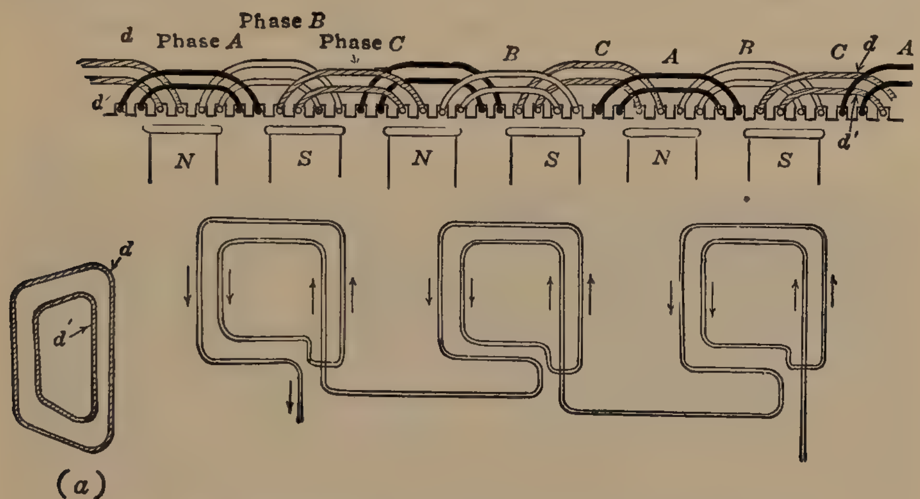


FIG. 130.—Three-phase chain winding, requiring special coils.

sary to carry a considerable variety of spares. The connections of the coils of phase B only are shown, the other phases being connected in a similar manner.

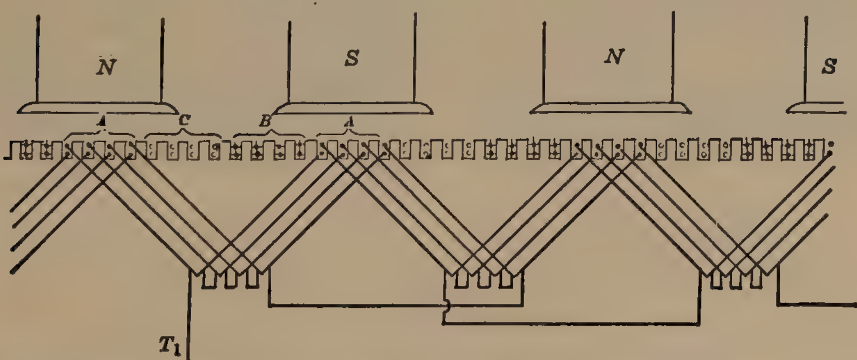


FIG. 131.—Three-phase, full pitch, two-layer lap winding.

Figure 131 shows a three-phase, full-pitch, lap winding, in which there are 12 slots per pole. The coil pitch is, therefore, equal to 12. For clearness, the connections of the A phase alone are shown, the connections of the B and C phases being similar.

It will be observed that in this type of winding, the two coil sides in any one slot belong to the same phase.

Except that the winding is now five-sixth pitch, Fig. 132 is similar to Fig. 131. A coil, instead of having a pitch of 12 slots, now has a pitch of 10 slots, so its spread is no longer equal to a full pole pitch. This is a *fractional-pitch winding.*

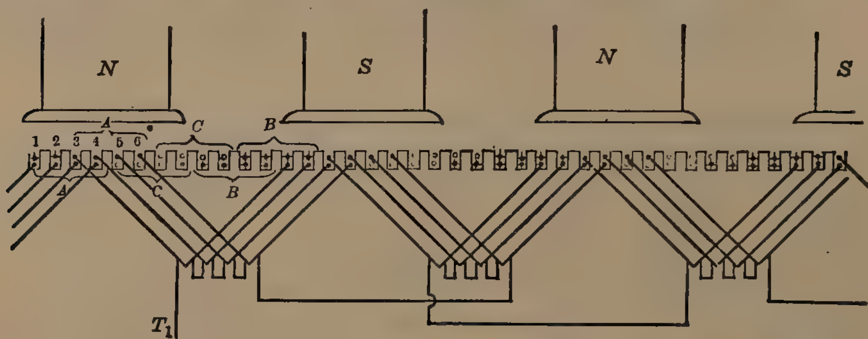
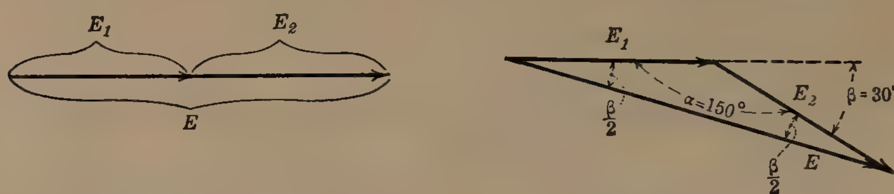


FIG. 132.—Three-phase, five-sixth-pitch, two-layer lap winding, four slots per pole per phase.

The advantages of this type of winding are that it improves the wave form, there is an appreciable saving of copper in the coil ends, and the inductance of the winding is reduced, because of the lesser mutual inductance between those conductors which lie in slots containing conductors of the other two phases (see Fig. 132). The coil-end inductance is also reduced because of



(*a*) Full pitch. (*b*) Fractional pitch.

FIG. 133.—Relation of coil-side voltages in full-pitch and in fractional-pitch windings.

the lesser length of free conductor. Such windings generate slightly less e.m.f. than full-pitch windings under the same conditions, since the two coil sides do not lie under corresponding parts of the poles at any given instant, and, therefore, their e.m.fs. are slightly less than 180° apart. This is illustrated in Fig. 133. E_1 is the e.m.f. induced in the conductors comprising

one side of a coil, and E_2 is the e.m.f. induced in the conductors comprising the other side of the coil: E_1 is equal to E_2 numerically, as each is induced by the same number of conductors cutting the same flux. Figure 133 (*a*) gives the relation of the induced e.m.fs. E_1 and E_2 in the two coil sides, respectively, when a full-pitch coil is used. When one side of a coil is under a north pole, the other side is in a corresponding position under a south pole. The induced e.m.fs., therefore, are 180° out of

FIG. 134.—Showing winding and end connections of alternator armature.

phase, but the coil connection is such that these e.m.fs. are additive, as shown in Fig. 133 (*a*).

When a five-sixth pitch is used, the coil spread is equal to $\frac{5}{6} \times 180°$ or 150 electrical space-degrees. The e.m.fs. E_1 and E_2 will, therefore, differ in phase by 150 electrical time-degrees, as shown by the angle α (Fig. 133 (*b*)). The total e.m.f. E, which is their vector sum, is slightly less than when a full-pitch coil is used.

The ratio $\dfrac{E}{E_1 + E_2} = \dfrac{E}{2E_1}$ is the pitch factor, k_P.

A study of Fig. 133 (*b*) shows that

$$k_p = \frac{E}{2E_1} = \frac{2E_1 \cos\frac{\beta}{2}}{2E_1} = \cos\frac{\beta}{2}$$

For example, for $\frac{5}{6}$ pitch, $\beta = 30°$

$$k_p = \cos 15° = 0.966 \text{ (see p. 154)}$$

It will be observed that with the fractional-pitch winding of Fig. 132, only two of the slots of each phase under any pole contain coil sides of the same phase. In the other slots, the two coil sides are of different phases. For example, slots 1 and 2 contain both phase *A* and phase *B* conductors; slots 3 and 4 contain phase *A* conductors only; and slots 5 and 6 contain both phase *A* and phase *C* conductors. Of this group, slots 3 and 4 contain phase *A* conductors only. The fact that certain slots contain conductors of different phases reduces slightly the inductance of the winding, as has already been pointed out.

Figure 134 shows a portion of a finished armature winding. The end connections, the binding down of the coil ends, the wooden slot wedges, and the ventilating ducts, are clearly shown.

ALTERNATOR CONSTRUCTION

81. Stator or Armature.—The stator or stationary member of the alternator is almost always the armature, the field structure being the rotating member or rotor. When the machine is in operation, the armature iron is continuously cut by the flux of the rotating field and must be laminated in order to reduce eddy-current losses. In machines of small diameter, each lamination is a single circular punching.

High-speed turbo-alternators have armatures of small diameter and are usually built up of single circular stampings, as shown in Fig. 135. The perforations back of the slots are ventilating channels. Engine-driven alternators must rotate at comparatively low speeds and so must have a large number of poles and armatures of comparatively large diameter. The pole pieces are made up of laminations riveted together and are dovetailed to the armature spider, as shown in Fig. 136. The armature is built up of small overlapping segments, dovetailed to the frame of the machine in much the same manner as the armature

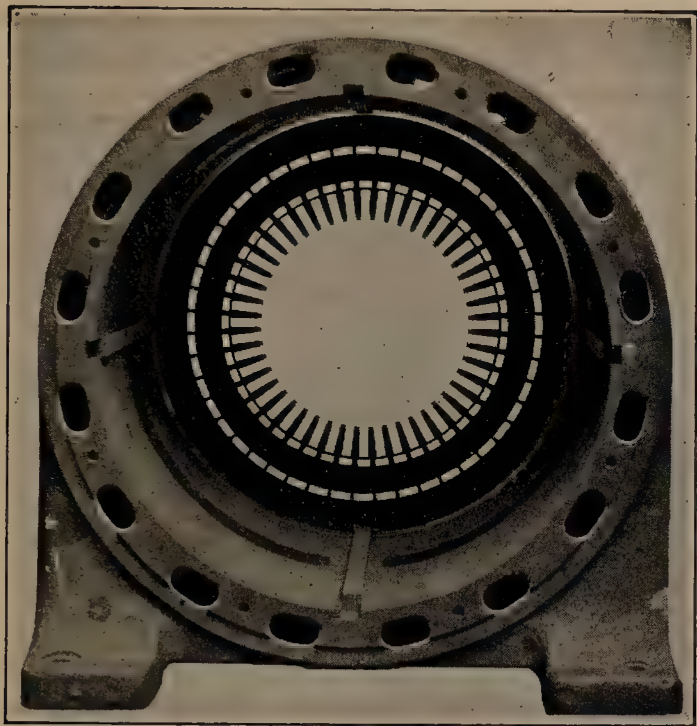


FIG. 135.—Punching and frame of turbo-driven alternator.

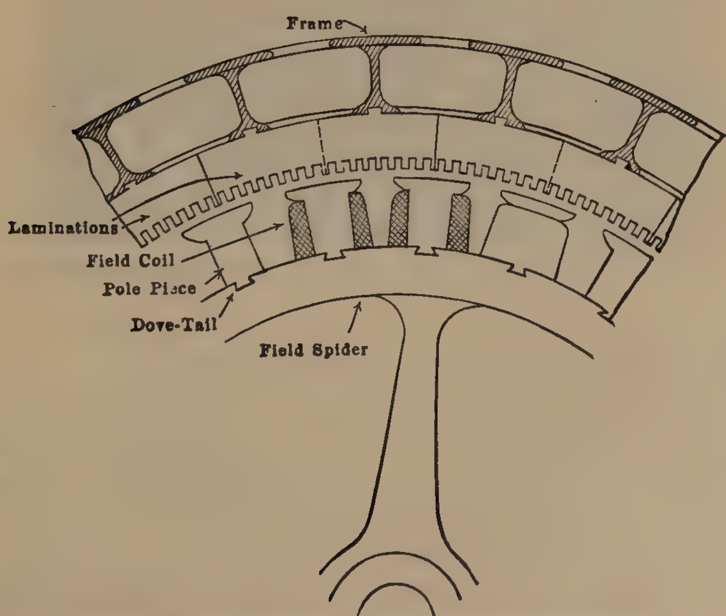


FIG. 136.—Cross-section of engine-driven alternator.

of engine-driven, direct-current generators are assembled (see Vol. I, p. 296, Fig. 242), except that in the alternator the armature laminations are a part of the stationary member. Figure 136 shows the general construction of such an alternator. The frame itself is usually a hollow box casting. This gives the necessary mechanical stiffness, with the minimum weight, and the space within the frame allows a free circulation of air for ventilating purposes. Figure 137 shows the complete armature of an

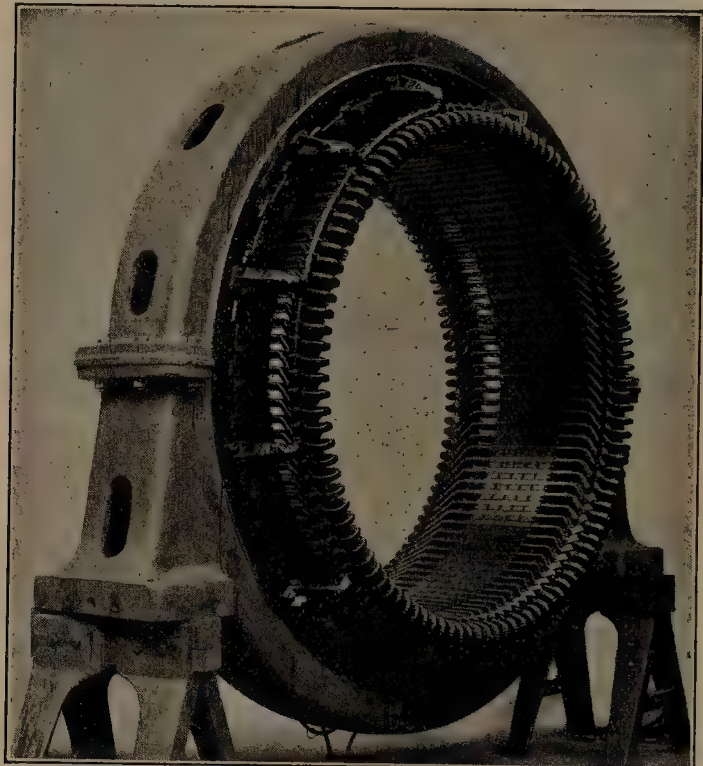


FIG. 137.—Completely wound stator of engine-driven alternator.

engine-driven generator. The ventilating ducts and the bracing of the coil ends should be particularly noted.

Large units must be so designed that they can withstand not only the stresses incident to normal operation but also the enormous mechanical stresses which occur at short circuit, due to the attraction and repulsion of the armature currents. The coil ends, unless well supported, are likely to be dragged out of position by electromagnetic stresses produced by the short-circuit currents. This is particularly true of turbo-alternators, whose

internal reactance is comparatively low, and whose short-circuit currents, therefore, may be of considerable magnitude. Figure

FIG. 138.—Bracing of end connections of turbo-alternator to withstand short-circuit stresses.

(a) Open Slot (b) Semiclosed Slot

FIG. 139.—Open and semiclosed slots.

138 illustrates the care taken in bracing the coil ends in one of the largest types of turbo-alternator.

Alternator slots are divided into two general classes, the open slot and the semiclosed slot. The open slot, shown in Fig. 139 (*a*), is the more common, because the coils can be form wound and insulated prior to being placed in the slots, giving the least expensive and most satisfactory method of winding.

The semiclosed or overhung type of slot, shown in Fig. 139 (*b*), is often necessary, especially in induction motors. The larger area of tooth face reduces the air-gap reluctance and also reduces the tufting of the flux which tends to produce ripples in the electromotive force wave. It is usually necessary to place the conductors in the slot one at a time, which is expensive and uneconomical of slot space. It is also difficult to apply insulation.

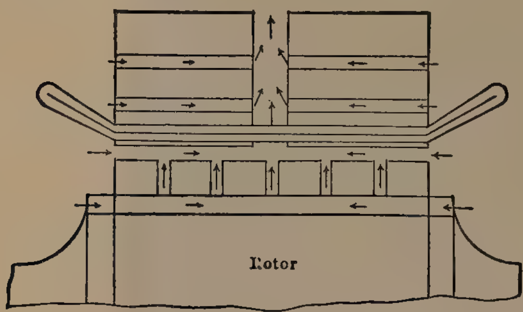


FIG. 140.—Passage of ventilating air through the ducts of turbo-alternator.

In both types of slot, the conductors are usually held in the slot by wooden or fibre wedges, as shown in the figures. The effect of the semiclosed slot may be obtained by the use of open slots and magnetic wedges. These wedges are only partly of iron, so that the slot is not entirely closed.

The internal temperatures of modern alternators are so high that built-up mica is found to be the insulation best able to withstand the high-temperature and high-voltage stresses simultaneously. Such mica is pressed around the active part of the conductor, forming a solid, homogeneous mass.

The problem of ventilating a large unit is not an easy one. A 20,000-kw. unit, having an efficiency of 96.5 per cent., requires 700 kw. to be dissipated. Such a unit might require from 60,000 to 70,000 cu. ft. of air per minute. This air is usually supplied by separate blowers, and to remove the dirt and increase the cooling properties of the air, it is often passed through an air washer consisting of a curtain of water. Figure 140 shows the passage of the air through the axial ducts back of the laminations (see Fig. 135) and out through a center radial duct, in a turbo-alternator. In modern practice, the cooling air is circu-

lated through a totally enclosed system, the same air being used over and over again after cooling. This reduces fire hazard by limiting the supply of oxygen blown through the windings.

FIG. 141.—Pole piece of 75-kv-a., 60-cycle, 360-r.p.m. alternator.

82. Rotating-field Structure.—From the standpoint of their field construction, alternators may be divided into three classes—

FIG. 142.—Thirty-six-pole rotor with strip-wound coils.

the very slow-speed engine-driven alternator (75 to 150 r.p.m.); the medium-speed belt-driven and waterwheel-driven type

(150 to 750 r.p.m.); and the high-speed turbo-alternator (750 to 3,600 r.p.m.).

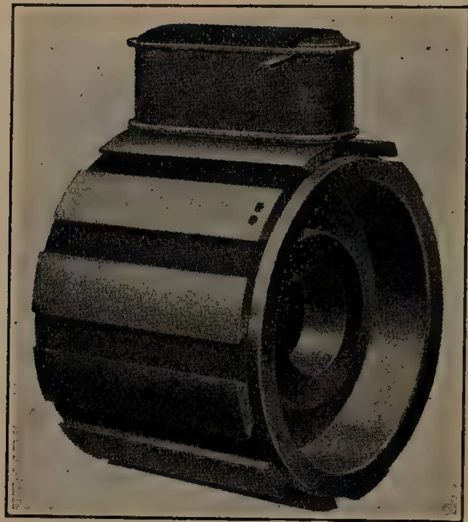


FIG. 143.—Revolving field spider with dovetailed poles.

The poles of practically all salient-pole generators have cores made up of laminations (Fig. 141), in order to reduce pole-face

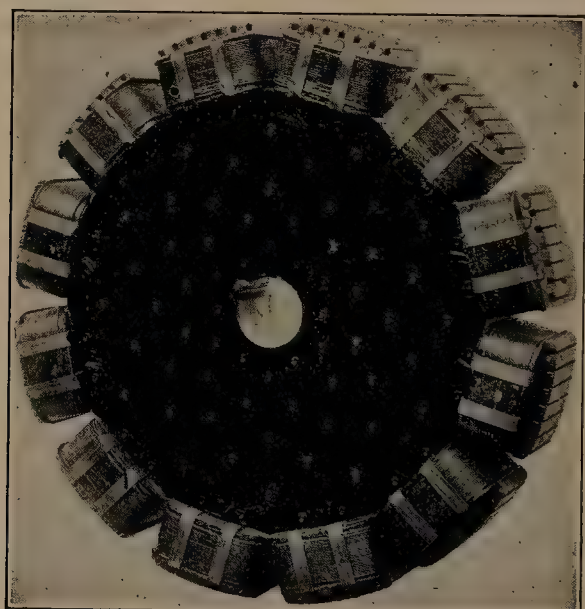


FIG. 144.—Twelve-pole, 600 r.p.m. rotor.

losses (Vol. I, p. 415). In slow-speed machines, the poles are often bolted to a cast-iron spider (Fig. 142), or they may be dove-

tailed to the spider in the manner indicated in Figs. 136 and 143.

At higher speeds, centrifugal forces require that the poles be dovetailed to the spider. In small machines, the spider may be of solid steel, as shown in Fig. 143. The pole pieces dovetail to this spider and are wedged in by keys driven one from each end.

In the larger types of generator, the spider is made of steel plates riveted together, as shown in Fig. 144. The poles are dovetailed to the spider in the manner indicated. The slots in the pole faces of this rotor should be noted. Damping or

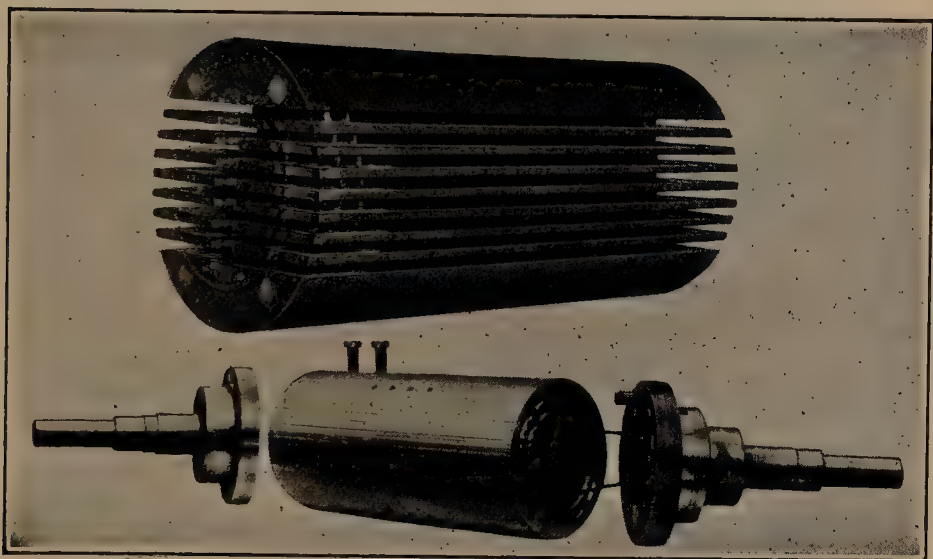


Fig. 145.—Parallel-slot, two-pole rotor for turbo-alternator.

amortisseur windings are placed in these slots, as will be described later (see p. 362, Fig. 319).

Salient poles cannot be used for high-speed turbo-generators, owing to the large centrifugal forces developed and to the excessive windage. A non-salient-pole rotor is used, therefore. There are two common types of such a rotor, the parallel-slot type shown in Fig. 145 and the radial-slot type shown in Fig. 146.

The winding in the parallel-slot type is of strip copper, wound by hand in the slots. The wires are held in the slots by means of non-magnetic metallic wedges. There is not sufficient space to run the shaft through the center of the rotor, so it is bolted to the ends by phosphor-bronze flanges (Fig. 145). These flanges

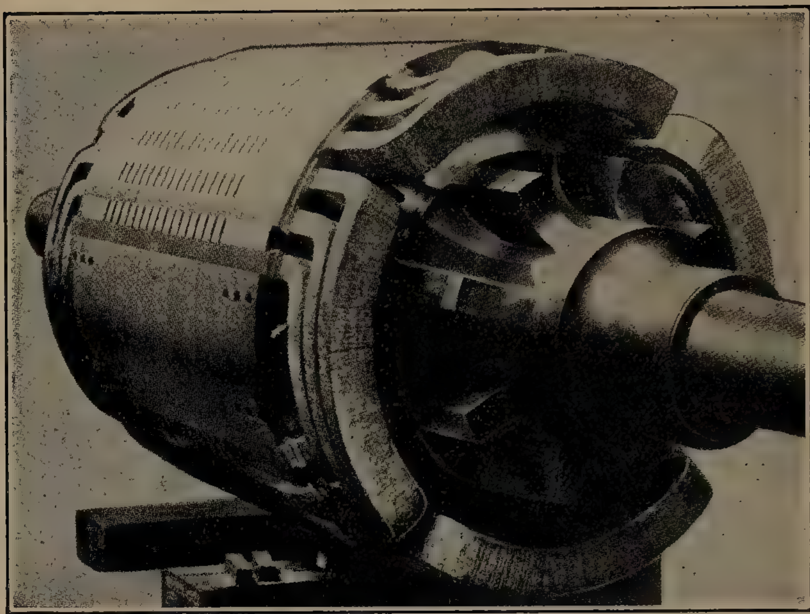


FIG. 146.—Radial-slot type of rotor, having four poles.

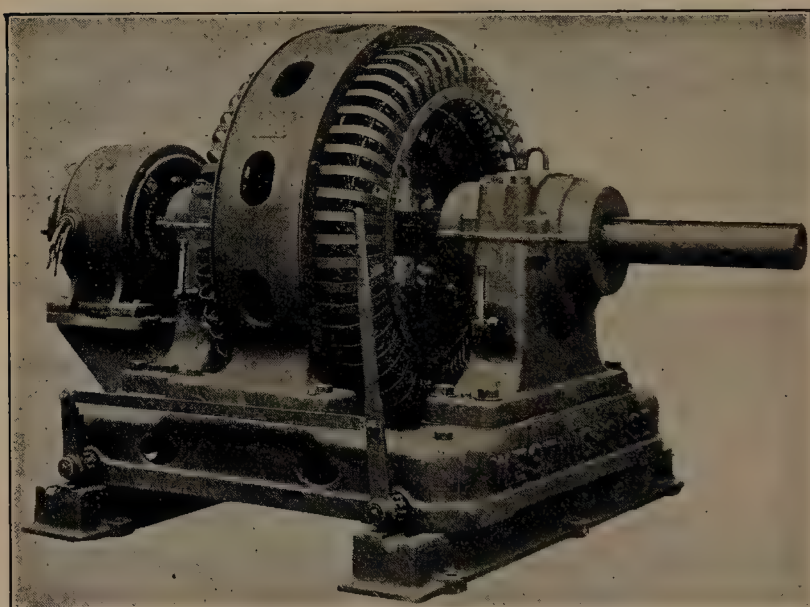


FIG. 147.—150-kv-a., 900-r.p.m., 2,400-volt, 60-cycle alternator, with direct-connected exciter.

must be non-magnetic, or they would short circuit the magnetic poles. The fact that they are of phosphor bronze makes them expensive. This construction gives a smooth rotor, with little windage loss, and strong mechanically, especially as regards the support of the coil ends. Parallel-slot rotors are seldom used except for two-pole rotors in small machines. The metal back of the slots becomes too small in cross-section to withstand the centrifugal forces, when attempt is made to adapt this type of rotor to more than two poles.

Figure 146 shows a four-pole, radial-slot rotor. Although the coil ends are not held so strongly in this type of rotor as they are in the parallel-slot type, the radial type is better adapted to rotors having more than two poles, because there is not the reduction of iron section behind the slots with increase in the number of poles, such as occurs in the parallel-slot type.

The winding on these types of rotor is called a *distributed* field winding (see Fig. 151).

The field connections are usually carried out through the center of the shaft to slip-rings. Two or more carbon brushes resting on the slip-rings carry the current to the winding. The excitation voltage is usually 120 or 250 volts and in the larger stations is supplied by bus-bars devoted to excitation only. In smaller installations, the exciter is mounted directly on the alternator shaft (Fig. 147) or else is belt driven from the alternator shaft. Large central stations usually have a storage battery floating on the exciter bus and, in addition, may have steam-driven exciters to be used in emergencies.

ALTERNATOR ELECTROMOTIVE FORCES AND OUTPUTS

83. Generated Electromotive Force.—Figure 148 (a) shows the magnetic flux between the armature surface and a north and a south pole of an alternator. Assume that the flux distribution is sinusoidal (Fig. 148 (b)), the flux density being a maximum under the center of the pole. Let B' be the average value of the flux density. B' is equal to $2/\pi$ times the maximum value B. Let a be a conductor cutting this flux with a velocity of v cm. per second. This conductor a has a length of l cm. perpendicular to the plane of the paper.

From Eq. (110) (Vol. I, p. 263), the maximum e.m.f. induced in conductor a occurs when it is directly under the center of the pole in the maximum flux density B. That is,

$$e_{max} = Blv10^{-8} \text{ volts.}$$

Let D be the pole pitch in centimeters, and f the frequency in cycles per second.

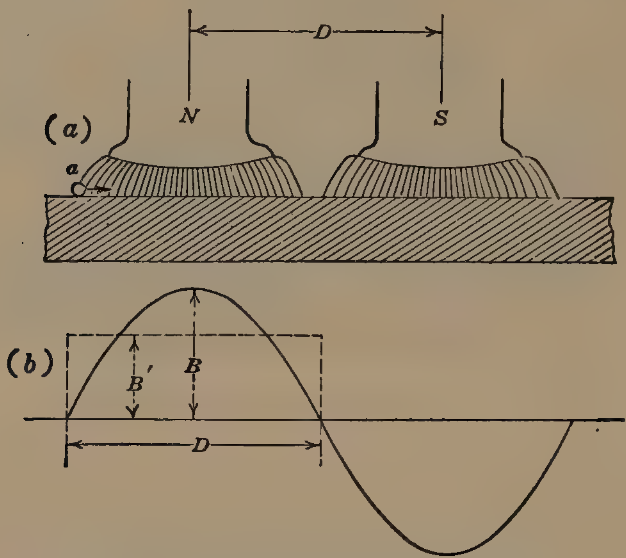


FIG. 148.—Generation of alternating electromotive force.

The time in seconds necessary for the conductor a to move the distance D is $1/2f$ sec. Therefore, $v = \dfrac{D}{1/2f} = 2fD$ cm. per sec. The total flux cut per pole

$$\phi = B'lD = \frac{2}{\pi}BlD$$

$$B = \frac{\pi\phi}{2lD}$$

The effective voltage is $1/\sqrt{2}$ times the maximum. The *effective* induced volts per conductor, by substitution in the above equation for e_{max},

$$e = \frac{e_{max}}{\sqrt{2}} = \frac{1}{\sqrt{2}}\left(\frac{\pi\phi}{2lD}\right)l(2fD)10^{-8} \text{ volts.}$$

If there are Z conductors in series per phase, the effective e.m.f. per phase

$$E = 2.22Z\phi f10^{-8} \text{ volts.} \tag{67}$$

(2.22 = 2 times the form factor, 1.11, for a sine wave (see p. 10).)

If the e.m.f. wave is not a sine wave, the form factor should be correspondingly changed.

Owing to the fact that the electromotive forces in the different coils of a phase belt are not in time phase with one another (Fig. 150), the conductor e.m.fs. do not add algebraically. A factor k_b, therefore, called the *breadth factor* or *belt factor*, must be introduced to correct for this relative phase displacement. This factor is unity for a concentrated winding and less than unity for a distributed winding. Its value is given by the equation

$$k_b = \frac{\sin \frac{n\alpha}{2}}{n \sin \frac{\alpha}{2}} \tag{68}$$

where n is the number of slots per pole per phase, and α is the angle between adjacent slots in electrical space-degrees.

Example.—Determine k_b for a three-phase winding in which there are 12 slots per pole.

$$n = 4 \quad \alpha = \frac{180°}{12} = 15°$$

$$k_b = \frac{\sin \frac{4 \times 15°}{2}}{4 \sin \frac{15°}{2}} = \frac{\sin 30°}{4 \sin 7.5°} = \frac{0.5}{0.522} = 0.958 \quad \textit{Ans.}$$

The table gives values of k_b for a few typical windings.

VALUES OF BREADTH FACTOR k_b

Slots per pole per phase	Single-phase	Two-phase	Three-phase
1	1.000	1.000	1.000
2	0.707	0.924	0.966
3	0.667	0.910	0.960
4	0.653	0.907	0.958

If fractional pitch is used, the electromotive forces in the two coil sides are out of phase, as shown in Fig. 133 (*b*) (p. 140).

This again reduces the voltage. Correction for this may be made by multiplying the voltage equation by k_p, the *pitch factor* (see p. 140).

$$k_p = \cos \frac{180°(1 - p)}{2} \tag{69}$$

where p is the pitch.

For example, with $\frac{5}{6}$ pitch,

$$k_p = \cos \frac{180°(1 - \frac{5}{6})}{2}$$

$$= \cos 15° = 0.966.$$

VALUES OF PITCH FACTOR k_p

Pitch	$\frac{9}{10}$	$\frac{6}{7}$	$\frac{5}{6}$	$\frac{4}{5}$	$\frac{3}{4}$	$\frac{2}{3}$
k_p	0.988	0.974	0.966	0.951	0.924	0.866

The complete equation becomes

$$E = 2.22 k_b k_p Z \phi f 10^{-8} \text{ volts.} \tag{70}$$

Example.—A six-pole, three-phase, 60-cycle alternator has 12 slots per pole and four conductors per slot. The winding is $\frac{5}{6}$ pitch. There are 2,500,000 lines entering the armature from each north pole, and this flux is sinusoidally distributed along the air-gap. The armature coils are all connected in series. The machine is Y-connected. Determine the open-circuit voltage of the alternator.

The total number of slots is equal to 72.

The series conductors per phase, therefore,

$$Z = \frac{4 \times 72}{3} = 96$$

Slots per pole per phase $= 72/(6 \times 3) = 4$. k_b (from table) $= 0.958$. $k_p = 0.966$.

The total induced voltage per phase, therefore,

$E = 2.22 \times 0.958 \times 0.966 \times 96 \times 2{,}500{,}000 \times 60 \times 10^{-8} = 296$ volts.

As the generator is Y-connected, the terminal voltage is

$$296\sqrt{3} = 513 \text{ volts. } \textit{Ans.}$$

Ordinarily, the flux distribution in a generator is not sinusoidal, especially with salient-pole machines, the wave being flat topped, as shown in Fig. 149. The e.m.f. wave *per conductor* has the same shape as the flux wave. If the coil is a full-pitch coil, the e.m.fs. in the two sides of each coil will be 180° out of phase and of the same magnitude, as these coil sides both lie at any instant

in corresponding parts of opposite poles. The e.m.f. wave of each coil, therefore, will have the same shape as the conductor e.m.f. wave. If but one slot per pole per phase is used, the resulting e.m.f. wave will have the same shape as the flux wave, which may be flat topped, as shown in Fig. 149.

Figure 150 (*a*) shows a phase belt, consisting of four coils, of a three-phase generator having 12 slots per pole, or four slots per pole per phase. The shape of the e.m.f. wave for each of the four full-pitch coils forming one phase of the winding is the same

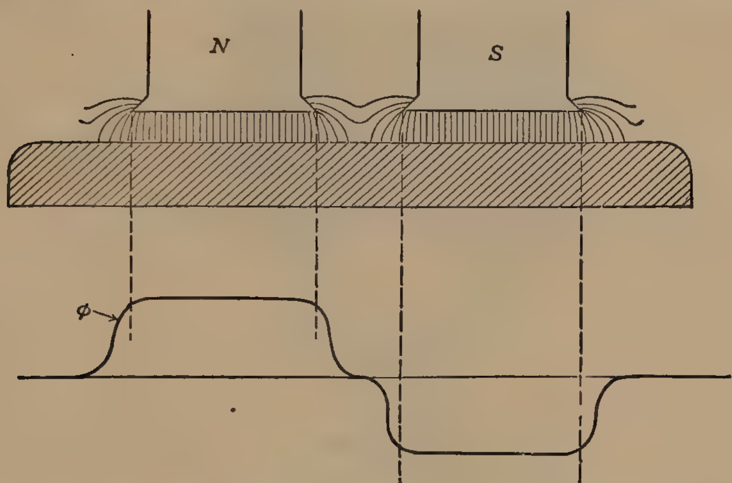


FIG. 149.—Flux density in air-gap of salient-pole machine.

as the shape of the flux wave, as is shown in Fig. 150 (*b*), at 1, 2, 3, and 4. As 12 slots represent 180 electrical space-degrees, 180/12, or 15, is the interval in electrical space-degrees between successive slots. The four e.m.fs., therefore, are 15 electrical time-degrees apart, as shown in Fig. 150 (*b*). As the coils are connected in series, the resultant electromotive force is found by adding the ordinates of the four waves, as shown. The resultant wave, instead of being a flat-topped one, like that of the individual coil, is well rounded and very nearly a sine wave. This is the reason why a distributed winding gives a better wave shape than a concentrated winding.

With a non-salient pole rotor, having a distributed winding (Fig. 146), the field coils lie in the rotor slots in the manner indicated in Fig. 151 (*a*). The m.m.f. of each coil is a rectangle, if the currents in the conductors be considered as concentrated at

their centers. This is shown in Fig. 151 (*b*), which gives the m.m.f. of a single coil The height of the rectangle is equal to the m.m.f. of this coil, which can be expressed in ampere-turns, gilberts, or any other convenient unit. The base of each rectangle

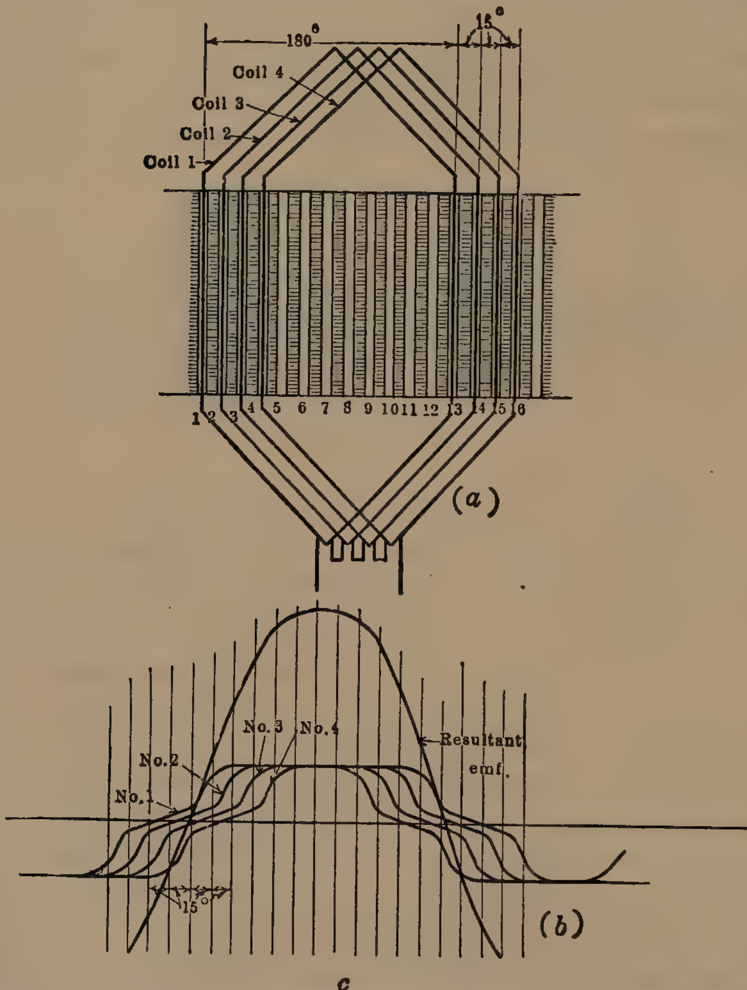


FIG. 150.—Resultant electromotive force wave in four-coil phase belt.

is equal to the coil spread. In Fig. 151 (*a*) and (*c*), there are three coils per pole. Each coil produces a rectangle of m.m.f. If there are the same number of turns in each of the three coils, the height of each m.m.f. rectangle is the same. The resultant m.m.f. is found by superposing the three m.m.f. rectangles, as shown in Fig. 151 (*c*). The resulting m.m.f. wave is "stepped,"

as shown. Due to fringing, the resultant flux wave is nearly sinusoidal, as indicated in the figure. A non-salient pole alternator, therefore, having a distributed winding, has usually a better wave shape than a salient-pole alternator.

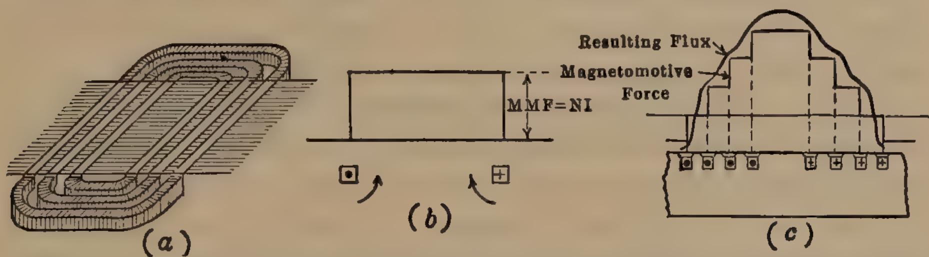


Fig. 151.—Distributed field winding and resulting magnetomotive force and flux waves.

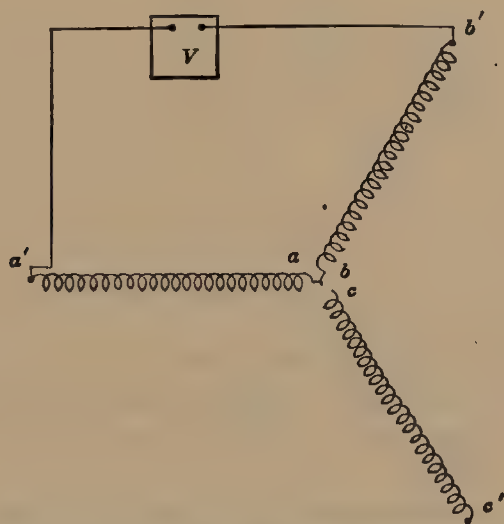


Fig. 152.—Connecting alternator coils in Y.

84. Phasing Alternator Windings.—Three-phase alternator windings may be connected either in Y or in delta. Instances often occur in practice where six leads come from the machine, these leads being the three pairs of terminals from the three phases. The proper phase relations must be observed in making the connections, whether they are to be connected in Y or in delta.

Let *aa′*, *bb′*, and *cc′* (Fig. 152) be the three coil windings of a three-phase machine.

Assume, first, that these three windings are to be connected in Y. First, connect ends a and b together. Measure $E_{a'b'}$, the voltage across their open ends. This should equal $\sqrt{3}$ times the coil voltage. It may be equal to the coil voltage, in which case, one coil should be reversed. Next, tie the end c of coil cc' to point ab. The voltages $E_{b'c'}$ and $E_{a'c'}$ should each be $\sqrt{3}$ times the coil voltage. If not, the coil cc' should be reversed.

If it be desired to connect the coils in delta, the ends a and b' (Fig. 153 (a)) should first be connected. The voltage $E_{a'b}$, across their open ends, should be equal to the coil voltage. If not, one of these two coils should be reversed. End c' of coil cc' should then be connected to b. The voltage $E_{ca'}$, across the open ends should be zero, as shown by the vector diagram in (b) (see

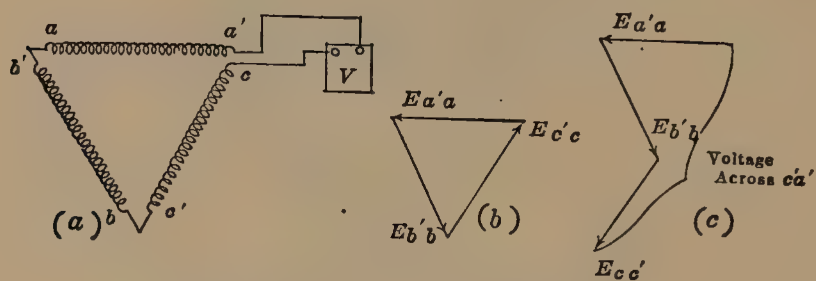


Fig. 153.—Connecting alternator coils in delta.

Par. 70, p. 111). If this voltage is practically zero, the two ends c and a' may then be closed. The voltage $E_{ca'}$ may be twice the coil voltage, as shown in (c). If this is found to be the case, coil cc' should be reversed.

85. Rating of Alternators.—The rating of electric machinery is determined, in general, by its temperature rise. This temperature rise is caused by the losses in the machine. The I^2R loss in the armature, due to the load current, limits the output of a machine. This loss depends upon the value of the armature current and is independent of power factor. For example, 100 amp. in a single-phase, 200-volt generator will produce the same I^2R loss, whether the load power factor be unity, 0.4, or any other value. The output in *kilowatts*, however, is proportional to the power factor. If the above generator is limited to 100 amp., its output will be 20 kw. at unity power factor but only 8 kw. at 0.4 power factor. The rating is 20 kv-a. regardless of power factor.

For the above reasons, alternators are ordinarily rated in kilovolt-amperes. If a machine is rated in kilowatts, unity power factor is assumed, unless otherwise specified. In stating the output of a machine, it is always well to state the power factor.

The rating of the prime mover driving an alternator is independent of the alternator power factor and is determined entirely by the kilowatt load. The same turbine could be used to drive a 200-kv-a. machine operating at 0.5 power factor or a 100-kv-a. machine operating at unity power factor, although the first alternator would have double the kilovolt-ampere rating of the second.

CHAPTER VII

ALTERNATOR REGULATION AND OPERATION

Alternator Regulation.—It is shown in Vol. I (Chap. XI) that the voltage of a shunt generator drops as load is applied. This is due to three causes—the I_aR_a drop in the armature, armature reaction, and the drop in field current which results from the decrease in terminal volts. As commercial alternators are excited from a separate source, there is no decrease of field current due to the drop in the alternator terminal voltage. Both the I_aR_a drop in the alternator armature and armature reaction, however, ordinarily cause a drop of terminal voltage as load is applied. Another factor which causes the alternator voltage to drop with application of load is the *reactance* of the alternator armature. This will be discussed later.

The regulation of direct-current generators is inherently better than the regulation of alternators. For example, shunt generators of commercial size regulate very closely, and it is usually possible to compound a shunt generator so that its terminal voltage is practically constant at all loads. In the alternator, the armature reactance drop, which is not present in the direct-current generator, and the greater effect of armature reaction result in poorer regulation. In addition, alternators cannot be compounded readily.

The regulation of the alternator depends not only on the magnitude of the current but on the power factor as well. A knowledge of the regulation of an alternator at various power factors is usually essential, since the amount by which the voltage varies with the load has an important bearing on the operation of the system as a whole. If the machine supplies incandescent lamps, it must regulate very closely, or else special regulators are necessary on the lighting circuits. Alternators, moreover, may regulate well at unity power factor, while, at low power factors, the regulation may be very poor, even if the *current* be the same in the two cases.

In the larger types of alternator, the large values of current which result from short circuit may cause serious damage to the machine and to the system. The value of this short-circuit current is closely related to the regulation of the machine, so that a knowledge of the regulation is helpful in designing the circuit breakers, switches, power-limiting reactances, etc.

The excitation power and the rating of the exciter also depend on the regulation. These are very important.

It is very desirable, therefore, to understand the factors and the reactions that affect the regulation and the operation of alternators. As it is usually impossible to obtain the requisite loads for testing an alternator under actual load conditions, it becomes necessary, in determining the regulation, to employ methods which do not require the actual loading of the machine. These methods will be described later.

86. Armature Reactance.—When a current flows in the conductors of an alternator armature, magnetic lines are set up around these conductors. Such lines are indicated around the conductors of one phase on a smooth-core armature, in Fig. 154. This magnetic leakage flux linking with the current gives *inductance* to the armature conductors. This inductance when multiplied by 2π times the frequency gives the *reactance* of the conductors. Alternating current flowing in these conductors will encounter not only resistance, therefore, but reactance as well.

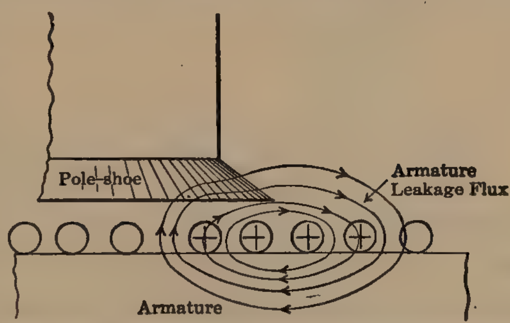


Fig. 154.—Flux linked with armature conductors on smooth-core armature.

Figure 155 (a) shows the conductors lying in a rather deep and narrow slot of an iron-clad armature. The current flowing in these conductors produces magnetic lines whose path is across the slot and back through the armature iron, as shown in the figure. The reluctance of this local magnetic circuit lies almost entirely in the slot itself, as the reluctance of that part of the path which lies in the iron is practically negligible. A deep, narrow slot will allow more lines per ampere-conductor to cross

it than the shallow and wider slot shown in Fig. 155 (*b*). Hence, an alternator with deep, narrow slots will have a much higher armature *reactance* than one with shallow, wide slots, other conditions being the same.

A semiclosed slot, such as that shown in Fig. 155 (*c*), will have considerably more magnetic flux per ampere-conductor than either of the slots of (*a*) or (*b*), because the overhanging tooth tips reduce the reluctance of the magnetic circuit. Thus, the reactance of a machine may be controlled in part by the design of the slot. In a smooth-core armature, like that shown in Fig. 154, the armature reactance will be small as compared with that of the slotted type of armature.

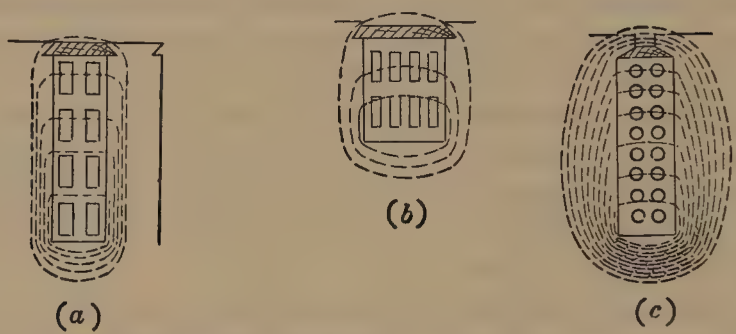


FIG. 155.—Slot leakage flux which produces armature reactance.

A certain amount of reactance is due to the magnetic flux linking the coil ends. Although this is small compared to the reactance due to the slot linkage, it cannot be neglected, as a rule.

It is pointed out in Vol. I (Chap. VIII) that the inductance varies as the *square* of the number of turns. This same rule applies to the conductors in alternator slots. If the number of series conductors in a slot is *doubled*, the reactance per slot is increased four times, other conditions remaining unchanged.

As the reactance is proportional to the frequency ($X = 2\pi fL$), the reactance of a 25-cycle alternator will be considerably less than that of a 60-cycle alternator, other conditions being the same.

87. Armature Resistance.—The armature iron forms a considerable portion of the path of the flux which links the armature conductors (Figs. 154 and 155). Since this flux is alternating, it is accompanied by hysteresis and eddy-current losses, which

occur in the iron immediately surrounding the slots. As this flux is produced by the armature current, the power represented by this loss must be supplied by the *armature current.* The eddy-current loss varies as the square of the flux density, and the hysteresis loss varies as the 1.6 power of the flux density. As the leakage flux is nearly proportional to the current, the eddy-current loss varies as the square of the current, and the hysteresis loss as the 1.6 power of the current, practically. The combined loss varies nearly as the square of the current.

The effect of these local iron losses is to increase the total loss due to the flow of current through the armature. As these local losses vary nearly as the current squared, their effect is practically the same as if the resistance of the armature were increased (see p. 52, Par. 27).

Unless the armature conductors are small in cross-section, the effect of the slot leakage flux is to force the current toward the top of the slot, so that the current density in the portions of a conductor near the top of the slot is greater than in those portions near the bottom of the slot. This also increases the effective resistance of the armature.

The effective resistance of an armature is, therefore, greater for alternating than for direct current, due to the alternating flux which accompanies the flow of the alternating current. The percentage increase depends, to a large extent, on the shape of the slots and the teeth and on the size of the conductors and ranges from 20 to 60 per cent. As the armature resistance drop is very small as compared with the voltage drops due to armature reactance and armature reaction, considerable error in determining the resistance introduces little error in most computations. The effective armature resistance may be determined by measuring the change in input with and without current flowing in the armature (see Par. 98). A more common, though less accurate, method is to measure the ohmic resistance with direct current and to increase this value by an estimated factor, such as 40 per cent., to cover the indeterminate losses.

88. Single-phase Armature Reaction.—In direct-current machines, the armature ampere-turns act on the magnetic circuit of the machine in such a way as to distort the air-gap flux and to change its magnitude. For a given armature current, the

direction and magnitude of this armature reaction depend on the position of the brushes (see Vol. I, p. 315). In an alternator, practically the same conditions exist. For a given armature current, the magnitude and direction of the armature reaction cannot depend upon brush position but do depend on the phase relation existing between current and voltage and, hence, on the power factor of the load.

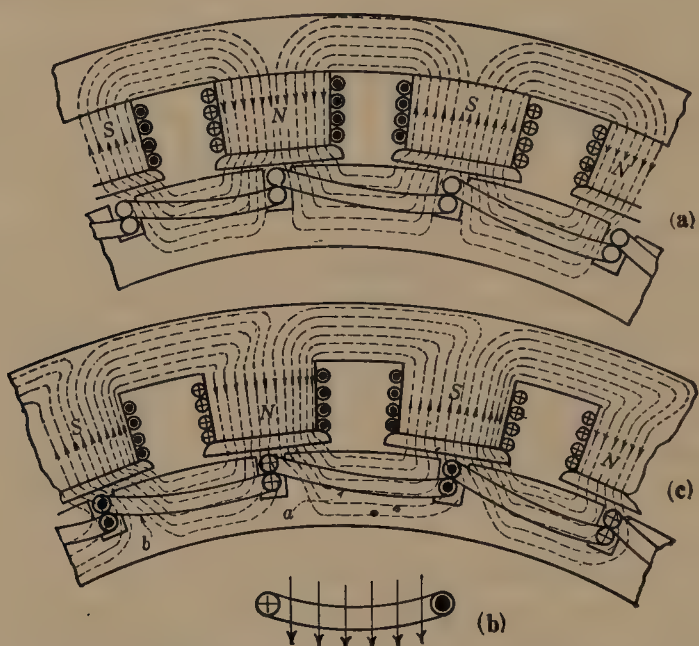
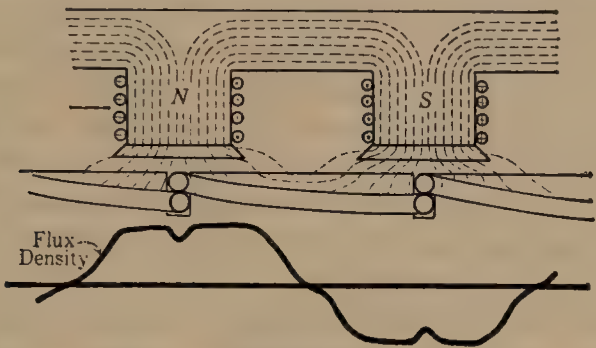


FIG 156.—Distortion of alternating flux by an in-phase current.

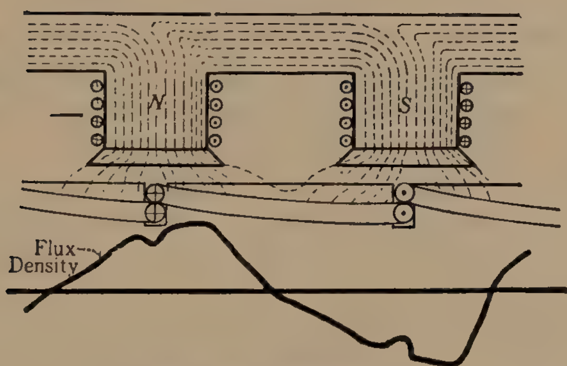
Figure 156 (*a*) shows the paths of the magnetic flux in the poles and armature of a single-phase, multipolar, salient-pole alternator, in which the armature moves from left to right. At the instant shown, the coil sides are directly under the pole centers, and the induced e.m.fs. must have their maximum values. Since the armature coils carry no current, they have no effect on the flux distribution. Hence, the flux distribution in (*a*) is determined entirely by the m.m.f. of the field coils.

If the armature circuit be closed, the armature will deliver current. If this current is in phase with the induced e.m.f., corresponding to a power factor of approximately unity (see Fig. 165, p 171), the current will have its maximum value when

the coil sides are directly under the centers of the poles (Fig. 156 (*b*)). The direction of the current will be as shown in (*b*) and (*c*), being inward in the conductors which lie under the *N*-poles. In coil *a*, the direction of the current flow is such that its magnetomotive force acts downwards, as shown in (*b*). On the other hand, the direction of the current in coil *b* is such that its m.m.f. acts upward. The effect on the main magnetic



(*a*) No Load



(*b*) Unity Power-factor Load

FIG. 157.—Flux distribution in air-gap of salient-pole alternator.

circuit of the current flowing in these coils is shown in Fig. 156 (*c*). The flux is increased on the right-hand side of each pole and decreased on the left-hand side. Were there no effect of saturation, the total flux would not be changed, as the increase on one side of the pole would be balanced by the decrease on the other side.

Figure 157 again shows the field structure and armature of a salient-pole multipolar alternator. The armature is now stationary, and the pole structure moves from right to left. Hence, the

relative directions of field and armature are the same as in Fig. 156. In (*a*) is shown the no-load flux distribution. As in Fig. 156 (*a*), this distribution is symmetrical about a vertical axis. Figure 157 (*b*) shows the flux distribution produced by the joint action of the field coils and the armature coils, when the armature current is in phase with the induced e.m.f., as in Fig. 156, (*b*) and (*c*). Although the total area under the flux wave is practically unchanged, the curve is now peaked on the side of the trailing pole tip and depressed on the other side. This occurs also in direct-current machines when the brushes are in the geometrical neutral (see Vol. I, p. 322, Fig. 268), and cross-magnetization alone results.

Fig. 158.—Vector diagram showing effect of armature reaction when current is in phase with induced e.m.f.

Because of this flux distortion, the maximum e.m.f. in a coil side no longer occurs when it is directly under the pole centers, but rather when it is to the right of this position (see Fig. 158).

It will be observed in Fig. 156, that the m.m.fs. of the armature coils are acting principally on the interpolar space, whose reluctance is high. In this position, therefore, the effect of the

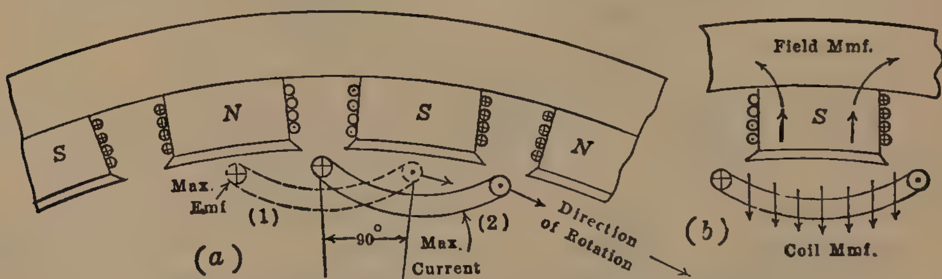


Fig. 159.—Armature reaction due to current lagging 90°.

coil ampere-turns upon the magnetic flux of the generator is a minimum. This does not apply to a non-salient pole machine, where the air-gap is uniform.

Figure 158 is a vector diagram representing these conditions. F_1 is the m.m.f. due to the field coils, A is the m.m.f. due to the armature coils, and F is the resultant of the two. When the current is in phase with the induced e.m.f., the space direction of the armature m.m.f. A is 90 electrical space-degrees from the resultant m.m.f. F. It will be observed (Fig. 158) that the prin-

cipal effect of A is to distort the alternator flux or to change it from its original position practically without altering its magnitude.

Figure 158 is a space-vector diagram of *m.m.fs.* The resultant *flux* is equal to the m.m.f. F divided by the reluctance of the magnetic circuit. In a non-salient pole machine, where the reluctance of the air-gap is uniform, ϕ may be found in terms of induced e.m.f. from the saturation curve. In salient-pole machines, this method is only approximate, as the reluctance of the magnetic circuit varies. The reluctance is a minimum when the space direction of F is along the pole centers and is a maximum when the space direction of F is midway between pole centers.

Figure 159 represents the conditions when the current lags 90° with respect to the induced e.m.f. When the coil is in position (1) (Fig. 159 (*a*)), the e.m.f. is a maximum, as in Fig. 156. The current is zero at this instant because it lags the induced voltage by 90°. The current does not reach its maximum value until the coil has traveled 90 electrical space-degrees farther and has reached position (2). The coil then lies directly under a south pole. It will be noted that the m.m.f. of this coil is *downward* and is, therefore, in direct opposition to the magnetic flux entering the south pole. *Therefore, when the current lags the induced electromotive force by* 90°, *it acts in direct opposition to the main field.* As a result, the field is materially weakened by a lagging current, and this is accompanied by a reduction of the induced electromotive force.

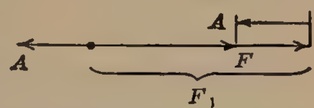


FIG. 160.—Vector diagram showing effect of armature reaction with current lagging 90°.

This result is similar to that of moving the brushes forward 90° in a direct-current generator. All the armature ampere-turns are then demagnetizing, tending to weaken the field.

It will be observed that this coil is acting directly upon a part of the magnetic circuit where there is iron, rather than on an interpolar space. For a given current in the armature, therefore, the coil in this position has a much greater effect upon the magnetic field of the machine than it had at unity power factor, shown in Fig. 157. Figure 160 shows vectorially the conditions which exist at zero power factor or 90° lagging current. The armature

reaction A acts in direct opposition to the impressed field F_1, so that the resultant field F is considerably smaller than F_1.

Figure 161 shows the conditions existing when the current leads the induced e.m.f. by 90°. As before, the electromotive force reaches its maximum value when the coil sides are directly under the pole centers (position (2), Fig. 161 (*a*)). The current, however, reaches its maximum value 90 electrical space-degrees ahead of this position or at (1). It will be observed that the ampere-turns of the coil now assist or strengthen the main field,

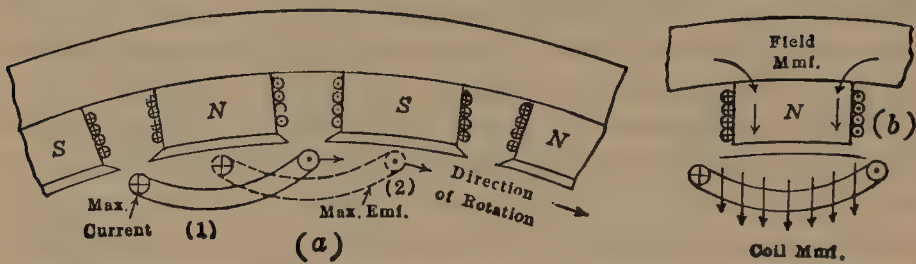


FIG. 161.—Armature reaction due to current leading 90°.

as they are acting directly in conjunction with it. The coil again is in the most favorable position so far as its effect upon the magnetic circuit of the machine is concerned. This condition may be represented by the vector diagram (Fig. 162). The m.m.f. of the field ampere-turns is F_1, that of the armature is A, and the resultant m.m.f. is their sum F, because the two are acting in the same direction.

It should be noted that for a given value of armature current, the effect of armature reaction varies with the power factor in a salient-pole type of machine, due to the varying reluctance caused by the salient poles. In a non-salient-pole generator, the air-gap is uniform around the periphery of the armature, so that the armature m.m.f. acts on a path of uniform permeance.

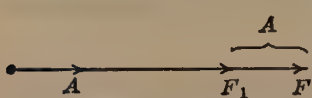


FIG. 162.—Vector diagram showing effect of armature reaction with current leading by 90°.

If the power factor has other than one of the three values just illustrated, the armature m.m.f. will add vectorially to the impressed m.m.f., according to the phase angle existing between the current and the induced voltage. This is illustrated in Fig. 163. The direction of the armature reaction is shown at a power-

factor cos θ, the current lagging, θ being the angle between the current and the *terminal* voltage. F_1 is the field m.m.f. When the current is in phase with the *induced e.m.f.*, the direction of the armature reaction is along A_1, 90° behind the resultant m.m.f. F (Fig. 158). When the current is in phase with the *terminal* voltage, the armature m.m.f. acts in the direction A_2, because the terminal voltage lags the induced e.m.f. by an angle α (see Fig. 165 (*b*)). If the current lags the terminal voltage by θ degrees, its m.m.f. must act along A, $\theta°$ behind A_2. Combining the m.m.f. A with the impressed m.m.f. F_1 gives the resultant m.m.f. F.

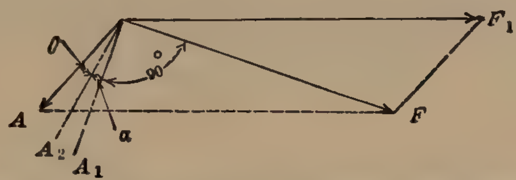


FIG. 163.—Vector diagram of armature reaction with current lagging $\theta°$.

Under the usual conditions of operation, the power factor is neither unity nor zero. Hence, the armature reaction may strengthen the field or may weaken it, according as the current leads or lags, and at the same time may distort it.

89. Polyphase Armature Reaction.—In Par. 88, the armature reaction was discussed under single-phase conditions and for maximum instantaneous current values. The current in the individual armature coils varies with time as the coils move along the air-gap (relative to field). Under these conditions, it can be shown that the armature reaction is pulsating with relation to the field poles. This produces double-frequency flux pulsation in the pole structure, which induces eddy currents. By Lenz's law, these eddy currents and the currents induced in dampers (see Fig. 319, p. 362), if such exist, tend to suppress these flux pulsations. The *average* effects of armature reaction, however, are those given in Par. 88.

If equal polyphase currents flow in polyphase windings and the space displacement of the windings in electrical degrees is equal to the phase displacement of these currents in time-degrees, a *rotating field* results. These conditions exist in alternator windings. The fundamental component of such rotating fields

is constant at all times. The production of such fields is described under Induction Motor (p. 267).

In Fig. 164 are shown such conditions in a three-phase alternator at the instant when the current in phase A is at its maximum positive value and the currents in phases B and C are one-half their maximum values and negative (see Fig. 95, p. 107). The flux distributions are assumed to be sinusoidal. The field poles are moving from left to right. The centers of the flux wave F_1,

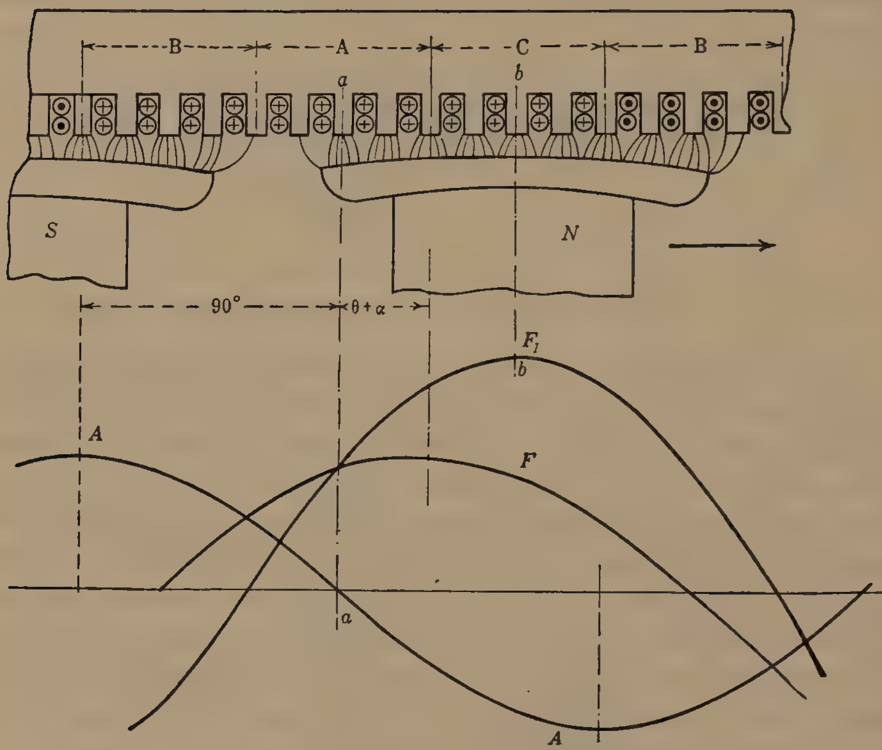


Fig. 164.—Field and armature m.m.fs. in polyphase alternator.

produced by these poles alone, occur at the center of the poles. The resultant flux wave is F (see Fig. 163). When the center of this resultant flux wave is at the center of the phase A belt (line aa), the induced e.m.f. in phase A is a maximum. The current lags this induced e.m.f. by an angle $(\theta + \alpha)$ (Fig. 166 (b)). Hence, F has moved along the air gap $(\theta + \alpha)$ degrees before the current in phase A reaches its maximum value. The currents in phases B and C are both negative and equal to one-half their maximum values. These conditions are shown by the crosses

and dots on the conductors (Fig. 164). The flux due to armature reaction, that is, due to the combined action of the currents in phases A, B, and C, is shown at A. This flux travels around the air-gap at synchronous speed, lagging the resultant flux F in space by $(90° + \theta + \alpha)$, (see Fig. 163) and is constant in value. The resultant flux density F is found by adding the ordinates of wave A to those of F_1. Hence, with constant load, the sine component of polyphase armature reaction is constant and is fixed in its relation to the field poles at all times.

90. Armature Impedance Drop.—In a direct-current generator, the induced armature voltage is obtained by adding *numerically* the IR drop in the armature and the terminal voltage. In the alternator, the armature reactance drop as well as the armature

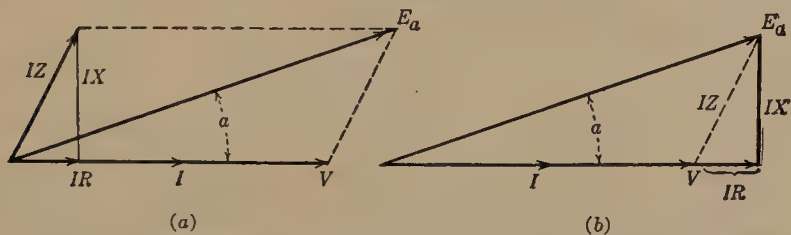


FIG. 165.—Alternator vector diagram for unity power factor.

resistance drop must be added to the terminal voltage in order to obtain the induced armature voltage. These voltage drops must be added *vectorially* to the terminal voltage, in order to obtain the induced e.m.f. That is, the e.m.f. induced in an alternator armature is the terminal voltage plus the armature *impedance drop*, this addition being performed vectorially.

Current in Phase with Terminal Voltage.—Figure 165 (a) shows the conditions existing when the load power factor is unity. V is the generator terminal voltage, and I is the armature current in phase with V. The IR drop in the armature is in phase with the current I, R being the *effective* resistance of the armature. The IX drop leads the current by 90° and is laid off at the end of IR. The vector sum of these two gives the IZ drop in the armature. This impedance drop when added vectorially to the terminal voltage V gives the e.m.f. E_a induced in the alternator armature. The vector addition is performed by completing the parallelogram having V and IZ for its adjacent sides. The

diagonal E_a is the vector sum of IZ and V and represents the induced e.m.f.

The same result is obtained by adding the IR drop directly to V (Fig. 165 (*b*)) and then adding the IX drop, at right angles to I and leading, at the end of IR. The vector addition in this case is made by the use of the triangle of vectors described in Chap. I (p. 13). The impedance drop IZ is shown dotted in Fig. 165 (*b*), as it is not used in obtaining E_a by this particular method.

It is to be noted that, with a load of unity power factor, the current is in phase with the *terminal* voltage but lags the generator *induced* voltage by an angle α.

It is a simple matter to find E_a if the other quantities are known. E_a is the hypotenuse of a right triangle of which $(V + IR)$ is one side and IX the other.

$$E_a = \sqrt{(V + IR)^2 + (IX)^2}. \tag{71}$$

Example.—A 60-kv-a., 220-volt, 60-cycle alternator has an effective armature resistance of 0.016 ohm and an armature reactance of 0.070 ohm. What is its induced e.m.f. when the machine is delivering its rated current at a load power factor of unity?

$$\text{The current } I = \frac{60{,}000}{220} = 273 \text{ amp.}$$

$$IR = 273 \times 0.016 = 4.37 \text{ volts.}$$
$$IX = 273 \times 0.070 = 19.1 \text{ volts.}$$
$$E_a = \sqrt{(220 + 4.4)^2 + (19.1)^2} = 225 \text{ volts.} \quad \textit{Ans.}$$

Lagging Current.—When the current lags the terminal voltage by the angle θ, the same method is employed to calculate the induced e.m.f. Figure 166 (*a*) shows the current I lagging terminal voltage V by the angle θ. The IR drop is along the current vector I, and the IX drop is in quadrature with I and leading, as before. The resulting impedance drop IZ is then found, being the resultant of IR and IX. This impedance drop is then added vectorially to V, giving the armature induced e.m.f., E_a. It will be noted (Figs. 165 and 166) that the position of the armature impedance triangle is determined by the current and not by the generator voltage. When, therefore, the current lags, this impedance triangle swings clockwise with the current.

As before, the impedance drop may be added at the end of V, if the proper phase relations are observed. The most direct method

of finding the induced e.m.f. E_a is to use the method described under the triangle of vectors (p. 14). IR, which is in phase with the current, is first added vectorially at the end of the terminal voltage V. Then the reactance drop IX, at right angles to the current and leading, is added at the end of IR. The resultant voltage found by completing the polygon is the induced e.m.f. E_a. This method is illustrated in Fig. 166 (*b*), where IR is parallel to I, and IX is at right angles to I and leading. The geometrical solution of this diagram is quite simple. If IR is projected on the current vector I, a right triangle of voltages,

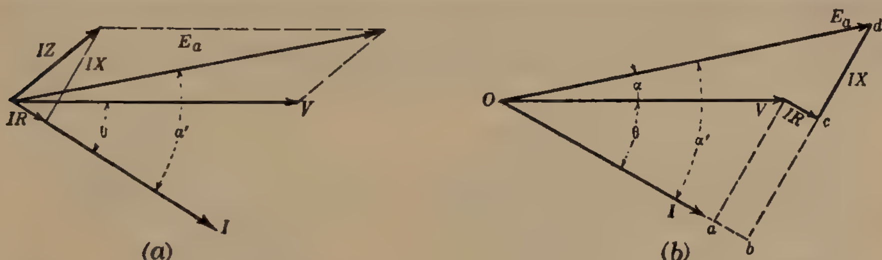


FIG. 166.—Alternator vector diagram for power factor $\cos\theta$, current lagging.

Obd, is formed, of which E_a is the hypotenuse. The values of the two legs of this right triangle may be found as follows:

$$Oa = V\cos\theta$$
$$ab = IR$$
$$aV = bc = V\sin\theta$$
$$cd = IX$$
$$E_a = \sqrt{\overline{Ob}^2 + \overline{bd}^2} = \sqrt{(Oa + ab)^2 + (bc + cd)^2}$$
$$= \sqrt{(V\cos\theta + IR)^2 + (V\sin\theta + IX)^2} \qquad (72)$$

The current now lags the induced voltage E_a by the angle α', which can be readily determined.

$$\tan\alpha' = \frac{bd}{Ob} = \frac{V\sin\theta + IX}{V\cos\theta + IR}$$

Example.—Determine E_a for a load in which the power factor is 0.7, current lagging, using the constants of the example on p. 172.

The rating of an alternator, as has already been pointed out, depends on the current or kilovolt-amperes rather than the kilowatts. The current rating of the generator, therefore, will remain unchanged, although the kilowatts in this problem are reduced to 0.7 of their former value.

$\cos\theta = 0.70$ $\qquad$ $IR = 4.37$ volts as before.

$\theta = 45.6°$

$\sin\theta = 0.7145$ $\qquad$ $IX = 19.1$ volts as before.

$$E_a = \sqrt{(220 \times 0.70 + 4.4)^2 + (220 \times 0.7145 + 19.1)^2} = 237 \text{ volts. } Ans.$$

It is to be noted that the induced e.m.f. is now higher than before, although the value of the impedance drop itself is the same. For a fixed value of induced e.m.f., therefore, the terminal volts become less with increasing lag of the current, even though the value of the current remains unchanged. This is due to the angle at which the impedance drop subtracts from the induced e.m.f. It would be expected, therefore, that the regulation of an alternator would be poorer for lagging current.

At unity power factor, the armature resistance drop is the important factor in determining the value of E_a. With a lagging current, the resistance drop plays but a small part, and the armature *reactance* drop becomes the important factor.

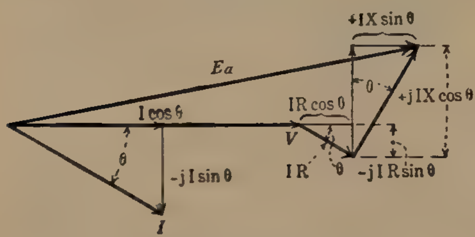


Fig. 167.—Alternator vector diagram in complex-lagging current.

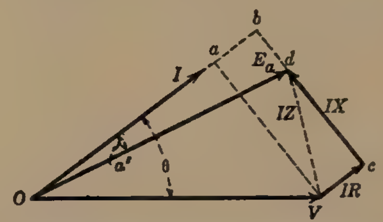


Fig. 168.—Alternator vector diagram for power factor cos θ, leading current.

The foregoing relationships may also be determined by the use of complex algebra. That is,

$$\begin{aligned} \underset{\cdot}{E}_a &= V + I(\cos\theta - j\sin\theta)(R + jX) \\ &= V + IR\cos\theta - jIR\sin\theta + jIX\cos\theta + IX\sin\theta. \qquad (73) \end{aligned}$$

Each of these quantities is given in Fig. 167.

$$\begin{aligned} \underset{\cdot}{E}_a &= (V + IR\cos\theta + IX\sin\theta) + j(IX\cos\theta - IR\sin\theta) \\ &= e_1 + je_2. \end{aligned}$$

In the foregoing example,

$$\begin{aligned} \underset{\cdot}{E}_a &= 220 + 273(0.70 - j0.7145)(0.016 + j0.070) \\ &= 220 + 3.06 - j3.12 + j13.38 + 13.65 \\ &= 220 + 16.72 + j10.26 = 236.7 + j10.3 \\ |E_a| &= \sqrt{(236.7)^2 + (10.3)^2} = 237 \text{ volts.} \quad \textit{Ans.} \end{aligned}$$

Leading Current.—Figure 168 shows the alternator vector diagram when the current *leads* the terminal voltage by an angle θ. As the current changes its phase relation with respect to the terminal voltage V, the impedance triangle swings with the current in a counterclockwise direction about the end of V. E_a is found in

the same manner as in Fig. 166. The voltage drop IR, parallel to the current, is projected on the current vector.

$$\begin{aligned} Oa &= V\cos\theta \\ ab &= IR \\ aV &= bc = V\sin\theta \\ cd &= IX \end{aligned}$$

$$E_a = \sqrt{\overline{Ob}^2 + \overline{bd}^2} = \sqrt{(Oa + ab)^2 + (bc - cd)^2} = \sqrt{(V\cos\theta + IR)^2 + (V\sin\theta - IX)^2} \quad (74)$$

This differs from Eq. (72) only in the sign of IX, which is now negative.

Example.—Repeat the foregoing problem when the power factor is 0.7, current leading.

$$\cos\theta = 0.70 \qquad IR = 4.37 \text{ volts}$$
$$\sin\theta = 0.7145 \qquad IX = 19.1 \text{ volts}$$

$$E_a = \sqrt{(220 \times 0.70 + 4.4)^2 + (220 \times 0.7145 - 19.1)^2} = 210 \text{ volts.} \quad \textit{Ans.}$$

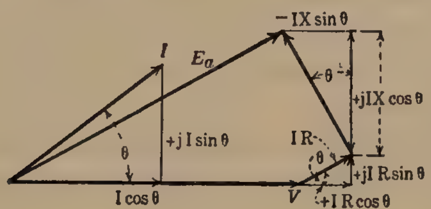


FIG. 169.—Alternator vector diagram in complex-leading current.

The induced e.m.f. in the armature is now *less* numerically than the terminal voltage. This is a condition which cannot exist in a direct-current generator. It results from the phase position of the IZ drop with respect to V.

The foregoing relationships may likewise be determined by complex algebra.

$$\begin{aligned} \dot{E}_a &= V + I(\cos\theta + j\sin\theta)(R + jX) \\ &= V + IR\cos\theta + jIR\sin\theta + jIX\cos\theta - IX\sin\theta. \end{aligned} \quad (75)$$

Each of these quantities is given in Fig. 169.

In the foregoing example,

$$\begin{aligned} \dot{E}_a &= 220 + 273(0.70 + j0.7145)(0.016 + j0.070) \\ &= 220 + 3.06 + j3.12 + j13.38 - 13.65 \\ &= 220 - 10.59 + j16.50 = 209.4 + j16.5 \\ |E_a| &= \sqrt{(209.4)^2 + (16.5)^2} = 210 \text{ volts.} \quad \textit{Ans.} \end{aligned}$$

91. Alternator Regulation.—The voltage E_a, determined in the preceding paragraphs, is the voltage *induced* in the alternator armature under load conditions. In practice, it is a quantity difficult to measure and can be calculated only approximately. There is no simple method of making a direct measurement of the armature reactance X. As a matter of fact, it is seldom necessary to know either the value of E_a or that of the armature reactance X.

A knowledge of the voltage regulation is very important, because it shows how closely a machine will maintain its voltage under the various conditions of load, from no load to full load.

If there were no armature *reaction*, E_a would be the no-load voltage of the machine, just as in a separately excited direct-current generator the induced voltage under load would be equal to the no-load voltage if there were no armature reaction. The effect of armature reaction is to change the value of the magnetic flux, and this is accompanied by a corresponding change in the value of the induced e.m.f. E_a. The effect of armature reaction on the operation of the machine is analyzed in the methods for determining regulation.

It is usually impossible to find the regulation of an alternator by actual loading, particularly in the larger sizes, until after the machine has been put into service, and even then it may be difficult to secure the desired adjustment of the load. To set up a generator for a load test requires a machine for driving purposes, and considerable power may have to be supplied and absorbed. With polyphase generators, there is the added difficulty of obtaining a balanced load.

The regulation of a machine, however, may be calculated with sufficient accuracy from data obtainable from open-circuit and short-circuit tests. These tests involve very little power supply and do not require any power-absorbing devices. There are three common methods for determining regulation—the *synchronous-impedance* or *e.m.f. method*, the *m.m.f. method*, and the *A. I. E. E. method*. The application and limitations of each method will be discussed in some detail.

92. Synchronous-impedance Method, or E.m.f. Method.—This method is often called the *pessimistic method*, because it gives a value of the regulation *poorer* than the actual regulation.

In the synchronous-impedance method, the effect of armature *reaction* is combined with the effect of armature *reactance*. That is, the armature reactance is increased a sufficient amount over its actual value to allow for the effect of armature reaction. That this may be done is shown as follows:

In Fig. 170, a sine distribution of flux along the air-gap is assumed. The line *ab* is the coil axis. When the coil axis lies

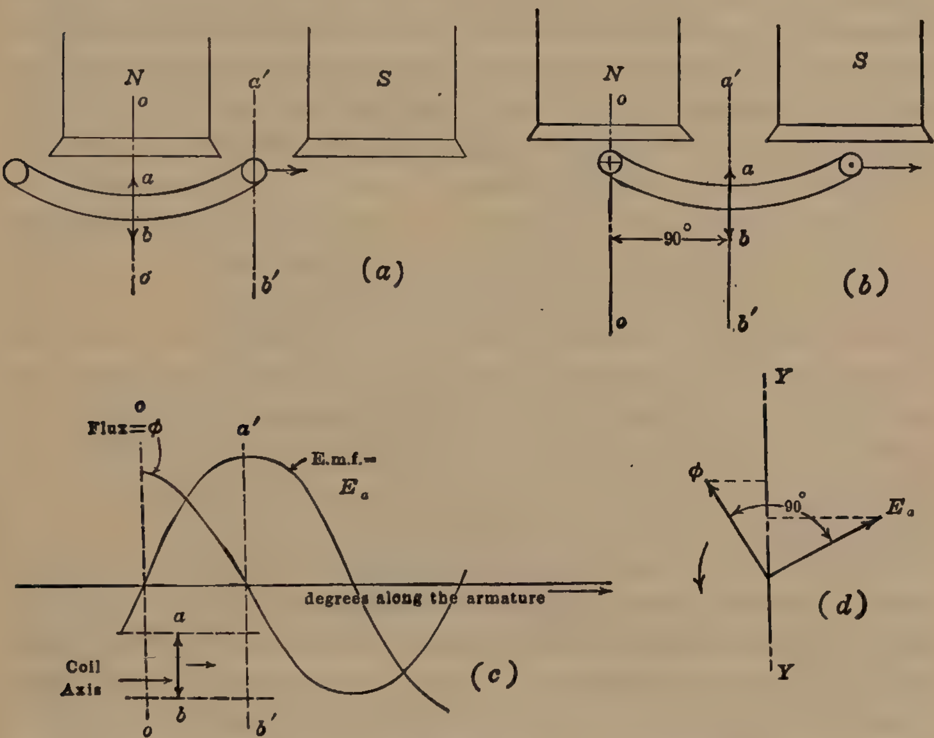


Fig. 170.—Relation of flux linking alternator coil to induced e.m.f. in coil.

along the pole axis *oo*, as is shown in (*a*), the flux linking the coil is a maximum. When the coil axis *ab* reaches position *a′b′* as shown in (*b*), the flux linking the coil is zero. The flux linking the coil varies, therefore, with the time as the coil moves along the air-gap. The frequency at which this flux varies is the same as the frequency of the induced e.m.f. In position (*a*), the flux linking the coil is a maximum, and the induced e.m.f. is zero. In position (*b*), the flux linking the coil is zero, and the induced e.m.f. is a maximum. It is seen that the e.m.f. induced in the coil reaches its maximum value 90 electrical space-degrees

later than the flux linking the coil and, therefore, later in time. The flux linking the coil may then be said to *lead* by 90° the e.m.f. which it induces.

This relationship of flux and e.m.f., as the coil moves along the gap, is shown graphically in Fig. 170 (*c*).

When the coil axis *ab* lies along the pole axis *oo*, the flux linking the coil is a maximum, and the induced e.m.f. E_a is zero. As the coil axis *ab* moves to the right, the flux ϕ linking the coil decreases sinusoidally, and the induced e.m.f. E_a increases sinusoidally. When the coil axis *ab* reaches *a′b′*, midway between pole centers, the flux linking the coil is zero, and the induced e.m.f. E_a is a maximum. Under the conditions assumed, the flux wave leads the e.m.f. wave by 90°, as shown.

These space relations may also be shown by rotating vectors (Fig. 170 (*d*)). The vector ϕ is equal to the maximum value of the flux linking the coil, and the vector E_a is equal to the maximum value of the induced e.m.f. Each position of these two rotating vectors represents a different position of the armature coil relative to the field poles. The instantaneous value of either quantity, ϕ or E_a, is found by projecting its vector on the vertical axis YY. It is seen that the flux ϕ reaches its maximum value 90 space-degrees in advance of the e.m.f. E_a.

Figures 170 (*c*) and (*d*) are *space-phase* diagrams. Figure 170 (*c*) shows the flux linking the coil and the induced e.m.f. in the coil for different space positions of the coil as it moves relative to the field poles. Figure 170 (*d*) shows these same quantities as rotating vectors.

Although ϕ, the flux linking the armature coil, and E_a, the induced e.m.f. in the coil, vary with the *space* position of the coil, they vary also with the *time*. When the coil moves through 360 electrical degrees in *space* with respect to the poles, the e.m.f. wave passes through 360 electrical degrees in *time*. The time of doing this is $1/f$ sec., where f is the frequency in cycles per second. The time required, therefore, for the coil to pass through a given number of electrical *space*-degrees is equal to the time required for the e.m.f. to pass through an equal number of electrical *time*-degrees. For this reason, a *space-phase* diagram and a *time-phase* diagram may often be combined, just as the angular variation of e.m.f. (Chap. I, Fig. 3, p. 4) was changed to the time varia-

tion of e.m.f. (Fig. 5, p. 6). The space-phase diagrams of Figs. 170 (*c*) and (*d*) may also be considered as time-phase diagrams.

Figure 171 shows the vector diagram of an alternator, in which the current I is in phase with the induced e.m.f. E_a. As F is the resultant field, E_a must lag F by 90°. It was shown in Fig. 158 (p. 166), that under these conditions the armature reaction acts at right angles to the resultant field F. The armature m.m.f., therefore, or the armature reaction A must have a space position of 90° behind the resultant field F. This brings it in phase with E_a and, therefore, in phase with the current I, as it should be, of course. As F is the resultant field, it must be the vector sum of the impressed field F_1 and the armature reaction field A, as shown in the vector diagram.

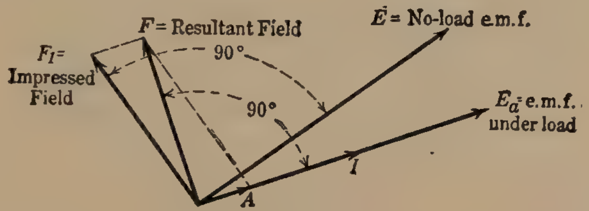


Fig. 171.—Vector diagram of alternator m.m.fs. and e.m.fs.

In a non-salient pole machine, the space direction of the resultant flux will be the same as that of the resultant m.m.f. F. In a salient-pole machine, the space direction of the resultant flux usually is *not* the same as that of the resultant m.m.f. vector F, due to the fact that the flux tends to seek the paths of minimum reluctance. The flux, therefore, is distorted in the direction of the pole pieces. In salient-pole machines, this introduces errors in the methods used for predetermining alternator regulation.

If the armature reaction were zero, due to there being no load on the machine, the resultant field would obviously be the impressed field F_1. The *no-load* induced e.m.f. must be 90° behind F_1, as shown at E (Fig. 171), because the no-load induced e.m.f. lags the no-load field by 90°.

It will be recognized (Fig. 171) that F_1, F, A constitute a *space diagram of m.m.f. vectors* taken from Fig. 158. E_a is also a space vector when considered as being combined with the m.m.f.

diagram shown in Fig. 170 (*c*) and (*d*). As the linking of the resultant flux F with the armature coils also varies with time, as described on page 177, F may be considered as being a *time* vector. E_a is also a *time* vector, just as I and E are time vectors, so that it may be combined with them, also. Hence, E_a and F are connecting links between the *space* diagram of m.m.fs. and the *time* diagram of currents and voltages. The space and the time diagrams, therefore, may be combined into the one given in Fig. 171.

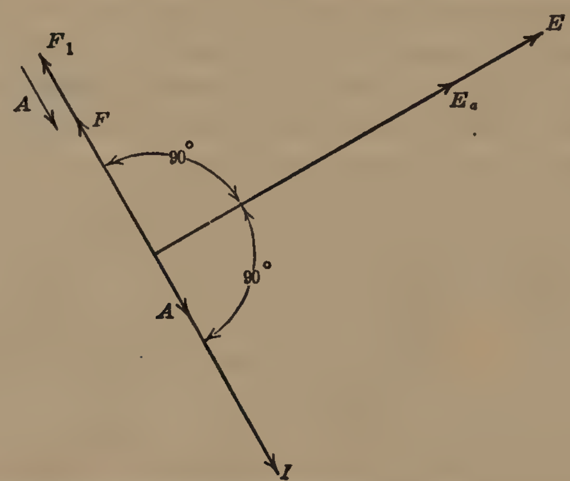


FIG. 172.—Relation of induced voltages to alternator field m.m.f., current lagging induced e.m.f. by 90°.

When the current lags the induced electromotive force by 90°, the armature reaction is in exact opposition to the resultant field (see Fig. 160, p. 167). Figure 172 shows the vector diagram for this condition. The current I lags the induced e.m.f. E_a by 90°. The armature reaction A being in direct opposition to F_1 is, therefore, in phase with the current. The resultant field F is found by subtracting the armature reaction A from the impressed field F_1. E is the no-load voltage due to field F_1.

In either Fig. 171 or Fig. 172, the no-load voltage E is found by adding vectorially a voltage E_aE to E_a, this voltage E_aE always being in quadrature with the current.

If the voltage E_aE adds in quadrature with the current, it must be in phase with the IX component of voltage already discussed. This is illustrated in Fig. 173. The current I is shown lagging the terminal voltage V by an angle θ. The inter-

nal voltage of the armature E_a is found by adding IR and IX vectorially to V. The resultant field F is 90° ahead of E_a. By adding voltage E_aE to E_a, and in quadrature with the current I, the no-load voltage E is found. The voltage E_aE does not actually exist under load, for E is the no-load induced e.m.f., and E_a the load induced e.m.f. E_aE, however, represents the drop in voltage due to the reduced flux caused by the *armature reaction* A. E_aE lags the armature m.m.f. vector $-A$ by 90° and would be proportional to $-A$ if there were no saturation of the iron. E_aE may then be considered as an e.m.f. induced by the armature reaction $-A$. *As a matter of fact, however, E_aE is a fictitious voltage which replaces the effect of change in flux due to armature reaction.*

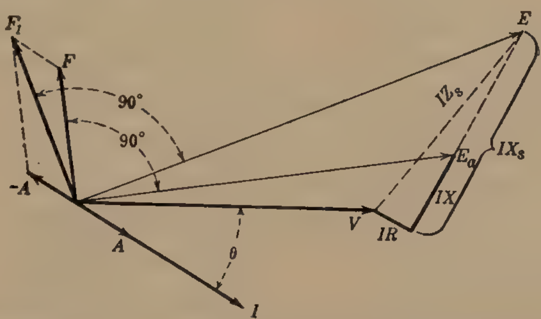


FIG. 173.—Complete vector diagram for synchronous-impedance method.

It is also evident that if IX be increased in value to IX_s, where $IX_s = IX + E_aE$, E may be computed without E_a being known. This assumes that the voltage E_aE is always proportional to the armature current, which is not strictly true.

The foregoing is the principle of the electromotive force or *synchronous-impedance* method. The *rational* or *general* method is first to compute E_a. Find, from the saturation curve, the field current F corresponding to E_a. Add $-A$ to F vectorially to find F_1, and then from the saturation curve find E corresponding to the field current F_1. One serious objection to this method is the difficulty of determining the armature leakage reactance X. It cannot be readily measured and can be only roughly calculated. These calculations and the general solution of the diagram are both laborious. The determination of the regulation is very much simplified if X be increased to the value X_s, so that E is found directly without knowing E_a. X_s is called the

synchronous reactance of the alternator. The corresponding impedance $Z_s(=\sqrt{R^2+X_s^2})$ is called the *synchronous impedance* of the alternator.

The synchronous reactance is determined experimentally as follows: The saturation curve of the alternator, E and I_f, is first determined in the usual manner and the curve plotted as shown in Fig. 174. The field is then made very weak, and the alternator armature is short circuited through an ammeter. The field is then gradually strengthened, and a new curve of armature current and I_f is determined. The field is increased until the armature current is almost twice its rated value. These two curves are shown plotted in Fig. 174.

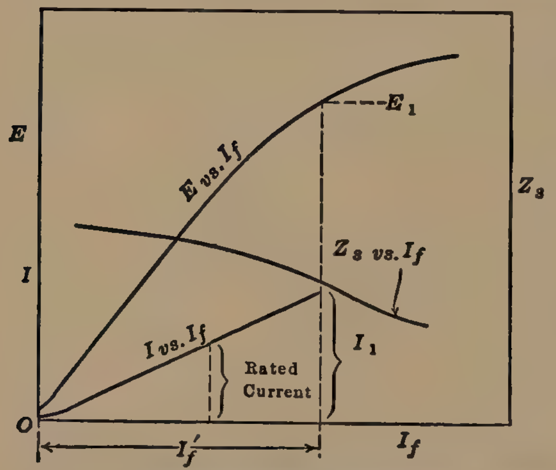


FIG. 174.—Open-circuit and short-circuit characteristics of alternator.

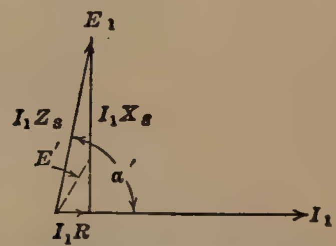


FIG. 175.—Short-circuit vector diagram of alternator.

Consider some value of field current I_f'. On open circuit, this field current produces a voltage E_1. On short circuit, the terminal voltage of the machine is practically zero. The voltage E_1 does not actually exist in the armature at short circuit, because of armature reaction. (The voltage actually induced is E', Fig. 175). If, however, the effect of the armature reaction is replaced by an armature reactance drop, the voltage E_1 may be considered as being entirely used in sending the current I_1 through the armature impedance. That is,

$$E_1 = I_1Z_s$$

where Z_s is the *synchronous impedance* of the armature. This short-circuit condition is represented vectorially in Fig. 175, where I is the short-circuit current and E_1 the assumed internal

e.m.f. of the armature. The synchronous impedance drop is made up of two components, I_1R, where R is the effective resistance of the armature, and I_1X_s, where X_s is the synchronous reactance of the armature.

Obviously,

$$Z_s = \frac{E_1}{I_1} \tag{76}$$

and

$$X_s = \sqrt{Z_s^2 - R^2}. \tag{77}$$

In practice, R is small compared with Z_s, and they combine almost in quadrature, so that

$$X_s = \frac{E_1}{I_1} \text{ very nearly.}$$

The value of the synchronous reactance depends, to a large, extent, upon the degree of saturation of the iron. For example, at low saturation the armature m.m.f. will have a much greater effect on the magnetic circuit than if the iron were saturated. Under short-circuit conditions, therefore, where the iron is operating at low saturation, the synchronous reactance will be *too large*. The variation of synchronous impedance with field current is shown in Fig. 174. As the iron becomes more saturated, the synchronous impedance *decreases*. Under operating conditions, the iron is considerably more saturated than it is under short-circuit conditions. In order to approach as nearly as possible to operating conditions, it is desirable to obtain the synchronous impedance at the highest possible value of armature current, as at I_1 (Fig. 174). Also, the synchronous impedance is determined at very low power factor, corresponding to short-circuit conditions, as shown by Fig. 175, where the angle α' between the current and the e.m.f. E_1 is nearly 90°. The armature current is a maximum, therefore, when the axes of the armature coils are almost opposite the pole centers, as shown in Fig. 159 (p. 166). As the armature m.m.f. has its maximum effect under these conditions, the value of the synchronous impedance so determined is too large for other positions of the coil, as shown, for example, in Fig. 157 (p. 165).

It will be seen that the value of synchronous impedance determined at short circuit is *too large* and will make the calculated

value of regulation too high. The synchronous impedance method is, therefore, called the *pessimistic* method. It is a safe method to use when making a guarantee, because the machine always regulates better than the computed values indicate.

The following example will illustrate the use of this method:

Example.—A 50-kv-a., 550-volt, single-phase alternator has an open-circuit e.m.f. of 300 volts when the field current is 14 amp. When the machine is short circuited through an ammeter, the armature current is 160 amp., the field current still being 14 amp. The ohmic resistance of the armature between terminals is 0.16 ohm. The ratio of effective to ohmic resistance may be taken as 1.2. Determine: (*a*) the synchronous impedance of the machine; (*b*) the synchronous reactance; (*c*) the regulation at 0.8 power factor, current lagging.

The rated current of the machine $I = 50{,}000/550 = 91$ amp.

(*a*) The synchronous impedance $Z_s = {}^{300}\!/_{160} = 1.87$ ohms.

The effective resistance $= 1.2 \times 0.16 = 0.192$ ohm.

(*b*) $X_s = \sqrt{(1.87)^2 - (0.192)^2} = 1.86$ ohms.

(*c*) $\cos \theta = 0.8 \qquad \sin \theta = 0.6.$

Applying Eq. (72) (p. 173).

$$E = \sqrt{[(550 \times 0.8) + (91 \times 0.192)]^2 + [(550 \times 0.6) + (91 \times 1.86)]^2}$$
$$= \sqrt{209{,}000 + 249{,}000} = 677 \text{ volts.}$$

The definition of *regulation* for an alternator is as follows:

In constant-potential alternators,[1] the regulation is the rise in voltage (when the specified load at specified power factor is reduced to zero) expressed in per cent. of rated voltage.

As the *synchronous* reactance was used in the foregoing problem, the armature reaction was taken into consideration, so that the no-load voltage of the machine is presumably 677 volts. The regulation, therefore, is

$$\frac{677 - 550}{550}100 = \frac{127}{550}100 = 23.1 \text{ per cent.} \quad \textit{Ans.}$$

It is to be noted in the foregoing problem that the synchronous impedance Z_s is practically equal to the synchronous reactance X_s, and in most cases it may be assumed, without appreciable error, as being equal to it.

93. Three-phase Application.—The preceding discussion and problems have all been applied to single-phase generators. This has been done merely to illustrate methods. Very poor results accompany the practical application of these methods to single-

[1] *A. I. E. E. Standards*, **7**, July, 1925, Rule 7–600.

phase alternators. In a single-phase alternator, the armature reaction is pulsating, even for a constant value of armature current. The flux in the poles pulsates, because of the variation of the current in the armature coils as they pass the poles (see p. 169). For this reason, the synchronous reactance is an indefinite quantity, and calculated results of regulation with single-phase machines are far from satisfactory.

In a polyphase machine, however, the armature reaction is substantially constant if the load be constant and balanced (see p. 169). When the m.m.f. of one phase has decreased, the resultant m.m.f. of the other two phases has increased, etc. The m.m.f. of the armature as a whole is practically constant in value, therefore, and is stationary in space with respect to the field poles. That is, if the field poles rotate, the armature m.m.f. follows them at rotor speed and is practically constant in magnitude for a fixed value of armature current.

Under these conditions, the synchronous reactance becomes a more definite quantity, and more satisfactory results are, therefore, obtainable with these various methods of testing. Figure 176 (*a*) shows the connections for making the open-circuit test of a three-phase alternator. This is substantially the same method as is used with direct-current generators. The field is excited from some direct-current source, and the field current is measured with an ammeter. The armature is driven at the rated or synchronous speed, and the open-circuit voltage measured for different values of field current. The voltage of one phase only need be measured as the phase voltages should all be equal. A frequency indicator F may be used for determining the speed of the machine. An additional resistance R_1 in the field circuit is often necessary for obtaining the points on the lower part of the saturation curve.

In the short-circuit test, all three phases must be short circuited. There are two methods of connecting the ammeters in this test. They may be connected in Y (Fig. 176 (*b*)), in which case the ammeters read the *line* current directly, or they may be connected in delta (Fig. 176 (*c*)), in which case the line current is obtained by multiplying the ammeter readings by $\sqrt{3}$ or 1.73. With delta connection, the ammeters need be only about half the range (1/1.73 or 0.58) necessary for the Y-connection. The

average of the ammeter readings is usually taken, although there should be but little difference in the three readings.

In calculating the regulation of a three-phase alternator, only one of its three phases is considered when making computations. The regulation, efficiency, etc., of one phase is determined, and,

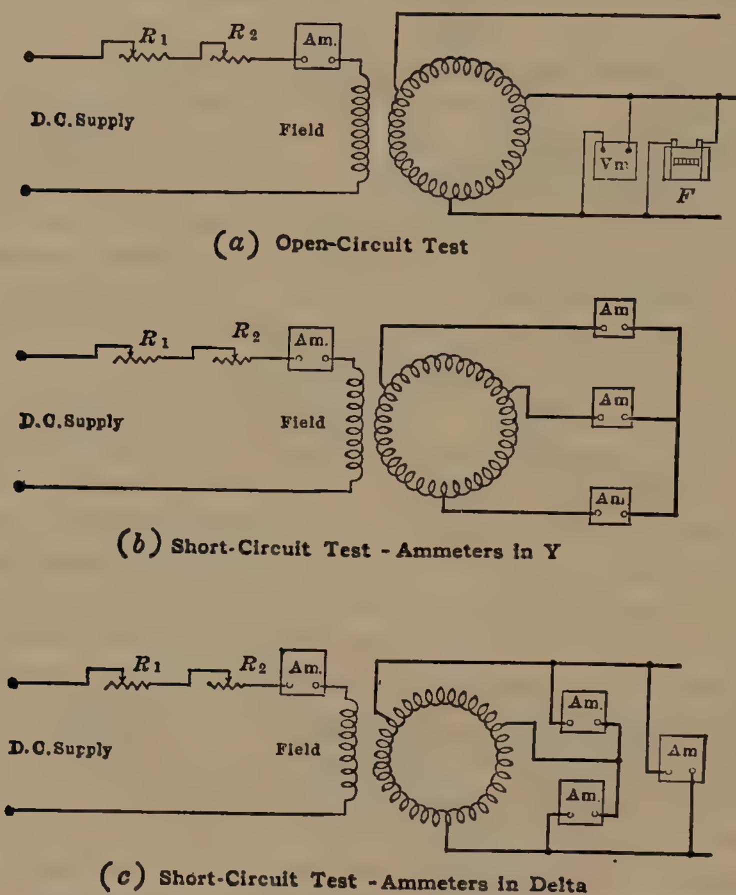


FIG. 176.—Connections for making open- and short-circuit tests of alternator.

the machine being symmetrical, the other phases will have similar characteristics. Only the single-phase calculations already described are necessary, therefore. Two conditions arise, one when the machine is considered as being Y-connected, and the other when it is considered as being delta-connected. In each case, only coil values of current and voltage are used.

94. Regulation of a Y-connected Generator.—It is impossible to determine whether a machine is Y-connected or delta-con-

nected unless the winding itself be inspected. Fortunately, it makes no difference, so far as calculation of the regulation is concerned, whether the machine be Y-connected or delta-connected. It may be assumed to be either, and the result is the same if the work is consistent.

If the machine is considered as being Y-connected, the coil voltage is equal to the line voltage divided by $\sqrt{3}$. The coil

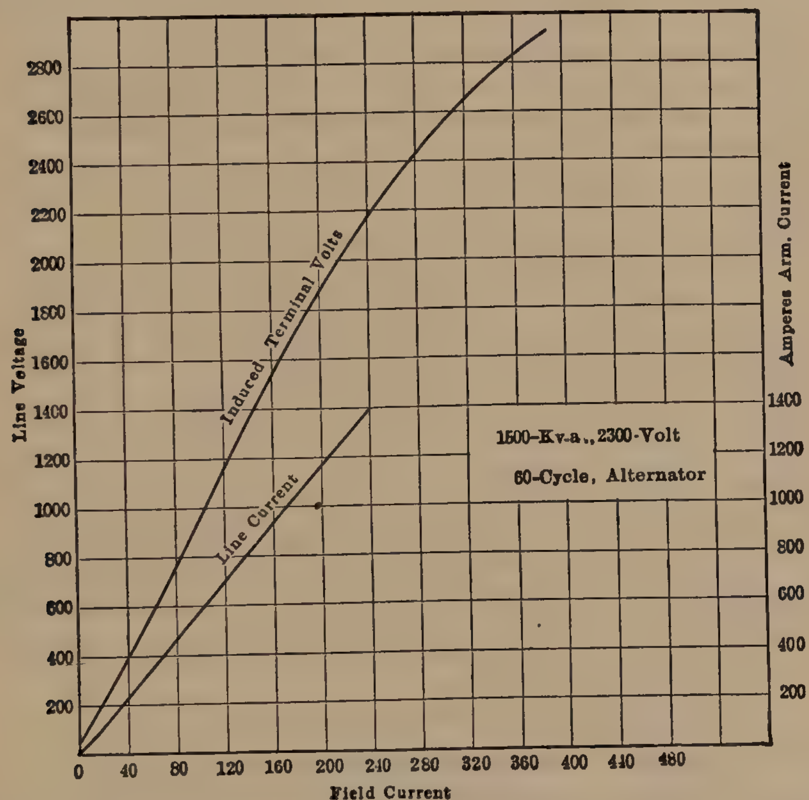


FIG. 177.—Open- and short-circuit characteristics of 1,500-kv-a. alternator.

current and the line current are the same. The method of dealing with such a problem is illustrated by the following example:

Example.—Figure 177 shows the open- and short-circuit characteristics of a 1,500-kv-a., 2,300-volt, 60-cycle alternator. Terminal volts and line current are plotted as ordinates with values of field current as abscissas. Assume that the machine is Y-connected. The resistance between each pair of terminals as measured with direct current is 0.12 ohm. Assume that the effective resistance is 1.5 times the ohmic resistance. Determine the

synchronous reactance of the generator and its regulation at 0.85 power factor, current lagging.

From Fig. 177, the maximum value of the short-circuit current is 1,400 amp., which is equal to the coil current. This corresponds to 240 amp. in the field, and at 240 amp. field current the open-circuit terminal voltage is 2,180 volts. The corresponding coil voltage is

$$\frac{2{,}180}{\sqrt{3}} = 1{,}260 \text{ volts}$$

$$Z_s \text{ (per coil)} = \frac{1{,}260}{1{,}400} = 0.90 \text{ ohm} = X_s \text{ nearly.}$$

If the resistance between terminals is 0.12 ohm, it includes two coils in series, as the Y-connection is assumed, so that the ohmic resistance per coil is $0.12/2 = 0.06$ ohm. The effective resistance per coil is equal to $1.5 \times 0.06 = 0.09$ ohm.

The rated current of the machine $= 1{,}500{,}000/2{,}300\sqrt{3} = 376$ amp. per terminal.

The rated voltage per coil $= 2{,}300/\sqrt{3} = 1{,}330$ volts.

The no-load volts per coil are found by applying Eq. (72) (p. 173).

$$\cos\theta = 0.850 \qquad \theta = 31.8° \qquad \sin\theta = 0.527$$

$$E = \sqrt{[(1{,}330 \times 0.850) + (376 \times 0.09)]^2 + [(1{,}330 \times 0.527) + (376 \times 0.90)]^2}$$
$$= 1{,}560 \text{ volts.}$$

$$\text{The percentage regulation per coil} = \frac{1{,}560 - 1{,}330}{1{,}330}\,100 = 17.4 \text{ per cent.}$$

Ans.

The open-circuit terminal voltage $= 1{,}560\sqrt{3} = 2{,}700$ volts.

$$\text{The percentage regulation using this value} = \frac{2{,}700 - 2{,}300}{2{,}300} = 17.4$$

per cent. *Ans.*

Or, applying Eq. (73) (p. 174),

$$|\dot{E}| = 1{,}330 + 376(0.85 - j0.527)(0.09 + j0.90)$$
$$= 1{,}537 + j269$$
$$|E| = \sqrt{(1{,}537)^2 + (269)^2} = 1{,}560 \text{ volts.}$$

95. Regulation of Delta-connected Generator.—In the delta machine, the line voltage and the coil voltage are equal, but the coil current is the line current divided by $\sqrt{3}$. The ammeters connected in delta, as shown in Fig. 176 (*c*), measure the coil current directly.

Let it be assumed in the problem of the preceding paragraph that the machine is delta-connected. Using 240 amp., the same value of field current as before, the coil voltage in the open-circuit test is now 2,180 volts, and the corresponding coil current in the short-circuit test is $1{,}400/\sqrt{3} = 808$ amp.

The synchronous impedance per coil

$$Z_s = \frac{2{,}180}{808} = 2.70 \text{ ohms,}$$

or three times its previous value.

Figure 178 shows the circuits of the delta when the ohmic resistance is measured with direct current. Let the resistance per coil be R, and the resistance measured between any two terminals be R_0. The circuit consists of two parallel branches, one of R ohms and the other of $2R$ ohms.

Therefore,

$$\frac{1}{R_0} = \frac{1}{R} + \frac{1}{2R}$$

$$R = \tfrac{3}{2}R_0$$

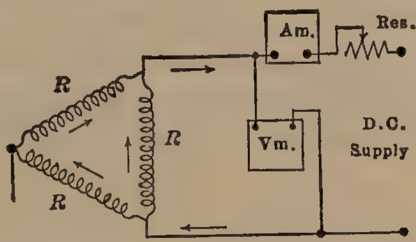


Fig. 178.—Measurement of delta coil resistance with direct current.

Therefore, the ohmic resistance per coil

$$R = (\tfrac{3}{2}) \times 0.12 = 0.18 \text{ ohm},$$

or three times its previous value. This must be increased 50 per cent., in order to obtain the effective resistance.

$$1.5 \times 0.18 = 0.27 \text{ ohm effective resistance.}$$

The rated *coil* current of the machine,

$$= \frac{376}{\sqrt{3}} = 217 \text{ amp.}$$

Applying Eq. (72) (p. 173).

$$E = \sqrt{[(2{,}300 \times 0.85) + (217 \times 0.27)]^2 + [(2{,}300 \times 0.527) + (217 \times 2.7)]^2}$$
$$= 2{,}700 \text{ volts}$$

which checks the result obtained when assuming that the machine was Y-connected.

Therefore, a machine may be assumed to be either Y- or delta-connected when it is desired to calculate the regulation.

96. Magnetomotive Force Method.—In the synchronous-impedance method of determining regulation, a voltage was

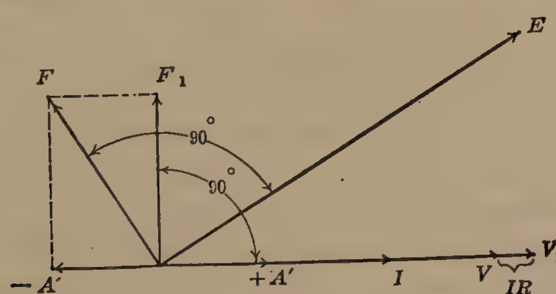


Fig. 179.—Vector diagram for magnetomotive force method at unity power factor.

substituted for armature reaction or for a m.m.f. In the m.m.f. method, a m.m.f. is substituted for a voltage, this voltage being the IX drop in the armature of the alternator. In other words, the armature *reactance* is considered as being zero, but the arma-

ture reaction is increased a sufficient amount to compensate for this.

The method involves a short-circuit and an open-circuit test and in this respect is similar to the synchronous-impedance method. Figure 179 shows the principle of the method. This diagram is constructed for unity power factor. V is the terminal voltage. To this is added the IR drop, giving the voltage V_1. A certain field m.m.f. F_1 is required to produce this voltage V_1.

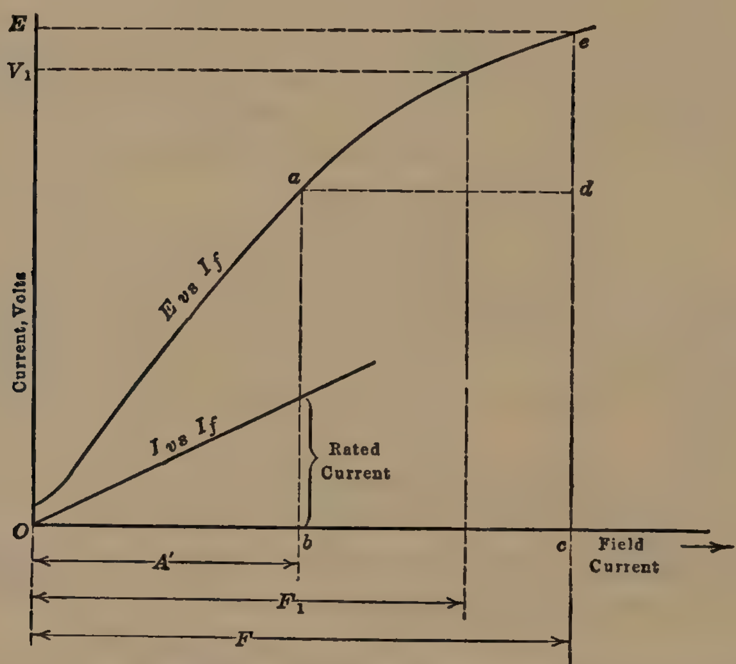


Fig. 180.—Open- and short-circuit tests, magnetomotive force method.

The value of this m.m.f. in terms of the field current is found on the saturation curve (Fig. 180). Corresponding to the value of V_1, the field current F_1 is found. F_1 is laid off at right angles to V_1 and leading it, as a m.m.f. leads by 90° the e.m.f. which its flux induces. In the short-circuit test, the field current is adjusted until the rated current flows. The corresponding value of field current A' (Fig. 180) is then read. The m.m.f. represented by this field current is necessary to send rated current through the armature reactance and at the same time overcome the armature reaction, if the resistance be neglected. This m.m.f. A' replaces the combined effect of the armature *reactance*

and the armature *reaction.* It is laid off 180° from the current, as shown at $-A'$ (Fig. 179). (The total m.m.f. which is assumed to produce the total voltage drop is $+A'$. The component which must balance this m.m.f. is $-A'$.) The resultant m.m.f. is F, which, at unity power factor, is the square root of the sum of the squares of F_1 and $-A'$. F is the m.m.f. which exists at no load under the assumptions made. The no-load voltage E lags F by 90° (Fig. 179) and is found on the saturation curve corresponding to field current F (Fig. 180).

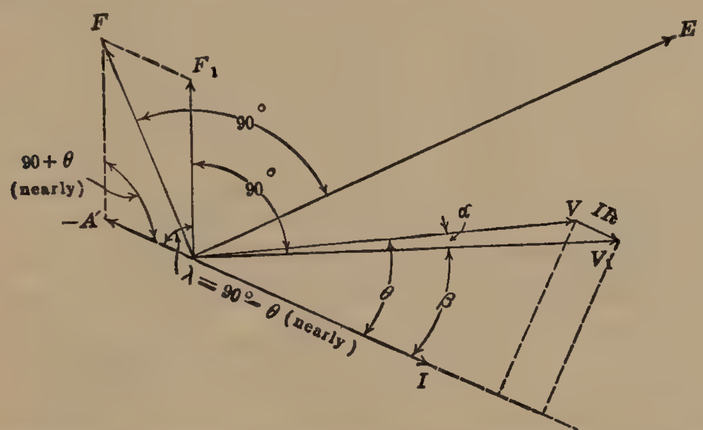


FIG. 181.—Vector diagram of magnetomotive force method, lagging current.

To summarize the method at unity power factor, the IR drop is added to the terminal voltage, and the field current corresponding to this sum is found on the saturation curve. The machine is then short circuited, and the field current necessary to send rated current through the armature is determined. The square root of the sum of the squares of these field currents is then found. The value of e.m.f. on the saturation curve corresponding to this resultant field current is assumed to be the no-load voltage of the machine.

When the power factor is less than unity, the diagram is similar to that shown in Fig. 181.

The voltage V_1 is the vector sum of V and IR. Its value is readily found by projecting these voltages on the current vector. Thus:

$$V_1 = \sqrt{(V\cos\theta + IR)^2 + (V\sin\theta)^2} \tag{78}$$

In most cases, a numerical addition of V and IR is sufficiently accurate.

The value of the angle α may be found by first finding the angle β.

$$\sin \beta = \frac{V \sin \theta}{V_1}$$

$$\alpha = \theta - \beta$$

α is usually so small that it may be neglected.

The vector F_1 leads V by $(90 - \alpha)$ degrees, but α is so small that it may be neglected. The armature reaction vector $-A'$ is 180° from the current vector. By geometry, the angle between $-A'$ and F_1

$$\begin{aligned} \lambda &= 180° - (90° + \theta - \alpha) \\ &= 180° - (90° + \theta) \text{ nearly} \\ &= 90° - \theta \text{ nearly.} \end{aligned}$$

By the cosine law,

$$F^2 = F_1^2 + A'^2 - 2F_1A' \cos (90° + \theta). \qquad (79)$$

The voltage E corresponding to F and found from the saturation curve (Fig. 180) is the no-load voltage of the generator.

Example.—Take the example of the preceding paragraph. The exact method will first be used. The machine will be considered as being Y-connected.

$$\text{The coil voltage} = \frac{2{,}300}{\sqrt{3}} = 1{,}330 \text{ volts.}$$

The IR drop is $376 \times 0.09 = 33.8$ volts.

$\cos \theta = 0.85 \qquad \sin \theta = 0.527$

$$V_1 = \sqrt{[(1{,}330 \times 0.85) + (34)]^2 + [1{,}330 \times 0.527]^2} = 1{,}359 \text{ volts.}$$

Algebraic addition would have given 1,364 volts, .

$$\sin \beta = \frac{1{,}330 \times 0.527}{1{,}359} = 0.516$$

$$\beta = 31.1° \qquad \theta = 31.8°$$

$$\alpha = 31.8° - 31.1° = 0.7°, \text{ which is negligible.}$$

From Fig. 177, the field current corresponding to 1,359 coil volts, or 2,350 volts on the saturation curve ($2{,}350 = 1{,}359\sqrt{3}$), is

$$F_1 = 266 \text{ amp.}$$

The rated current of the coils is 376 amp. Corresponding to this current (Fig. 177), the field current is 64 amp. from the short-circuit test.

$$F^2 = \overline{266}^2 + \overline{64}^2 - 2 \times 266 \times 64 \cos (90° + 31.8°)$$

$$F^2 = 92{,}840 \qquad F = 305 \text{ amp.}$$

From the saturation curve, the terminal voltage corresponding to 305-amp. field current is 2,580 volts across the terminals, or 1,490 coil volts.

$$\text{The regulation} = \frac{1{,}490 - 1{,}330}{1{,}330} = 12.0 \text{ per cent.} \quad \textit{Ans.}$$

Because of the low saturation on short circuit, a given m.m.f. will produce a greater increase of flux than an equal m.m.f. will produce under operating conditions, where the iron is saturated. The e.m.f. corresponding to a given m.m.f. at short circuit will be much greater, therefore, than the e.m.f. corresponding to an equal m.m.f. taken higher up on the saturation curve. This is illustrated in Fig. 180. On short circuit, the voltage *ab* corresponds to the m.m.f. A'. The additional voltage *de* corresponds to a m.m.f. *bc* equal to A' but taken higher up on the saturation curve. The voltage *de* is obviously much less than the voltage *ab*. Hence, that part of the m.m.f. A which replaces a voltage is too small under load conditions. The no-load e.m.f. E found on the saturation curve is, therefore, too low, and the regulation as determined by this method is ordinarily less than the actual regulation. For this reason, this method is often called the *optimistic method.* This is illustrated by the foregoing example, where the regulation as obtained by the synchronous-impedance method is 17.4 per cent., whereas that obtained by the m.m.f. method is 12.0 per cent.

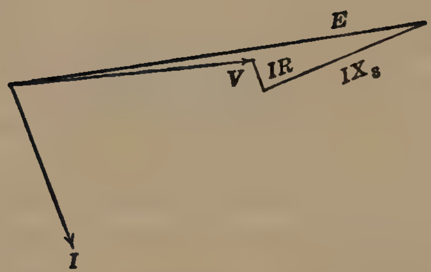
That part of the m.m.f. A which actually is armature reaction is too high on short circuit, due to the favorable position of the armature coils with respect to the field poles. As in the synchronous-impedance method, this factor tends to give a too high value of regulation. These two sources of error tend to offset each other in the m.m.f. method, whereas they both produce errors in the same direction in the synchronous-impedance method. The m.m.f. method, therefore, usually gives results closer to the actual regulation than does the synchronous-impedance method. The actual value of the alternator regulation probably lies between the two values just determined. Were the saturation curve of the machine a straight line, both methods would give nearly the same result.

97. American Institute of Electrical Engineers Method.—This method, recommended by the A. I. E. E., has an advantage over the other two methods, in that the synchronous impedance is measured when the machine is operating at full voltage and, therefore, at normal saturation. This is accomplished by applying a load of very low power factor, usually an underexcited synchronous motor. The vector diagram for this condition is shown

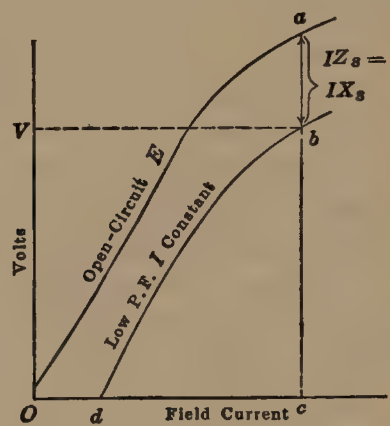
in Fig. 182 (a). V is the terminal voltage under these conditions, E the open-circuit voltage, and IX_s the synchronous reactance drop at rated current. As the IR drop is small and is nearly in quadrature with V, the open-circuit voltage is substantially equal to the numerical sum of the terminal voltage V and IX_s. Numerically, therefore,

$$IX_s = E - V$$

$$X_s = \frac{E - V}{I}. \tag{80}$$



(a) Vector diagram for a load of very low power-factor.



(b) Curves used for A. I. E. E. method.

FIG. 182.—A. I. E. E. method.

X_s is ordinarily determined from a no-load saturation curve and a curve taken at low power factor and rated current. Thus, in Fig. 182 (b), Oa is the no-load saturation curve and db is a curve taken at low power factor and rated current. This low-power-factor load is ordinarily obtained by using underexcited synchronous motors as the load. The rated terminal voltage of the machine is cb, and when the load is thrown off, the open-circuit voltage obviously becomes ca, as the field current remains constant.

The synchronous reactance X_s, which is practically equal to the synchronous impedance, is determined by dividing ab by the rated current of the machine, this being the current at which curve db was determined.

$$X_s = \frac{ac - bc}{I} = \frac{ab}{I}.$$

When the value of X_s is determined, it may be utilized in finding the regulation in the same manner as described under the synchronous-impedance method of Pars. 94 and 95.

If it is not possible to load the machine, the distance *Od* may be found from a short-circuit test, and the curve *db* determined from a knowledge of machines having similar constants.

The A. I. E. E. method gives a too large value of regulation for machines having salient poles, as the armature reaction is

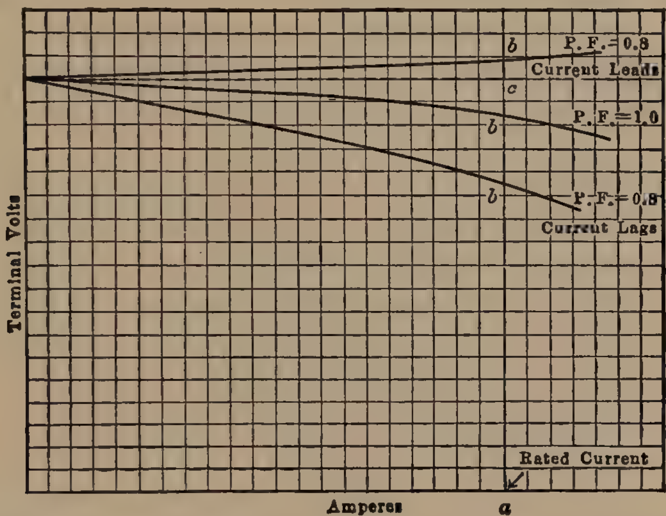


FIG. 183.—Characteristics of alternator at different power factors.

too large at low power factors, since the coil is acting directly upon the magnetic circuit of the generator, as has been shown in Fig. 159 (p. 166).

From the foregoing, it is obvious that for a given current the regulation depends on the *power factor*. The regulation has the greatest values at low power factors, lagging current. At unity power factor, the regulation is usually some nominal value, that is, from 8 to 15 per cent. With leading current, the voltage tends to *rise* as load is applied, and the regulation may be zero or even negative. Figure 183 shows three typical load curves of an alternator, one being taken at unity power factor, the second at 0.8 power factor, lagging current, and the third at 0.8 power factor, leading current. The regulation in each case is as follows:

$$\text{Regulation} = \frac{ac - ab}{ab}.$$

It should be kept in mind that for a fixed *kilowatt* output the regulation with lagging current is even poorer than the values obtained for fixed *current* output.

98. Efficiencies of Alternators.—As with direct-current generators, it is not simple to measure the input to alternators (see Vol. I, p. 417). The direct measurement of efficiency by actual loading is accompanied by the difficulties of providing the necessary power and finding suitable load. Hence, the efficiencies of alternators are usually determined from their losses. The losses are friction and windage; core loss due to the *resultant field;* armature resistance (effective) loss; and field-excitation loss. The machine may be run as a synchronous motor, and the stray power, which includes the first two losses, determined. It is, however, preferable and more accurate to determine these two losses separately by driving the machine with a small direct-current motor. The input to the *armature* of this motor is measured, and its output is calculated, knowing its stray-power and armature losses. These are easily determined. In this way, the friction and windage loss and the core loss of the alternator are readily measured. The core loss is plotted as a function of field current.

It is not simple to determine the effective resistance of the alternator armature. The simplest and most common method is to measure by means of a direct-current motor the input to the alternator with the armature short circuited and with sufficient field excitation to cause the desired current to circulate in the armature. Both the armature and field circuits are then opened, and the motor input again measured. Neglecting the loss due to the resultant field at short circuit, which is small, the difference of the two inputs gives the loss due to the current flowing through the effective resistance of the alternator armature. If the difference of inputs is P_a and the current per phase, I, the effective resistance

$$R = \frac{P_a}{nI^2} \tag{81}$$

where n is the number of phases.

The efficiency of the alternator is then

$$\eta = \frac{nVI}{nVI \cos \theta + nI^2R + P_{fw} + P_{cl} + V_fI_f} \tag{82}$$

where V and I are the voltage and current per coil; $\cos\theta$, the power factor; P_{fw}, the friction and windage; P_{cl}, the core loss; and V_fI_f, the field voltage and current. P'_{cl} must be determined for the *resultant* field F (Fig. 173, p. 181).

The following table gives efficiencies and other data for a few typical alternators:

CHARACTERISTICS OF THREE-PHASE 60-CYCLE ALTERNATORS

Single continuous rating, 50° C. 80 per cent. Power factor, Voltage, 240, 480, 600, 2,400. Manufactured by Westinghouse Electric & Manufacturing Company

Capacity, kilovolt-amperes	Poles	Speed, r.p.m.	Exciter kilowatts at 125 volts	Load ½	Load ¾	Load Full	Net weight, pounds
				Efficiency at 80 per cent. power factor			
Horizontal Engine-driven Type							
50	24	300	5.25	82.6	83.3	82.6	2,500
100	24	300	7.25	87.3	88.7	87.7	3,400
450	40	180	17.5	90.6	91.9	91.6	10,500
1,000	52	138	28.5	91.8	92.8	92.6	20,000
Horizontal High-speed Coupled Type							
50	6	1,200	2.25	83.2	85.5	86.1	2,400
100	6	1,200	3.00	87.8	89.2	89.2	2,700
400	10	720	7.75	91.6	92.7	92.9	6,300
1,100	10	720	13.5	92.8	94.0	94.2	12,000
2,000	10	720	20.0	93.5	94.7	95.1	20,000
4,000	8	900	22.5	93.9	95.2	95.7	29,000
Turbo-driven, Direct-connected Type							Cubic feet of air per minute
625	2	3,600	13.4	87.8	90.9	92.4	4,200
1,250	2	3,600	13.5	90.6	93.1	94.3	6,500
2,500	2	3,600	20.0	90.4	93.2	94.5	11,000
6,250	2	3,600	39.0	90.5	93.2	94.2	20,000
15,625	4	1,800	65.0	92.2	94.4	95.4	42,500
37,500	4	1,800	115.0	93.8	95.4	96.1	90,000

99. Tirrill Regulator.—An automatic voltage regulator of the Tirrill type for direct-current machines is described in Vol. I (Chap. XI, p. 361). An automatic voltage regulator is much more essential in the smaller alternating-current stations than in direct-current stations. The voltage changes in the generator and throughout the system are greater with alternating current than with direct current, because of the added reactance drop in the generator armature, transformers, feeders, etc. Alternating-current generators cannot readily be compounded to compensate for voltage drop as direct-current generators are.

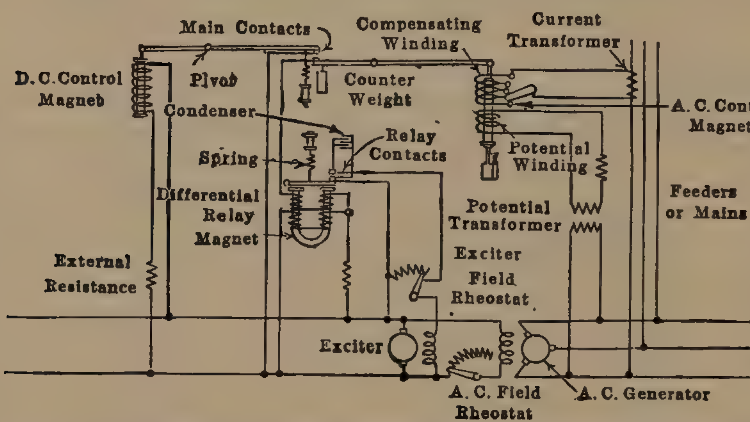


FIG. 184.—Elementary connections of alternating-voltage regulator.

The Tirrill regulator is also designed to be used with alternating-current generators. As with large, direct-current machines, the regulator acts through the field of an exciter. The underlying principle of the regulator is the same whether used for alternating or for direct current, the voltage being controlled in each case by the rapid short circuiting of the exciter field rheostat. Figure 184 shows the connections of the alternating current type as applied to a three-phase alternator.

There are two control magnets, an alternating-current control magnet and a direct-current control magnet.

The alternating-current control magnet is operated primarily by a potential winding connected across one phase of the generator, usually through a potential transformer. The plunger of this magnet acts upon one end of a pivoted lever. On the other end of the lever there are an adjustable counterweight and the lower contact of the main contacts.

The direct-current magnet is operated by a winding connected across the exciter terminals. The plunger of this magnet acts upon another pivoted lever. On the other end of this lever there is the upper contact of the main contacts, so that these main contacts are not fixed but "floating." A spring on the contact end of the lever tends to keep the main contacts closed.

There is a differential relay magnet just as in the direct-current regulator. One coil of this magnet always is connected across the exciter terminals, and the other coil is connected across the exciter terminals when the main contacts are closed. A series resistance limits the current in each to its proper value. The armature of this relay magnet, when released, is pulled upward by a spring and closes the relay contacts. These relay contacts short circuit the exciter field rheostat. A condenser is shunted across these contacts to minimize arcing.

The field of the exciter is first adjusted so that the alternator voltage is about 65 per cent. below normal. This weakens both control magnets so that the floating main contacts are closed. This closes the circuit of the second winding on the relay magnet which opposes the other winding. The relay armature is, therefore, released, and the relay contacts closed. These contacts short circuit the exciter field rheostat, and the voltage for both the exciter and the alternator rises. When the voltage has reached the value for which the regulator has been adjusted, the control magnets open the main contacts, the relay contacts open, and the voltage of the exciter and the alternator both drop. The cycle is then repeated. When in operation, the main contacts and the relay contacts vibrate continuously, so that voltage fluctuations are scarcely noticeable.

A compensating winding, supplied by the secondary of a current transformer in series with the line, compensates for line drop. It increases the pull of the alternating-current control magnet so that the main contacts are drawn closer together and the duration of short circuit of the field rheostat is increased. The General Electric Company also manufactures a counter e.m.f. regulator for alternators, which operates on the same principle as that of the regulator described in Vol. I (p. 363).

100. Parallel Operation of Alternators.—The same reasons which make it necessary to operate direct-current generators in

parallel (see Vol. I, p. 434) apply to alternators. Alternators, however, are made in units of very much greater capacity than it is possible to make direct-current machines, since there are no commutation difficulties. The largest single alternating-current unit at the present time is of 160,000 kv-a. capacity (see p. 2).

In order to operate satisfactorily in parallel, direct-current generators must have drooping voltage characteristics. In order that alternators may operate satisfactorily in parallel, their *prime movers* must have *drooping* speed-load characteristics. Otherwise, the operation will be unsatisfactory. The reason for this is as follows:

Two alternators 1 and 2 are operating in parallel, as shown in Fig. 185 (*a*). If they are operating in parallel, they must have

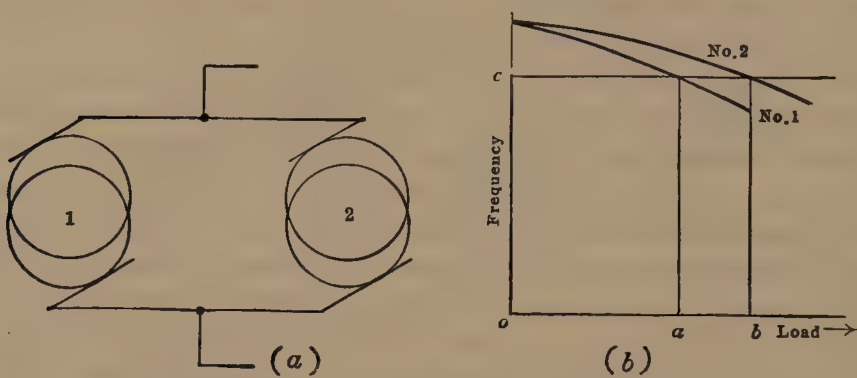


Fig. 185.—Alternators in parallel and speed-load characteristics.

the same frequency and the same terminal voltage. Figure 185 (*b*) shows the speed-load curve of each of the prime movers driving the alternators. (Instead of plotting speed in r.p.m., the frequency or electrical speed is plotted. For example, a six-pole alternator running at 1,200 r.p.m. would have the same electrical speed as an eight-pole alternator running at 900 r.p.m.) The speed-load curves of the prime movers are determined by their respective governors, if they are steam-, water-, or gas-driven units. If motor driven, the speed-load characteristics depend upon the motor speed-load characteristics.

Let *oc* (Fig. 185 (*b*)) be the frequency at which the system is operating. By projecting horizontally to intersect the speed-load curves, the load taken by each machine at this frequency is obtained. *oa* is the load on machine 1, and *ob* is the load on

machine 2, as both machines are operating at system frequency. Let the field of 1 be strengthened by means of its field rheostat. At the same time, weaken the field of 2 so that the line voltage does not change. If these were direct-current generators, machine 1 would immediately take more load. But 1 *cannot take more load* because its prime mover can deliver only the load *oa* at this frequency. Machine 2 cannot drop any load because its prime mover can deliver only the load *ob* at this frequency. Both machines must always operate at the same frequency, which is not true of direct-current machines. *Therefore, the kilowatt load delivered by alternators in parallel cannot be shifted appreciably by means of the generator fields.*

To change the kilowatt load of either machine, the speed-load characteristic of its prime mover must be changed. In engine-driven units, this is done by changing the tension in the governor spring or altering in some manner the governing device. Assume, in Fig. 185 (*b*), that it is desired to make generator 1 take the same load as 2. The governor spring of 1 is so adjusted that the characteristic of 1 is raised, as shown in Fig. 186. Both machines now deliver the same load *oa′* at a frequency *oc′*. Under the conditions shown (Fig. 186), the frequency *oc′* is higher than the original frequency *oc* (Fig. 185). If the original frequency is to be maintained, the speed-load characteristic of 2 must be lowered at the same time that the characteristic of 1 is raised. To adjust the load, therefore, between alternators in parallel, the speed-load characteristics of the prime movers must be changed. If the alternators are driven by shunt motors, the speed-load characteristics of the motors may be changed by adjusting the motor field rheostats. It will be noted, in Fig. 186, that the loads of the two machines are equal at one frequency only.

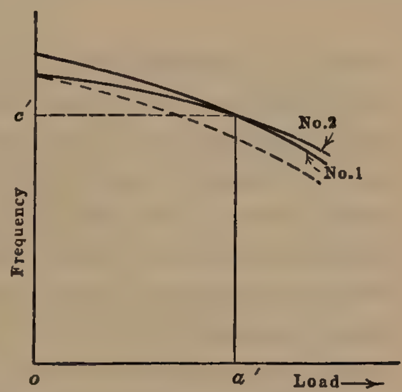


Fig. 186.—Sped-load curves of alternators in parallel—effect of changing governor control.

If the prime movers had flat speed-load characteristics, the operation of the alternators would be unstable. That is, very

small disturbances or changes of frequency would cause very large fluctuations in the kilowatt load delivered by each machine. This condition would result in serious operating difficulties.

It has been shown that direct-current shunt generators operating in parallel are in *stable equilibrium* (see Vol. 1, p. 435). That is, any circumstance which tends to throw machines out of parallel is counteracted by reactions opposing this tendency. In the same way, any action tending to throw alternators out of parallel is opposed by reactions which tend to prevent the

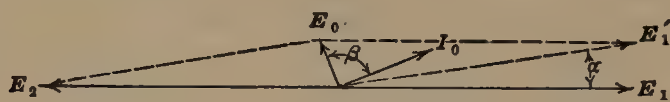


FIG. 187.—Synchronizing current of alternators in parallel.

alternators' pulling out. This is most clearly illustrated by the conditions existing when neither alternator is supplying external load. If the two alternators are considered as a local series circuit, their voltages are in *opposition*. These voltages are represented in Fig. 187 by E_1 and E_2, respectively. E_1 and E_2 are equal and opposite, so that the net voltage acting in the local circuit of the two alternators is zero. There is, therefore, no current flowing between the alternators, just as there is no current circulating between two batteries having equal e.m.fs. and connected with terminals of like polarity together.

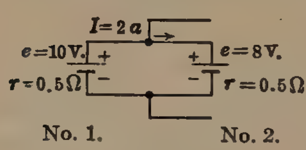


FIG. 188.—Batteries in parallel.

Assume that the prime mover of generator 1 speeds up temporarily. The internal induced voltage of this generator will advance an angle α with respect to E_2. That is, E_1 will advance to position E_1'. The vector sum of the two alternator e.m.fs. E_1' and E_2 will no longer be zero, but, due to the change in their relative phase positions, the vector sum of E_1' and E_2 will be E_0.

The result is the same as with the two batteries of Fig. 188. No. 1 has an e.m.f. of 10 volts, and 2 has an e.m.f. of 8 volts. If the load current is zero, the current circulating between these batteries is then found by dividing the sum of the two

voltages, giving each the proper sign, by the sum of the resistances of the two batteries. That is,

$$I_0 = \frac{10 + (-8)}{0.5 + 0.5} = 2 \text{ amp.}$$

In the same way, the current circulating between the two alternators is the *resultant* voltage divided by the sum of the impedances of the two machines.

$$I_0 = \frac{\dot{E}_1' + \dot{E}_2}{\dot{Z}_1 + \dot{Z}_2} = \frac{E_0}{\sqrt{(R_1 + R_2)^2 + (X_1 + X_2)^2}} \tag{83}$$

where Z_1, Z_2, R_1, R_2, and X_1, X_2 are the respective impedances, resistances, and reactances of the two machines. As the resistance of an alternator armature is very small compared to its reactance, this circulatory current will lag by an angle β, nearly 90°, with respect to the voltage E_0 producing it, as shown in Fig. 187.

It will be observed that I_0 is nearly in phase with the voltage E_1'. It, therefore, puts a power load on generator 1, and this tends to slow down this generator. On the other hand, I_0 is nearly 180° from E_2, that is, it is acting in opposition to E_2. I_0, therefore, develops motor action in generator 2, as the induced e.m.f. acts in opposition to the current. This motor action tends to speed up machine 2. *Therefore, if two alternators in parallel attempt to pull out of step, a current is developed which circulates between the two machines. This current tends to accelerate the lagging machine and to retard the leading machine and so acts to prevent the alternators from pulling out of synchronism.*

If the machines are operating under load, I_0 merely puts more load on the machine which tends to lead and takes load off the machine which tends to lag. This last machine will not ordinarily operate as a motor, as it did under no-load conditions; but as its load is reduced, its angular position will be advanced.

Because I_0 tends to hold the two machines in synchronism, it is called the *synchronizing current.*

It has already been stated that changing the field current does not vary the distribution of load between two alternators. It does, however, affect the current delivered by the two machines. Figure 189 (a) shows the vector diagram for two similar alternators having a common terminal voltage V. Both machines are

delivering equal currents I_1 and I_2, respectively, which are in phase with the terminal voltage V. The resultant load current is their sum I', which is in phase with V. As both machines have have equal resistances and reactances, their respective internal voltages E_1 and E_2 are the same. (In this diagram, the machines are treated with reference to the *external* circuit, in which case the voltages and currents are acting in *conjunction*.)

Let the field of generator 1 be weakened and that of 2 be strengthened. It has already been shown that this cannot affect the division of the kilowatt load between the machines. When the field of generator 1 is weakened, its internal voltage *decreases*, and when the field of 2 is strengthened, its internal voltage

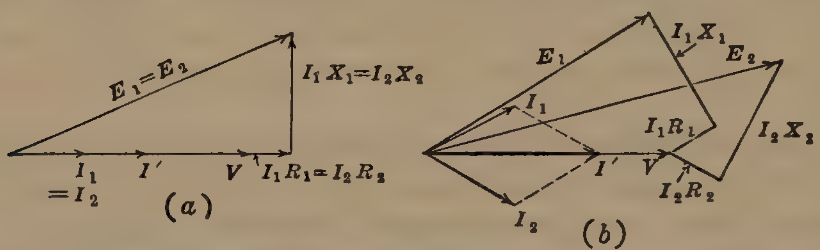


FIG. 189.—Vector diagram of currents and voltages when alternators operate in parallel.

increases. Now, both machines must continue to have equal terminal voltage. It has already been shown that if a machine delivers a leading current, its internal voltage is less than when the machine delivers a lagging current (see Par. 90, p. 171). A leading current in an alternator tends, moreover, to strengthen the field, and a lagging current tends to weaken the field, through armature reaction.

For generator 1 to operate with a reduced internal voltage, it must deliver a leading current, making E_1, shown in Fig. 189 (*b*), less than its previous value, shown in Fig. 189 (*a*). On the other hand, E_2 (Fig. 189 (*b*)), is greater in magnitude than in Fig. 189 (*a*), because generator 2 now delivers a lagging current. Also, the leading current in generator 1 tends to strengthen its field, and the lagging current in generator 2 tends to weaken its field, through armature reaction. In both cases, the change of flux produced by change in field current is *opposed* by armature reaction. The load current I' cannot change in phase or in mag-

nitude, as the phase and magnitude of I' is determined entirely by the character of the load which is connected to the system. Since, therefore, I_1 and I_2 are equal, they must make equal angles with V so that their resultant I' will still lie along V.

It will also be observed that each machine is delivering a larger current than it did before and yet that the kilowatt output of each has not changed. This means that the heating (I^2R) loss in each machine has been increased without any compensating advantages. This is not, therefore, the best condition of operation.

Figure 190 shows the diagram of Fig. 189 (*b*) with the voltage drops eliminated. E_0 is now the *difference* of E_1 and E_2, and I_0,

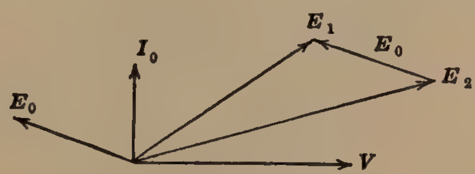


FIG. 190.—Vector diagram showing effect of excitation upon alternator circulatory current.

the circulating current, lags E_0 by nearly 90°, as in Fig. 188. It will be observed that I_0 is nearly in quadrature with the terminal voltage V, so that it transfers practically no power from one machine to the other. This substantiates what has already been demonstrated, that changing the field current cannot transfer appreciable load from one machine to the other.

101. Synchronizing.—Before direct-current generators can be safely connected in parallel, two conditions must be fulfilled. The two terminal voltages must be equal, or substantially so, and the proper polarity must be observed.

These same two conditions must be fulfilled when alternators are connected in parallel. The equality of voltages can be readily determined by connecting a voltmeter first to one machine and then to the other. The voltmeter, when so connected, does not give any indication as to polarity, as the indications of an alternating-current voltmeter are independent of its polarity.

Lamps, however, can be used to determine the correct polarity. Figure 191 shows the connections for phasing a three-phase alternator with the bus-bars. A lamp is connected across each pole

of the three-pole switch which connects the machine to the line. The voltage rating of the lamps should be 15 per cent. greater than that of the machine or line. For example, if the system is 220 volts, two 115-volt lamps in series may be used across each pole, although these lamps will be subjected to overvoltage during a part of the synchronizing period. If the machines are properly connected, the three lamps should all become bright and dim together. If they brighten and grow dim in sequence,

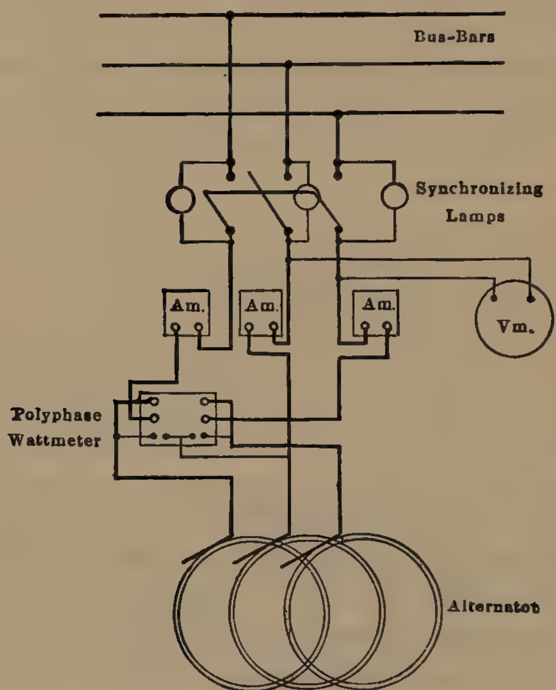


FIG. 191.—Connections for "three-dark" method of synchronizing with lamps.

it means that the phase rotation of the two machines is opposite, so that one phase must be reversed.

The lamps flicker at a frequency equal to the *difference* in the frequencies of the two machines. As the machines approach synchronism, the flicker becomes slower and slower. When the lamps are all dark, the switch may be closed. The fact that the lamps are all dark indicates that the potential difference between each switch blade and its clip is nearly zero and the two alternators are in *opposition* so far as their local series circuits are concerned. Two points across which the potential difference is

zero may be connected without any resulting disturbance, so that the switch may now be safely closed, and the two alternators are in parallel.

The disadvantage of this method is that lamps are dark even although a very considerable voltage may exist across their terminals, and the machines may be connected in parallel, therefore, when considerable voltage difference exists between them. This may do no harm with slow-speed or small-capacity units, but with high-speed turbo-units, which have little armature reactance and are quite "sensitive," there may be considerable disturbance if there exists a substantial phase difference at the

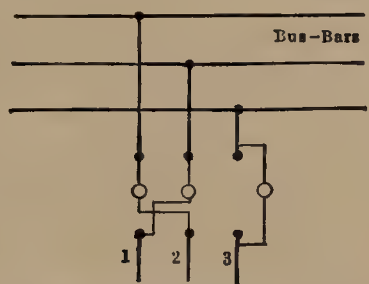


FIG. 192.—Connections for "two-bright-and-one-dark" method of synchronizing with lamps.

time of connecting in parallel. Another objection to this "three-dark" method is that the lamps do not show whether the incoming machine is fast or slow.

The foregoing difficulties may be in part eliminated if the connections of two of the lamps, as 1 and 2 (Fig. 192), be crossed. When the machines are in synchronism, 1 and 2 are bright, and 3 is dark. As one of the bright lamps is increasing and one is decreasing in brilliancy near the point of synchronism, it is possible to determine very accurately the instant at which the switch should be closed. This is called the *Siemens-Halske* or *two-bright-and-one-dark* method. By noting the sequence of brightness of the lamps, it can be determined whether the incoming machine is fast or slow.

The best method is the use of the synchronism indicator or synchroscope described in Chap. III (p. 97). Such an instrument shows very accurately the position of synchronism. The synchroscope is connected across but one phase. It is possible

that one phase of each machine may be in synchronism, but the other two out of phase due to wrong phase rotation. The correct phase rotation must be determined by lamps or by other means before depending entirely upon the synchroscope. Synchronizing lamps are often used in conjunction with a synchroscope so that the operator has a check on the instrument.

102. Hunting of Alternators.—The driving torque of a reciprocating engine, or of a gas engine, is not uniform during a revolution of the flywheel but varies from zero at the dead centers to a maximum at some intermediate position. Even with a heavy flywheel, this variation of torque may impart impulses to the induced e.m.f., causing it to be ahead of its proper position at some instants and behind it at other instants. This causes heavy synchronizing currents to flow between machines in parallel and often causes their rotating members to "oscillate" as they are rotating. The angular effect of the crank position can be appreciated when it is realized that in a 60-pole alternator a displacement of 1 mechanical or space-degree in the rotating member makes a difference of 30 electrical degrees in the phase angle of the e.m.f. The above impulses are often communicated to the system, causing synchronous motors and converters to oscillate. These oscillations are called *hunting*. Hunting may become serious if the engine governors have a natural frequency of oscillation nearly the same as that of the machine rotors. The oscillations may then become cumulative and may even cause the machines to go out of synchronism.

Remedies for hunting are to use heavy flywheels, put dashpots on the engine governors, and to use amortisseur or squirrel-cage windings around the field (see Fig. 319, p. 362). Where several engine-driven units are used, they are often paralleled when their cranks occupy different angular positions. This minimizes the effect of the engine impulses on the system, although their effect is increased in the local interchange currents between generators.

CHAPTER VIII

THE TRANSFORMER

The static transformer is a device for transferring electrical energy from one alternating-current circuit to another without a change in frequency. This transference is usually, but not always, accompanied by a change of voltage. A transformer may receive energy at one voltage and deliver it at a *higher* voltage, in which case it is called a *step-up* transformer. A transformer may receive energy at one voltage and deliver it at a *lower* voltage, in which case it is called a *step-down* transformer. A transformer may receive energy at one voltage and deliver it at the *same* voltage, in which case it is called a *one-to-one* transformer.

A static transformer has no rotating parts, and, therefore, it requires little attention, and its maintenance is low. The cost per kilowatt of transformers is low as compared with other apparatus, and the efficiency is much higher. As there are no teeth, slots, or rotating parts, and the windings can be immersed in oil, it is not difficult to insulate transformers for very high voltages.

Because of these many desirable characteristics, the transformer is a very useful piece of apparatus, and as it can transform from low to high voltage, and from high to low voltage, economically, it is largely responsible for the extensive use of alternating current.

103. Transformer Principle.—The transformer is based on the principle that energy may be efficiently transferred by induction from one set of coils to another by means of a varying magnetic flux, provided both sets of coils are on a common magnetic circuit.

Electromotive forces are induced by a change in flux linkages. In the generator, the flux is substantially constant in magnitude. The amount of flux linking the armature coils is changed by the

relative *mechanical* motion of flux and coils. In the transformer, the coils and magnetic circuit are all stationary with respect to one another. The e.m.fs. are induced by the change in the *magnitude* of the flux with time. This is illustrated in Fig. 193.

A core is made up of rectangular stampings of sheet steel, clamped or bolted together.

A continuous winding P is placed on one side or leg of the iron core. Another continuous winding S, which may or may not have the same number of turns as P, is shown diagrammatically as being placed on the opposite side or leg.[1] An alternator A supplies current to the primary winding P. As this winding is linked with an iron core, its m.m.f. produces an alternating flux in the core. This alternating flux links the turns of the winding

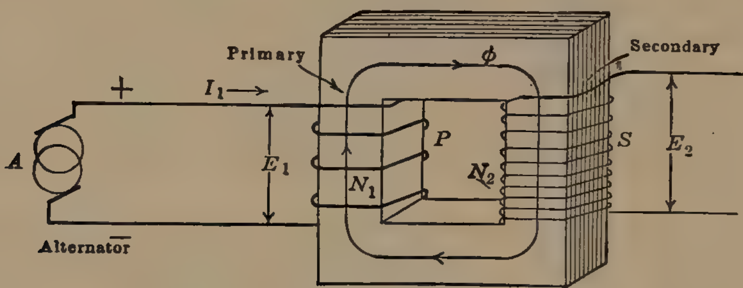


Fig. 193.—Simple transformer, secondary open circuited.

S. As this flux is alternating, it induces in the winding S an e.m.f. of the same frequency as its own. Because of this induced e.m.f., the secondary winding S is capable of *delivering* current and energy. The energy, therefore, is transferred from P, the primary, to S, the secondary, by means of the magnetic flux.

The winding P which *receives* energy is called the *primary*. The winding S which *delivers* energy is called the *secondary*. In a transformer, either winding may be the primary, the other being the secondary, depending upon which winding receives and which delivers energy.

104. Induced E.m.f.—The flux ϕ, called the *mutual flux*, in passing through the magnetic circuit formed by the iron core, links not only the turns of the secondary winding S but also the turns of the primary winding P. An e.m.f., therefore, must be

[1] Actually, the primary and the secondary will be on the same leg to reduce magnetic leakage (see p. 235).

induced in both the windings S and P. As this flux ϕ is the same for each of the two windings, it must induce the *same e.m.f. per turn* in each winding. The *total induced e.m.f.* in each winding must then be proportional to the number of turns in that winding. That is,

$$\frac{E_1}{E_2} = \frac{N_1}{N_2} \tag{84}$$

where E_1 and E_2 are the primary and secondary *induced* e.m.fs. and N_1 and N_2 are the number of turns in primary and secondary, respectively. In the ordinary transformer, the terminal voltage differs from the induced e.m.f. only by a very small percentage, so

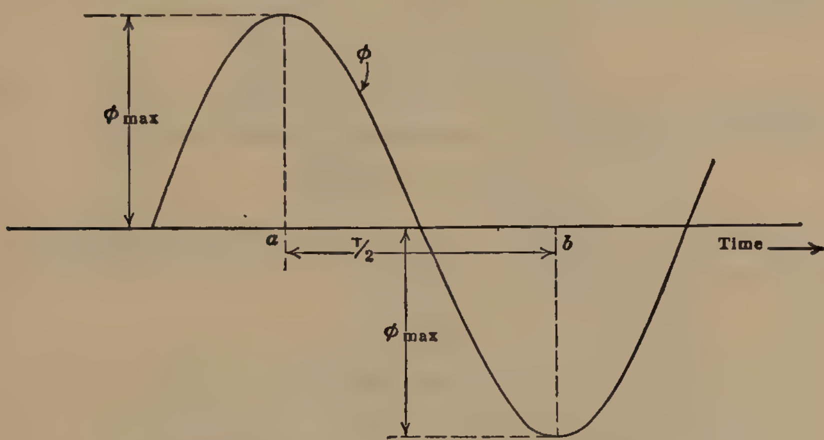


FIG. 194.—Sinusoidal variation of flux with time.

that for most practical purposes it may be said that the primary and secondary terminal voltages are proportional to the respective number of turns.

The induced e.m.f. in a transformer is proportional to three factors—the flux, the frequency, and the number of turns. The complete equation for the induced e.m.f., assuming a sine wave, is as follows:

$$E = 4.44fN\phi_{max}10^{-8} \text{ volts} \tag{85}$$

where f is the frequency in cycles per second, N is the number of turns, and ϕ_{max} is the maximum value of the flux in the core. The factor 4.44 is four times the form factor, which is 1.11 for a sine wave (see Chap. 1, Par. 5, p. 10).[1]

[1] Compare Eq. (85) with Eq. (67), p. 152, using N instead of Z in Eq. (67).

This equation is derived as follows:

Figure 194 shows the mutual flux ϕ varying sinusoidally with the time. Between points a and b, the total change of flux is $2\phi_{max}$ lines or maxwells. This change of flux occurs in half a cycle or in a time $T/2$ sec., where T is the *period* or the time required for the wave to complete one cycle. The time $T/2$ is obviously equal to $1/(2f)$ sec. From Eq. (74) (Vol. I, p. 211), the *average* induced e.m.f. becomes

$$\begin{aligned} e' &= -N\frac{2\phi_{max}}{T/2}\,10^{-8} \text{ volts} \\ &= -N\frac{2\phi_{max}}{1/(2f)}\,10^{-8} \text{ volts} \\ &= -4fN\phi_{max}\,10^{-8} \text{ volts.} \end{aligned}$$

Since with a sine wave the ratio of effective to average volts is 1.11 (see p. 10, Par. 5), the *effective* induced e.m.f. is

$$E = 4.44fN\phi_{max}10^{-8} \text{ volts.} \tag{85}$$

If the flux varies other than sinusoidally with the time, a factor k_f called the *form factor* must be substituted for 1.11 in the above equation.

Equation (85) may be proved more rigorously as follows:

$$\begin{aligned} \varphi &= \phi_{max}\sin \omega t \\ e &= -N\frac{d\varphi}{dt}10^{-8} = -N\phi_{max}\omega \cos \omega t(10^{-8}) \text{ volts.} \\ E_{max} &= N\phi_{max}\omega\ 10^{-8} = 2\pi fN\phi_{max}10^{-8} \text{ volts.} \\ E &= \frac{2\pi}{\sqrt{2}}fN\phi_{max}10^{-8} = 4.44fN\phi_{max}10^{-8} \text{ volts.} \end{aligned}$$

The maximum flux $\phi_{max} = B_{max}A$, where B_{max} is the maximum flux *density* and A is the core cross-section. Equation (85) may then be written,

$$E = 4.44fNB_{max}A10^{-8} \text{ volts} \tag{86}$$

This equation is the more convenient to use, for transformer cores are designed on the basis of permissible flux density. The use of Eq. (86) is illustrated in the following example:

Example.—The core of a 60-cycle transformer has a net cross-section of 20 sq. in., and the maximum flux density in the core is 60,000 lines per square

inch. There are 700 turns in the primary and 70 turns in the secondary. What is the rated voltage of the primary and of the secondary?

$E_1 = 4.44 \times 60 \times 700 \times 60{,}000 \times 20 \times 10^{-8} = 2{,}230$ volts. *Ans.*
$E_2 = 4.44 \times 60 \times 70 \times 60{,}000 \times 20 \times 10^{-8} = 223$ volts. *Ans.*

Also,

$$E_2 = \frac{2{,}230}{10} = 223 \text{ volts. } \textit{Ans.}$$

105. Ampere-turns.—Figure 195 shows a transformer having a primary and a secondary winding. The directions of the flux, of the voltages, and of the currents, as indicated on the figure, are those existing at the instant when the upper primary conductor is positive and the current is increasing. Assume, first, that there is no load on the secondary. Under these conditions,

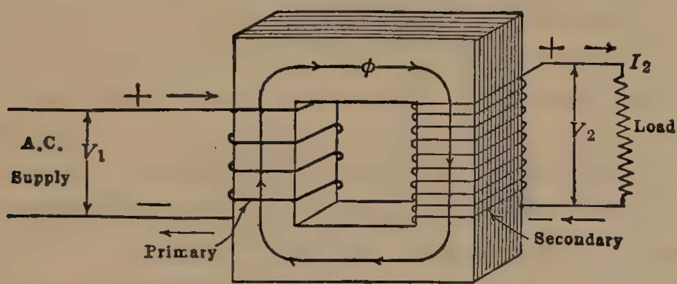


FIG. 195.—Simple transformer, load applied to secondary.

a very small current flows in the primary, usually from 3 to 8 per cent. of the rated current. This no-load current can be resolved into two components, one supplying the no-load losses, and the other in quadrature with the first and producing the flux ϕ (see p. 219). This quadrature current is called the *exciting* or *magnetizing current* of the transformer. As the energy current which is in phase with the back e.m.f. is small, the quadrature current is very nearly equal, numerically, to the total no-load current. The no-load current, therefore, is often called the *exciting current* of the transformer. The back e.m.f. is nearly constant for all loads, as it differs from the terminal voltage only by the primary impedance drop, which is small. The flux, therefore, and, hence, the exciting current are practically independent of the load.

This exciting current produces a flux ϕ in the core, the direction of the flux being as shown (corkscrew rule). The value of this flux must be such as to make the induced primary e.m.f. practi-

cally equal to the primary line voltage. This primary induced e.m.f. is a *back* e.m.f. and is, therefore, in opposition to the primary impressed voltage.

Now apply a load to the secondary. As a result, a current I_2 flows in the secondary. The direction of this current must be such as to *oppose* the increase in the flux ϕ. This is in accordance with Lenz's law that an induced current always has such a direction as to oppose the cause which produces it. If the secondary current I_2 were producing the flux ϕ, then by the corkscrew rule the current would flow *in* at the upper terminal (Fig. 195). Since I_2 opposes the flux ϕ, it must actually flow *out* at the upper terminal. The secondary current I_2 then tends to reduce the value of the flux in the transformer core. If the flux is reduced, the back electromotive force of the primary is also reduced. This permits more current to flow into the primary which supplies the increase in power due to loads, being applied to the secondary. This is the sequence of reactions which follow the application of load to the secondary, enabling the primary to take from the line the increased power demanded by the secondary.

The change in the back e.m.f. in the primary from no load to full load is ordinarily about 1 or 2 per cent. As the back e.m.f. is proportional to the mutual flux ϕ, *the value of ϕ, therefore, does not change appreciably over the working range of the transformer.* If this flux does not change appreciably, the *net* ampere-turns acting on the core cannot change appreciably. The increased ampere-turns due the secondary load must be just balanced, therefore, by the additional ampere-turns due to the increased primary current. Since the flux remains practically constant, it follows that the exciting current must remain substantially constant.

The effect of any *increase* of primary ampere-turns, when not opposed by equal secondary ampere turns, would be to increase the mutual flux. This would increase the back e.m.f. and might cause the primary to deliver power back into the power source, which is in violation of the law of the conservation of energy. Any primary ampere-turns in excess of the exciting ampere-turns, therefore, must be balanced by equal and opposing secondary ampere-turns.

The exciting current is of small magnitude and differs considerably in phase from the total primary current, as shown by I_0 in Fig. 197 (p. 218). It is usually neglected, therefore, in comparison with the total primary current. If it be neglected, the *primary* and *secondary ampere-turns* are *equal*, and

$$N_1I_1 = N_2I_2.$$

Therefore,

$$\frac{I_1}{I_2} = \frac{N_2}{N_1}. \tag{87}$$

That is, *the primary and secondary currents are inversely as the respective turns.*

The above relation also follows from the law of the conservation of energy. If the transformer losses be neglected and unity power factor be assumed,

$$V_1I_1 = V_2I_2$$

$$\frac{I_1}{I_2} = \frac{V_2}{V_1} = \frac{N_2}{N_1}.$$

106. Leakage Reactance.—In the preceding discussion, it has been assumed that *all* the flux which links the primary also links

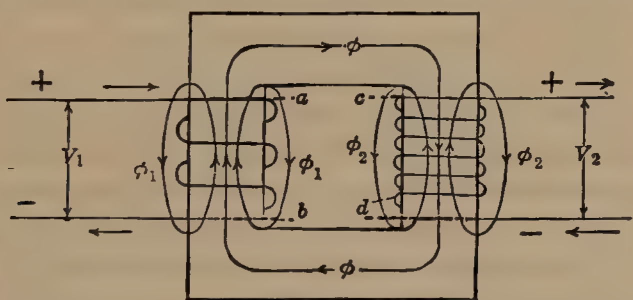


FIG. 196.—Mutual flux, primary leakage flux, and secondary leakage flux in transformer.

the secondary. In practice, it is impossible to realize this condition. All the flux produced by the primary does not link the secondary, but a part completes its magnetic circuit by passing through the air rather than around through the core, as shown by ϕ_1 (Fig. 196). That is, between planes a and b (Fig. 196), there is a m.m.f. due to the primary ampere-turns, plane a being at a higher magnetic potential than plane b at the instant shown. This m.m.f. is proportional to the primary current and tends to

send flux from a to b both through the air and around through the core. That part of the flux which passes from a to b through the air follows a magnetic circuit which is acted upon by the primary ampere-turns only. This flux ϕ_1 is called the *primary-leakage* flux. It is proportional to the total ampere-turns of the primary alone, as the secondary turns do not link the magnetic circuit of ϕ_1. ϕ_1, therefore, induces an e.m.f. in the primary but not in the secondary. The flux ϕ_1 is in time phase with the total primary current I_1. The e.m.f. induced by ϕ_1 must lag ϕ_1 and I_1 by 90° (see p. 28, Par. 16). The e.m.f. necessary to balance this counter e.m.f. is opposite and equal to it and, therefore, leads the current I_1 by 90°. As this counter e.m.f. is proportional to the current and lags it by 90°, it is nothing more than a reactance voltage and is denoted by $-I_1X_1$. The component of line voltage which balances this e.m.f. is $+I_1X_1$. A reactance drop exists in a transformer primary, therefore, in precisely the same manner that a reactance drop exists in an alternator armature. The effect of the primary leakage flux, therefore, is to oppose the flow of current *into* the transformer.

The m.m.f. of the secondary coil, acting alone, is such that the top of the coil is at a higher magnetic potential than the bottom of the coil. That is, plane c is at a higher magnetic potential than plane d, and, therefore, a flux ϕ_2 tends to pass from c to d through the air, as shown. Flux ϕ_2 is called the *secondary leakage flux.* As its path is not linked by the primary, the secondary leakage flux is proportional to the secondary ampere-turns only. ϕ_2 induces an e.m.f. in the secondary, lagging the secondary current I_2 by 90°. This is also a reactance voltage, and the component which balances it leads the secondary current by 90°. This last voltage is denoted by I_2X_2. The secondary reactance opposes the current flowing *out* of the secondary, just as the armature reactance of an alternator opposes the current flowing out of the armature. Both the primary and the secondary reactances of the transformer have the same effect on the regulation of the transformer as the armature reactance of the alternator has on the regulation of the alternator.

In that part of the core which is surrounded by the secondary winding, the mutual flux ϕ and the secondary leakage flux ϕ_2 are shown in opposition. As ϕ is produced by the joint ampere-

turns of primary and secondary, and ϕ_2 by the ampere-turns of the secondary alone, ϕ and ϕ_2 are almost never in time phase with each other but are usually out of phase by an angle greater than 90°, as shown in Fig. 197 (*a*). Two separate fluxes in the core do not actually exist at the same instant, but merely the resultant flux, found by combining ϕ and ϕ_2. The primary leakage flux ϕ_1 and the secondary leakage flux ϕ_2 have the same general direction in the space between the primary and secondary coils.

In actual transformers, the primary and secondary windings are not placed on separate legs, as shown in Figs. 193, 195, and 196, for, as they are widely separated, large primary and secondary leakage fluxes would result. These large leakage fluxes would cause the transformer regulation to be too poor for commercial use. To reduce the leakage, the primary and secondary should be interleaved. Each is usually split, therefore, into a number of coils, and alternate primary and secondary coils are placed together, as shown in Figs. 207, 208, and 209 (pp. 234, 235 and 236).

Hence, in actual transformers, the leakage-flux paths are not so simple as those indicated in Fig. 196. Because of the small spaces between primary and secondary windings, the paths of the leakage fluxes are much more restricted. The entire primary and secondary leakage fluxes ϕ_1 and ϕ_2, moreover, do not link all the turns of their respective windings but only a portion of them. The equivalent effect of ϕ_1 and ϕ_2, however, is readily determined in the ordinary transformer by simple measurements, as is described later.

107. Transformer Vector Diagram.—Figure 197 (*a*) shows the relations existing among the currents and voltages in a transformer, when the secondary is delivering a current I_2, at terminal voltage V_2 and power factor $\cos \theta_2$. A one-to-one ratio of transformation is assumed, in order that the lengths of all the vectors in the diagram shall be of the same order of magnitude. This same diagram may be made applicable to any ratio of transformation, merely by multiplying the proper vectors by the ratio of transformation.

The secondary current I_2 is laid off at phase angle θ_2 from the secondary terminal voltage V_2. The secondary leakage flux ϕ_2 is in time phase with I_2 and induces the e.m.f. which is balanced

by I_2X_2, leading I_2 by 90°. The induced voltage E_2 of the secondary is determined by adding vectorially to V_2 the secondary resistance drop I_2R_2, in phase with I_2, and the secondary reactance drop I_2X_2, due to ϕ_2, in quadrature with I_2 and leading. As both the primary and secondary induced voltages are induced by the same flux, and both windings have the same number of turns, since the ratio is one-to-one, the primary and secondary induced voltages will be equal in magnitude and will be in phase with each other. Therefore, $E_1 = E_2$. It has already been demonstrated that an e.m.f. induced by a flux varying sinusoidally with time is a sine wave and lags the flux 90° (p. 28, Par. 16;

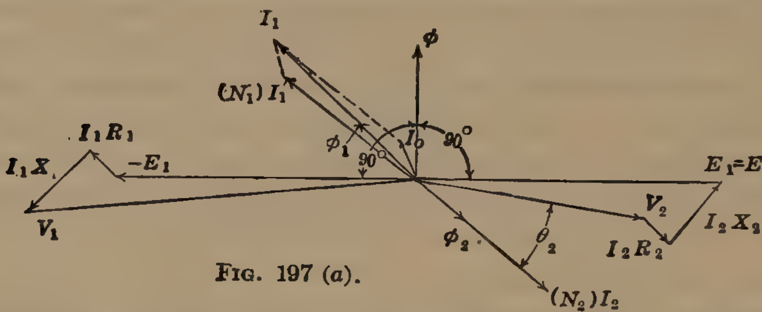


Fig. 197 (a).

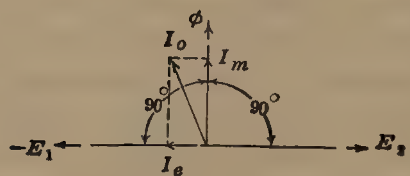


Fig. 197 (b).

Fig. 197 (a).—Complete vector diagram for transformer.

Fig. 197 (b).—Energy and magnetizing components of no-load or exciting current.

also, rigorous proof, p. 29). In Fig. 197 (a), therefore, the mutual flux ϕ *leads* the induced e.m.fs. by 90°, as shown.

The line must first supply a voltage at least equal to the primary induced voltage, and in opposition thereto, before current can flow *into* the primary. This is analogous to the direct-current motor, where the line must first supply a voltage equal to the back e.m.f. and in opposition thereto, before any current can flow into the armature. A voltage $-E_1$, therefore, opposite and equal to E_1 must first be supplied by the line. The primary must furnish at least a sufficient number of ampere-turns to balance the ampere-turns of the secondary. These

primary ampere-turns and the secondary ampere-turns are equal and opposite. If there are N_2I_2 ampere-turns in the secondary, therefore, there must be an equal number of ampere-turns in the primary to balance these. These primary ampere-turns N_1I_1' (Fig. 197 (*a*)) are 180° from N_2I_2. It is not customary to show the ampere-turns on the diagram, however, but only the currents, as in Fig. 197. The ampere-turns may then be obtained by multiplying each current by its proper number of turns.

In addition to I_1', the no-load current I_0 must exist to produce the mutual flux ϕ and to supply the no-load losses. This current would be in phase with the flux ϕ were it not for the core losses. These losses require that I_0 have an energy component shown by I_e in Fig. 197 (*b*). That is, I_0 is resolved into two components, a magnetizing component I_m in phase with ϕ, and an energy component I_e in phase with the primary e.m.f. $-E_1$ and leading I_m by 90°.

The total primary current is I_1, the vector sum of I_0 and I_1'.

The primary leakage flux ϕ_1 is in phase with I_1 and induces the e.m.f., which is balanced by I_1X_1.

The primary terminal voltage V_1 may now be found by adding I_1R_1 and I_1X_1 vectorially to $-E_1$.

The transformer regulation is defined as the rise in secondary voltage divided by the rated-load voltage, when rated load is removed from the transformer. (See p. 229.) The primary voltage is assumed to be constant.

The regulation for a one-to-one transformer is given by

$$\frac{V_1 - V_2}{V_2}.$$

In most transformer vector diagrams, it is necessary to exaggerate greatly the magnetizing-current and voltage-drop vectors. For example, I_0 is about 3 per cent. I_1; I_2R_2, 1 per cent. V_2; etc. If these quantities be drawn in their actual proportions, they will be too small to be significant.

108. Simplified Diagram.—The diagram of Fig. 197 may be materially simplified if the magnetizing current I_0 be neglected. As I_0 is usually from 3 to 8 per cent. of I_1 and the two are considerably out of phase, I_0 may ordinarily be neglected without serious error. Figure 198 shows the diagram of Fig. 197 with

I_0 omitted. The ratio of transformation, however, is no longer one-to-one. The primary has N_1 turns, and the secondary N_2 turns. Hence, the ratio of transformation is N_1/N_2. With usual transformers it is not practicable to use the same scale for primary and secondary voltages and currents. For example, with only a 20/1 ratio of transformation, either one scale would be so small as to be useless or the other so large as to be impracticable.

In Fig. 198, each of the primary voltages is multiplied by the inverse ratio of transformation N_2/N_1, and the primary current is multiplied by N_1/N_2, the ratio of transformation. Hence,

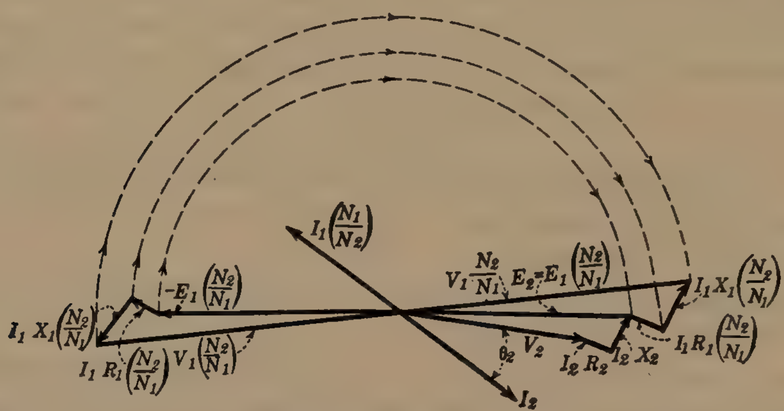


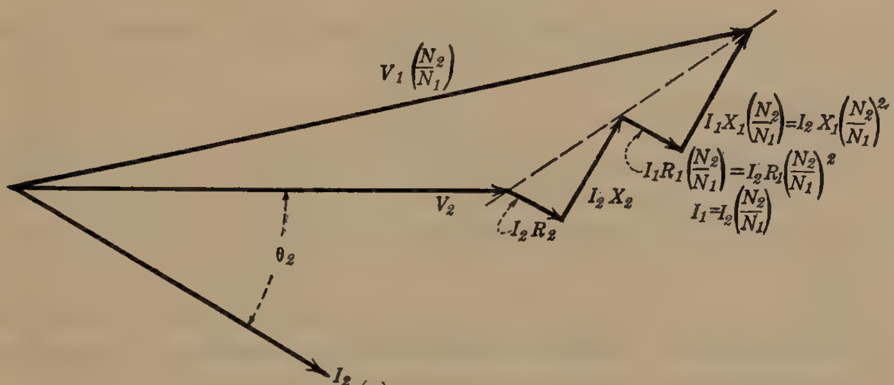
FIG. 198.—Transformer diagram with primary voltages rotated to secondary side of diagram.

$E_1(N_2/N_1) = E_2$, etc., and both primary and secondary vectors are of the same order of magnitude. It is thus seen that the diagram for *any* ratio of transformation can be made substantially the same as that for a one-to-one ratio.

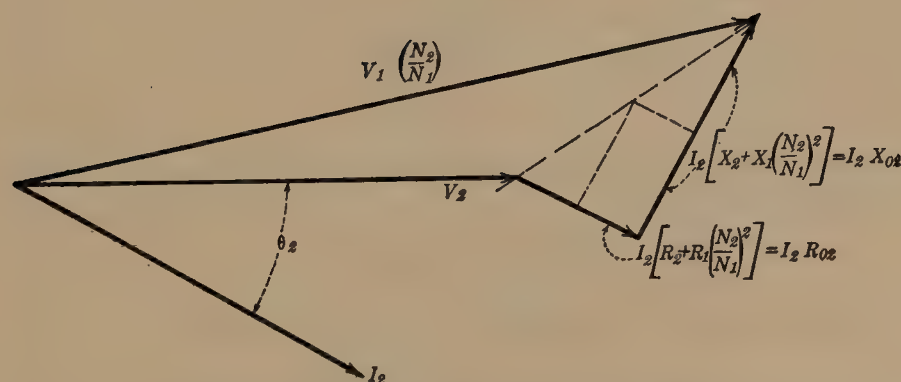
Referring to Fig. 198, $-E_1(N_2/N_1)$ is 180° from E_2 and is equal to E_2; $I_1(N_1/N_2)$ is 180° from I_2; $I_1R_1(N_2/N_1)$ is 180° from I_2R_2; and $I_1X_1(N_2/N_1)$ is 180° from I_2X_2. If, therefore, the entire left-hand side of the diagram be rotated through 180° with the origin as a center, as shown in Fig. 198, $E_1(N_2/N_1)$ and E_2 coincide, $I_1R_1(N_2/N_1)$ and $I_1X_1(N_2/N_1)$ become parallel to I_2R_2 and I_2X_2, respectively.

The right-hand side of the diagram (Fig. 198) gives a simple method for determining the regulation of the transformer, as will be shown in the following paragraph:

109. Equivalent Resistance and Reactance.—Figure 199 (a) gives the right-hand side of the transformer diagram of Fig. 198, E_2 and $E_1(N_2/N_1)$ being omitted. The resistance drop $I_1R_1(N_2/N_1)$ is parallel to I_2R_2, and since $I_1 = (N_2/N_1)I_2$, this resistance drop is also equal to $I_2R_1(N_2/N_1)^2$. Likewise, the



(a)



(b)

FIG. 199.—Equivalent diagram of transformer.

reactance drop $I_1X_1(N_2/N_1)$, parallel to I_2X_2, is equal to $I_2X_1(N_2/N_1)^2$.

It is obvious that the two separate resistance drops may be combined into a single resistance drop; and that the two separate reactance drops may likewise be combined into a single reactance drop, as is done in Fig. 199 (b), without in any way affecting the relations of V_2 and $V_1(N_2/N_1)$.

The equivalent resistance drop, for the transformer as a whole, becomes $I_2[R_2 + R_1(N_2/N_1)^2]$; and the equivalent reactance drop becomes $I_2[X_2 + X_1(N_2/N_1)^2]$.

The quantity

$$R_2 + R_1\left(\frac{N_2}{N_1}\right)^2 = R_{02} \tag{88}$$

is the *equivalent resistance* of the transformer referred to the *secondary.*

The quantity

$$X_2 + X_1\left(\frac{N_2}{N_1}\right)^2 = X_{02} \tag{89}$$

is the *equivalent reactance* of the transformer referred to the *secondary.*

It is obvious, in Fig. 198, that secondary voltage vectors on the right-hand side might equally well be rotated to combine with the primary voltage vectors on the left-hand side. Since primary and secondary are interchangeable,

$$R_1 + R_2\left(\frac{N_1}{N_2}\right)^2 = R_{01} \tag{90}$$

$$X_1 + X_2\left(\frac{N_1}{N_2}\right)^2 = X_{01} \tag{91}$$

are the *equivalent resistance* and *reactance* referred to the *primary.*

It is obvious that

$$\frac{R_{02}}{R_{01}} = \frac{X_{02}}{X_{01}} = \left(\frac{N_2}{N_1}\right)^2. \tag{92}$$

The equivalent impedance referred to the primary

$$Z_{01} = \sqrt{(R_{01})^2 + (X_{01})^2}.$$

The equivalent impedance referred to the secondary

$$Z_{02} = \sqrt{(R_{02})^2 + (X_{02})^2}.$$

Also,

$$\frac{Z_{01}}{Z_{02}} = \left(\frac{N_1}{N_2}\right)^2. \tag{93}$$

That is, the equivalent resistance, reactance, and impedance, referred to the primary, are to the equivalent resistance, reactance, and impedance referred to the secondary as the ratio of primary to secondary turns *squared.*

Consider the copper loss in a transformer

$$P_c = I_1^2R_1 + I_2^2R_2.$$

Since $I_1 = I_2(N_2/N_1)$ and $I_2 = I_1(N_1/N_2)$

$$P_c = I_1^2\left[R_1 + R_2\left(\frac{N_2}{N_1}\right)^2\right] = I_1^2R_{01} \tag{94}$$

$$= I_2^2\left[R_2 + R_1\left(\frac{N_1}{N_2}\right)^2\right] = I_2^2R_{02}. \tag{95}$$

That is, the total copper loss in a transformer is equal to the primary current squared multiplied by the equivalent resistance of the transformer referred to the primary; likewise, the total copper loss in a transformer is equal to the secondary current squared multiplied by the equivalent resistance of the transformer referred to the secondary.

It is thus seen that the equivalent resistance of a transformer, when used in conjunction with the current in the side to which this resistance is referred, may be used to determine the equivalent resistance drop in both primary and secondary; the equivalent resistance may be used to determine the total copper loss in the transformer. The equivalent reactance may be used in a similar way to determine the equivalent reactance drop in the transformer.

It is interesting to note that, if the copper losses of primary and secondary are equal,

$$I_1^2R_1 = I_2^2R_2$$

$$\frac{R_1}{R_2} = \frac{I_2^2}{I_1^2} = \left(\frac{N_1}{N_2}\right)^2. \tag{96}$$

That is, under the foregoing assumption, the primary and secondary resistances are proportional to the squares of their numbers of turns. Equation (96) also holds when the mean lengths of primary and secondary turns are equal, the primary and secondary current densities in the copper also being equal.

Example.—A 50-kv-a. 4,400 to 220-volt transformer has a primary resistance and reactance of 3.45 and 5.40 ohms, respectively. The secondary resistance and reactance are 0.0085 and 0.014 ohm, respectively. Find: (*a*) the equivalent resistance referred to the primary; (*b*) the equivalent resistance referred to the secondary; (*c*) the equivalent reactance referred to both primary and secondary; (*d*) the equivalent impedance referred to

both primary and secondary; (e) the total copper loss, using the individual resistances of the two windings and using the equivalent resistance referred to each side.

The primary current

$$I_1 = \frac{50{,}000}{4{,}400} = 11.36 \text{ amp.}$$

The secondary current

$$I_2 = \frac{50{,}000}{220} = 227 \text{ amp.}$$

The ratio of transformation

$$\frac{N_1}{N_2} = \frac{4{,}400}{220} = \frac{20}{1}$$

(a) $R_{01} = 3.45 + (20/1)^2 0.0085 = 3.45 + 3.40 = 6.85$ ohms. *Ans.*

(b) $R_{02} = 0.0085 + (1/20)^2 3.45 = 0.00850 + 0.00863 =$ 0.0171 ohm. *Ans.*

Also,

$R_{02} = R_{01}(1/20)^2 = 6.85/400 = 0.0171$ ohm (*check*).

(c) $X_{01} = 5.40 + (20/1)^2 0.014 = 5.40 + 5.60 = 11.00$ ohms. *Ans.*

$X_{02} = 0.014 + (1/20)^2 5.40 = 0.014 + 0.0135 = 0.0275$ ohm. *Ans.*

Also,

$X_{02} = (1/20)^2 X_{01} = 11.00/400 = 0.0275$ ohm (*check*).

(d) $Z_{01} = \sqrt{(6.85)^2 + (11.0)^2} = 12.96$ ohms. *Ans.*

$Z_{02} = \sqrt{(0.0171)^2 + (0.0275)^2} = 0.0324$ ohm. *Ans.*

Also,

$Z_{02} = Z_{01}(1/20)^2 = 12.96/400 = 0.0324$ ohm (*check*).

(e) $P_c = (11.36)^2 3.45 + (227)^2 0.0085 = 883$ watts. *Ans.*

$P_c = I_1^2 R_{01} = (11.36)^2 6.85 = 883$ watts. *Ans.*

$P_c = I_2^2 R_{02} = (227)^2 0.0171 = 883$ watts. *Ans.*

The equivalent resistance, reactance, and impedance referred to either side may be used in determining the transformer characteristics, such as regulation, efficiency, etc. (see Par. 112).

110. Open-circuit Test.—Figure 200 shows a transformer having the low side connected to an alternating source of supply and the high side open circuited. Either an auto-transformer or a drop wire is shown as a means of varying the voltage supplied to the low side of the transformer. A voltmeter, an ammeter, and a wattmeter are connected in the primary circuit. The voltmeter reads the voltage across the primary terminals, the ammeter reads the no-load current, and the wattmeter reads the power taken by the transformer under these conditions. This power goes to supply the primary I^2R loss and the core loss of the transformer. As the exciting current is very small, the primary I^2R loss due to it may be neglected. The wattmeter,

therefore, reads the transformer core loss. If the primary voltage be varied and the core loss be determined for different values of voltage, a curve is obtained showing the relation of core loss to voltage. At no load, the flux is practically proportional to the terminal voltage, as the primary impedance drop due to the

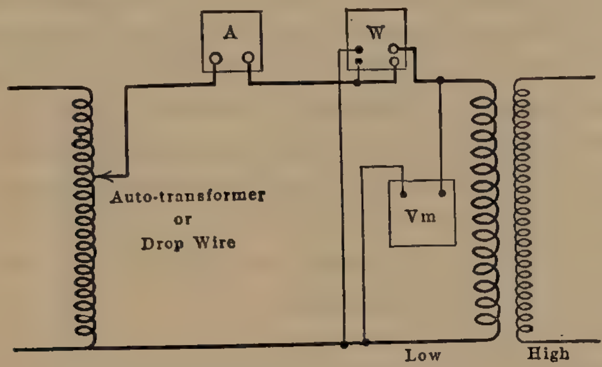


FIG. 200.—Connections for open-circuit test.

no-load current is negligible (see Eq. (85), p. 212). The eddy-current loss varies as the square of the voltage, and the hysteresis loss as the 1.6 power of the voltage. The core loss will increase, therefore, nearly as the square of the voltage, as shown in Fig. 201 (*a*).

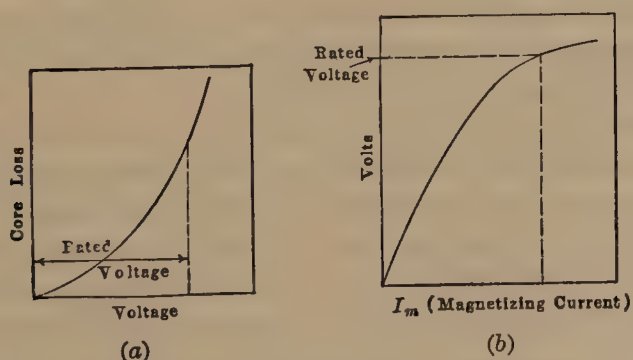


FIG. 201 (*a*).—Relation of core-loss to voltage in transformer.
FIG. 201 (*b*).—Relation of magnetizing current to voltage in transformer.

Transformers are usually so designed that the most economical use of materials is obtained. The core is operated, therefore, at a flux density as high as the allowable core loss will permit. A study of Fig. 201 (*a*) shows that a slight increase of voltage, above rated voltage, produces a very large percentage increase

in core loss. As transformers are rated by their maximum safe operating temperatures, this increased core loss may cause overheating of the transformer. The effect, therefore, of operating transformers at overvoltage is to produce a large increase in temperature.

If the magnetizing current be plotted as abscissas, and the voltage as ordinates, a saturation curve similar to that of Fig. 201 (*b*) is obtained. The point marked "rated voltage" is the point on the saturation curve at which transformers are generally operated and is well beyond the knee of the curve. Outside the question of increased core loss, the usual transformer cannot be operated at a voltage very much in excess of its rated voltage, for the exciting current increases very rapidly with small increase in voltage, as indicated in Fig. 201 (*b*).

The flux density in the core is determined primarily by the permissible core loss. Open-hearth annealed sheet steel, such as is used in dynamos, can be used for transformer cores. For a given flux density and frequency, however, silicon steel has much less core loss per unit volume than open-hearth steel, the effect of the silicon being to increase the electrical resistance and, hence, reduce the eddy-current loss. Because of its small core loss, silicon steel may be operated safely at very high flux densities. The greater cost of silicon steel is more than offset by the saving in iron and copper and in the general reduction of the transformer dimensions.

To obtain the true value of the exciting current, the current I_0, measured by the ammeter, in Fig. 200, should be resolved into two components, one of which lies along the voltage $-E_1$, or V, and is shown as I_e in Fig. 202 ($-E_1$ and V are practically equal at no load). This current $I_e = I_0 \cos\theta$ is the *energy* component of the current and supplies the core losses. The quadrature component $I_m = I_0 \sin\theta$ is the true magnetizing current, shown plotted in Fig. 201 (*b*). In most commercial transformers, $I_0 = I_m$, very nearly.

111. Short-circuit Test.—Figure 203 shows the transformer of Fig. 200 reversed and the low side short circuited. The reversal is made in order that the line current may not be excessive and, also, in order that a reasonable voltage drop may be obtained. In a transformer, the impedance drop seldom exceeds 5 per cent.

of the rated voltage. If the 2,200-volt side of a transformer (Fig. 203) be used as the primary, the voltage necessary to send rated current through the windings on short circuit is about 5 per cent. of 2,200, or 110 volts, which is a standard voltage for instrument coils. If the secondary of the transformer were rated at 220 volts, the voltage at short circuit would be only 11 volts, and the current would also be high. At this low voltage, high precision could not be obtained with ordinary instruments.

When a primary current I_1 flows (Fig. 203), the secondary current I_2 is equal to $I_1(N_1/N_2)$. There is, therefore, no need of using an ammeter for measuring I_2. The power delivered to the

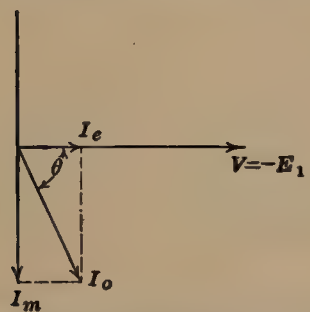


FIG. 202.—Magnetizing and core-loss currents in transformer.

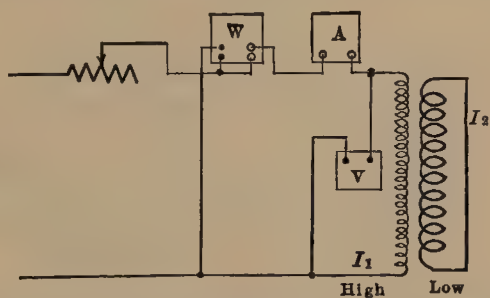


FIG. 203.—Connections for short-circuit test of transformer.

transformer (Fig. 203) goes to supply three losses; the primary copper loss, $I_1^2R_1$; the secondary copper loss, $I_2^2R_2$; and the core loss, at short circuit. The core loss is negligible, as 5 per cent. primary voltage means only about 2.5 per cent. of the rated value of flux, since half the impressed voltage on short circuit is consumed in the primary impedance drop. The core loss at 2 or 3 per cent. of the rated flux is so small as to be negligible, for the core loss varies nearly as the square of the flux. The power at short circuit, therefore,

$$P = I_1^2R_1 + I_2^2R_2 = I_1^2R_{01} = I_2^2R_{02}$$

where R_{01} and R_{02} are the transformer equivalent resistances referred to the primary and secondary, respectively.

$$R_{01} = \frac{P}{I_1^2} \tag{97}$$

$$R_{02} = \frac{P}{I_2^2} \tag{98}$$

The value of equivalent resistance as found in this manner may be checked with the value determined by measuring the resistance of each winding with direct current. The ratio of effective to ohmic resistance is only a few per cent. greater than unity in most transformers.

Figure 204 shows the equivalent-circuit vector diagram for the short-circuit test. This diagram is merely that of Fig. 199, except that V_2 now equals zero and all quantities are now referred to the primary side. It will be recognized, in Fig. 204,

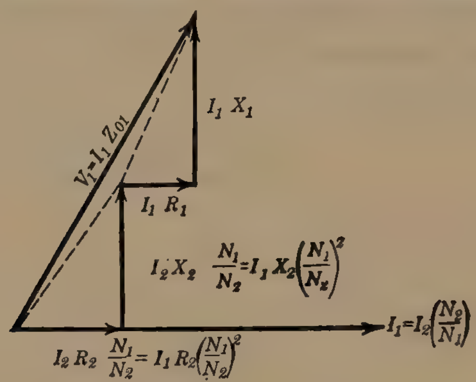


Fig. 204.—Vector diagram for short-circuited transformer.

that the entire voltage V_1 is consumed in the impedance drops of the two windings.

That is,

$$V_1 = I_1 Z_{01} = \sqrt{I_1^2\left[R_1 + R_2\left(\frac{N_1}{N_2}\right)^2\right]^2 + I_1^2\left[X_1 + X_2\left(\frac{N_1}{N_2}\right)^2\right]^2}$$
$$= I_1\sqrt{R^2_{01} + X^2_{01}}$$

Hence,

$$Z_{01} = \frac{V_1}{I_1}. \tag{99}$$

Where Z_{01} is the equivalent impedance of the transformer, referred to the primary side. Also, from Eq. (93) (p. 222),

$$Z_{02} = Z_{01}\left(\frac{N_2}{N_1}\right)^2. \tag{100}$$

The equivalent resistance (Eqs. (97) and (98)) and the equivalent impedance (Eqs. (99) and (100)) being known, the equivalent reactance is readily found.

$$X_0 = \sqrt{Z_0^2 - \mathrm{R}_0^2} \tag{101}$$

for either primary or secondary side.

In making the short-circuit and the open-circuit tests, the question of instrument losses should be investigated, and correction made if this be found necessary. As the losses in a transformer are very small, the power taken by the instruments may be a considerable percentage of the power being measured.

112. Regulation.

The regulation of a constant-potential transformer is the difference between the no-load and rated-load values of the secondary terminal voltage, expressed in per cent. of the rated-load secondary voltage, with the primary impressed terminal voltage adjusted to such a value that the transformer delivers rated kilovolt-amperes output at a specified power-factor and at rated secondary voltage.[1]

Knowing the equivalent resistance and the equivalent reactance of the transformer, it is possible to determine the regulation. Referring to Fig. 199 (*b*) (p. 221), the no-load secondary voltage is $V_1\ (N_2/N_1)$, since at no-load the impedance drop in the transformer is negligible. Hence, with lagging current,

$$V_1\left(\frac{N_2}{N_1}\right) = \sqrt{(V_2 \cos \theta_2 + I_2R_{02})^2 + (V_2 \sin \theta_2 + I_2X_{02})^2} \quad (102)$$

$$\text{Regulation} = \frac{V_1(N_2/N_1) - V_2}{V_2}\,100. \quad (103)$$

Also,

$$V_1\left(\frac{N_2}{N_1}\right) = V_2 + I_2(\cos \theta_2 - j \sin \theta_2)(R_{02} + jX_{02}). \quad (104)$$

Equations (102) and (104) should be compared with (72) and (73), (p. 173 and 174; also, see (74) and (75)).

Equations (102), (103), and (104) are applicable to the primary side if the subscripts are changed. The regulation is the same in either case.

The vector diagram (Fig. 199 (*b*)), shows that after the correction for ratio is made, the transformer is a low impedance in series with its load. This is illustrated in Fig. 205, in which the secondary quantities are expressed in terms of the primary (see example, Par. 113).

[1] *A. I. E. E. Standards*, **13,** Rule 13–117, August, 1925.

113. Efficiency.—The ordinary transformer has a very high efficiency (see table, p. 233). Hence, the efficiency cannot be determined with high precision by direct measurement of output and input, since the losses are only of the order of 2 or 3 per cent. The difference between the readings of the output and input instruments is then so small that an instrument error as low as one-half of one per cent. would cause an error of the order of 15 per cent. in the losses. It is easier, therefore, and more precise to determine the efficiency from the losses.

It has been pointed out that with constant voltage the mutual flux of the transformer is practically constant from no load to full load. It usually does not vary more than from 1 to 3 per cent. The core loss, therefore, is practically constant at all loads and may be determined by the open-circuit test (Fig. 200). For

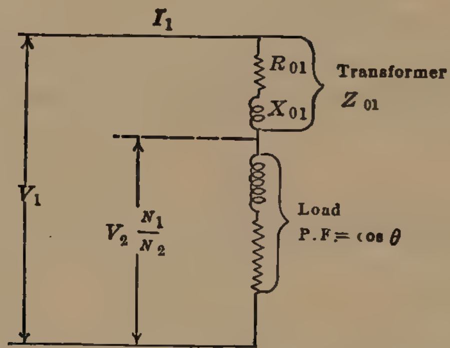


FIG. 205.—Equivalent circuit of transformer.

most purposes, it is necessary merely to measure the loss at the rated voltage of the transformer.

The only other losses are the primary and secondary copper losses. These can be calculated readily, knowing the resistances of primary and secondary, or they may be computed from the equivalent resistance determined at short circuit. The efficiency of the transformer may then be computed, since the losses are known. That is, the efficiency

$$\eta = \frac{V_2I_2(\text{P.F.})}{V_2I_2(\text{P.F.}) + \text{core loss} + I_1^2R_1 + I_2^2R_2} \qquad (105)$$

$$= \frac{V_2I_2(\text{P.F.})}{V_2I_2(\text{P.F.}) + \text{core loss} + I_2^2R_{02}}. \qquad (106)$$

Example.—A 20-kv-a., 2,200 to 220-volt, 60-cycle distributing transformer is tested for efficiency and regulation as follows: A wattmeter, an ammeter, and a voltmeter are used to measure the input to the low side, the high side being open circuited, as shown in Fig. 200. The wattmeter reads 148 watts, the ammeter 4.2 amp., and the voltmeter 220 volts. The transformer is then reversed, the low side being short circuited and 220 volts applied to the high side. Instruments having the proper ranges are connected in circuit, as shown in Fig. 203. The ammeter now reads 10.5 amp., the wattmeter 410 watts, and the voltmeter 220 volts.

Find: (*a*) transformer core loss; (*b*) equivalent resistance referred to high side; (*c*) equivalent resistance referred to low side; (*d*) equivalent reactance referred to high side; (*e*) equivalent reactance referred to low side; (*f*) regulation of transformer at 0.8 power factor, lagging current; (*g*) efficiency of transformer at full and at half-load, load being at 0.8 power factor, lagging current.

(*a*) Core loss is indicated directly by the wattmeter and is equal to 148 watts. *Ans.*

(*b*) $R_{01} = 410/(10.5)^2 = 3.72$ ohms. *Ans.*

(*c*) $R_{02} = 3.72(220/2{,}200)^2 = 0.0372$ ohm. *Ans.*

(*d*) $Z_{01} = 220/10.5 = 21.0$ ohms.

$X_{01} = \sqrt{(21.0)^2 - (3.72)^2} = \sqrt{427} = 20.7$ ohms. *Ans.*

(*e*) $X_{02} = 20.7(220/2{,}200)^2 = 0.207$ ohm. *Ans.*

(*f*) High-side quantities will first be used.

The rated high-side current is $20{,}000/2{,}200 = 9.1$ amp.

$$V_1 = \sqrt{(2{,}200 \times 0.8 + 9.1 \times 3.72)^2 + (2{,}200 \times 0.6 + 9.1 \times 20.7)^2}$$
$$\sqrt{5{,}492{,}000} = 2{,}340 \text{ volts}$$

(See Eq. (102).)

$$\text{Regulation} = \frac{2{,}340 - 2{,}200}{2{,}200} = 6.36 \text{ per cent.} \quad \textit{Ans.}$$

The same result is obtained using the low-side constants.

$$V_1\left(\frac{N_2}{N_1}\right) = \sqrt{(220 \times 0.8 + 91 \times 0.0372)^2 + (220 \times 0.6 + 91 \times 0.207)^2}$$
$$= \sqrt{54{,}920} = 234 \text{ volts}$$
$$\text{Regulation} = \frac{234 - 220}{220} = 6.36 \text{ per cent.} \quad \textit{Ans.}$$

Also, using Eq. (104).

$$V_1\left(\frac{N_2}{N_1}\right) = 220 + 91(0.8 - j0.6)(0.0372 + j0.207)$$
$$= 220 + 14.0 + j13.0 = 234 + j13.0$$
$$\left|V_1\left(\frac{N_2}{N_1}\right)\right| = \sqrt{(234)^2 + (13.0)^2} = 234 \; (\textit{check}).$$

(*g*) Full-load efficiency (using high-side constants)

$$= \frac{20{,}000 \times 0.80}{20{,}000 \times 0.80 + 148 + (9.1)^2 \times 3.72} = \frac{16{,}000}{16{,}460} =$$

97.2 per cent. *Ans.*

Half-load efficiency.

$$= \frac{10{,}000 \times 0.80}{10{,}000 \times 0.80 + 148 + (4.55)^2 \times 3.72} = \frac{8{,}000}{8{,}225} =$$

97.3 per cent. *Ans.*

The same values of efficiency are obtained if the low-side current and resistance are used.

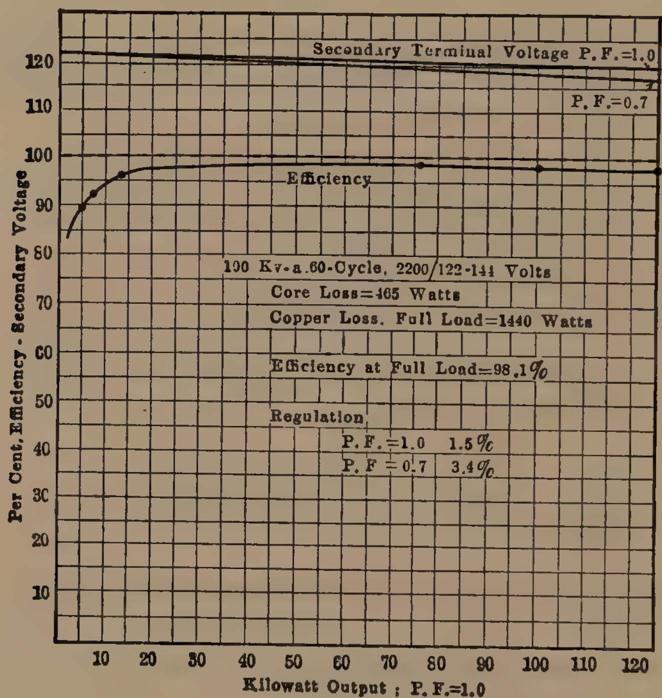


FIG. 206.—Characteristics of a 100-kv-a., 60-cycle transformer.

Figure 206 shows the voltage characteristic and the efficiency of a 100-kv-a., 60-cycle, 2,200/122 to 144-volt transformer plotted against load. It will be noted that the efficiency is high and is practically constant from one-eighth load to 25 per cent. overload.

Below are given regulations and efficiencies of a few typical transformers.

Single-phase, 55°, Self-cooled, Oil-insulated Transformers
Manufactured by
Westinghouse Electric & Manufacturing Company
(60 cycles)

Kilovolt-amperes	Net weight, pounds	Efficiency, per cent.			Regulation	
		One-half load	Three-quarters load	Full load	P.F. = 1.0	P.F. = 0.8
2,300-volt primaries; 460, 230, 115 secondaries						
5	200	97.3	97.4	97.2	2.08	2.8
15	375	97.9	97.9	97.7	1.82	2.7
50	1,200	98.3	98.4	98.3	1.30	2.4
200	3,900	98.2	98.3	98.2	1.30	2.9
500	8,800	98.1	98.3	98.3	1.10	3.2
13,200-volt primaries; 2,300-volt secondaries						
1,000	16,500	98.3	98.5	98.6	1.00	4.0
2,500	26,000	98.6	98.6	98.8	0.90	4.2
5,000	32,000	98.65	98.8	98.8	0.89	4.1
25 cycles						
5	300	95.9	96.1	95.8	3.10	3.9
15	700	97.1	97.2	97.0	2.30	3.0
200	6,000	97.4	97.6	97.4	1.80	3.2
1,000	24,400	98.1	98.3	98.2	1.20	3.8
5,000	56,000	98.5	98.75	98.7	1.20	5.7

114. All-day Efficiency.—Transformers frequently must be connected to give service 24 hr. a day, although the load may be light or practically nothing for a considerable portion of the time. This is particularly true of lighting transformers, which must be ready always to give service but which are lightly loaded except during the house-lighting period. The performance of a transformer under these conditions must be judged by its *all-day* efficiency. This is equal to the ratio of the *energy* output over 24 hr. to the *energy* input over the same period.

Example.—Determine the all-day efficiency of the transformer (p. 231) with the following unity power-factor loads: five-fourths load, 2 hr.; full load, 3 hr.; one-half load, 5 hr.; no load, 14 hr.

The energy output

$$W_1 = 25{,}000 \times 2 + 20{,}000 \times 3 + 10{,}000 \times 5 = 160{,}000 \text{ watt-hr.}$$

The energy input

$$W_2 = 160{,}000 + \left[\left(\frac{5}{4} \times 9.1\right)^2 \times 3.72 \times 2\right] + (\overline{9.1}^2 \times 3.72 \times 3) + \left[\left(\frac{9.1}{2}\right)^2 \times 3.72 \times 5\right] + [148 \times 24] = 160{,}000 + 9{,}640 + 9{,}240 + 3{,}850 + 3{,}550 = 186{,}300.$$

$$\eta = \frac{160{,}000}{186{,}300} = 0.860 \quad \textit{Ans.}$$

as compared with 0.973 at rated load with 0.80 power factor.

115. Core- and Shell-type Transformers.—Transformers are divided into two general types—the core type and the shell type. These two types differ in the arrangement of the iron and copper with respect to each other.

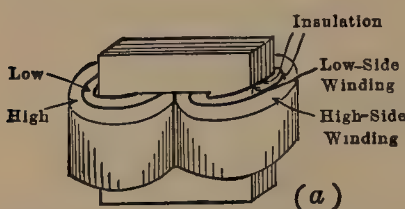


FIG. 207 (*a*).—Arrangement of coils and core in a core-type transformer.

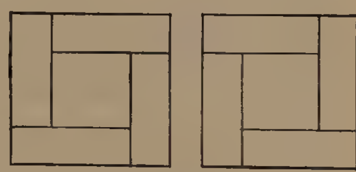


FIG. 207 (*b*).—Arrangement of joints in adjacent lamination layers.

In the core type of transformer, the winding or the copper surrounds the iron core. Figures 193, 195, and 196 are diagrammatic merely, but they represent core-type transformers. Figure 207 (*a*) shows the general arrangement of the core-type transformer. The core is in the form of a hollow square made up of sheet-steel laminations about 14 mils thick. These laminations are usually built up with rectangular strips the joints of which butt, in the individual layers. The joints lap in alternate layers, however, as indicated by Fig. 207 (*b*), which shows the arrangement of joints in two adjacent layers. When a large number of transformers of a single type are being manufactured, the laminations are often made of L-shaped stampings stacked so

that the joints alternate. Figure 208 shows a core-type transformer assembled, with leads, etc., but without the case.

If a transformer were made with the primary and secondary coils on separate legs, as indicated in Figs. 193, 195, and 196, an unsatisfactory transformer would result, as the large leakage flux for both primary and secondary would give very poor regulation. By having both a primary and a secondary on each leg, as shown in Fig. 207 (*a*) and Fig. 208, the leakage flux

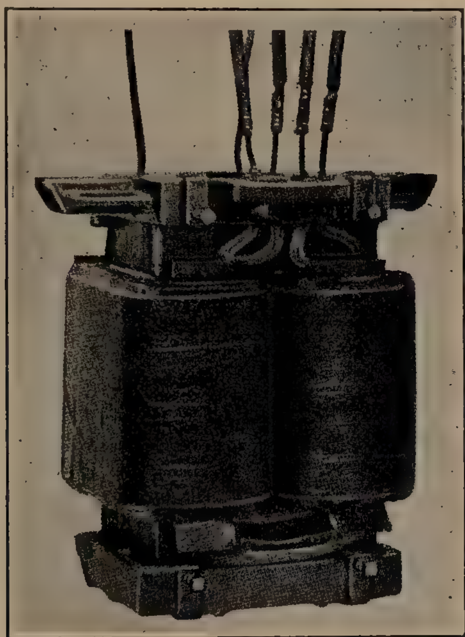
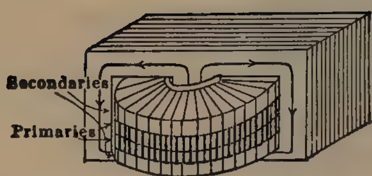


FIG. 208.—Coil and core assembly of Wagner core-type distribution transformer.

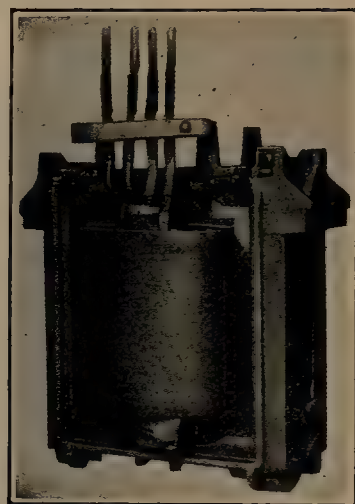
is reduced to a very small value. If the high-voltage winding were placed next the core, it would be necessary to insulate it, both from the core and from the low-voltage winding. Thus, two layers of high-voltage insulation would be necessary. By placing the high-voltage winding outside and around the low-voltage winding, only the one layer of high-voltage insulation, that between the high- and low-voltage windings, is necessary.

In the core type of transformer, the mean length of turn is less, but the mean length of magnetic path is greater than it is in the shell type. The core type of transformer is well adapted to high voltages, especially in the smaller capacities, because the insulation problem is not difficult.

In the shell type of transformer, the iron surrounds the copper, as shown in Fig. 209. The core has the form of a figure 8. The entire flux passes through the central part of the core, but outside this central core it divides, half going in each direction, as shown in Fig. 209 (*a*). This results in a much shorter effective length of magnetic path but a greater mean length of turn. The coils are made in the shape of pancakes, usually wound with strip copper. These coils are taped, and the primary and secondary are usually stacked so that each primary is adjacent to a secondary. In this manner, the leakage flux of both primary and



(*a*).—Arrangement of coils and core in shell-type transformer.



(*b*).—Coil and core assembly of Wagner shell-type distribution transformer.

FIG. 209.—Shell-type transformer.

secondary is reduced to a very small value. In Fig. 209 (*a*), the primaries are the high side and the secondaries are the low side. The secondaries, or low-side coils, are placed adjacent to the iron in order to minimize the amount of high-voltage insulation required.

116. Type H Transformer.—In designing a transformer, it is desirable that the mean length of turn be as short as possible. This reduces both the weight of copper and the resistance and reactance of the winding. This is accomplished in the type H transformer of the General Electric Company, by making a shell-type transformer in which the core is cruciform in shape, as

shown in Fig. 210. The central core around which the coils are wound is operated at much higher flux density than the four wings. Although the reluctance and losses in this core are high, they are not excessive when the entire magnetic circuit is considered. These transformers are used mostly as distribution transformers for stepping down from 2,200 and 1,100 volts to 220 and 110 volts, so that the primary is the high side. It will be observed that the low side, the secondary, is next the iron. That is, one of the two low-side coils is next the central core, and the other is next the iron of the four wings. The two high-side coils lie between the two low-side coils and are not adjacent to the iron. The two high-side coils are insulated from the low-side coils by the mica shields represented by heavy lines. The advantage of this design is that only moderate insulation is required between the low-side coils and the core. As high-voltage insulation need be used only between the high- and the low-voltage coils, a minimum amount of high-voltage insulation is required. This transformer is oil cooled, channels being provided between coils to permit the circulation of oil.

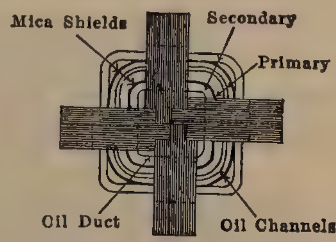


Fig. 210.—Core and windings of type H transformer.

Figure 211 shows a Type H transformer assembled but removed from its case.

117. Cooling of Transformers.—All the energy lost in a transformer must be dissipated as heat. Although this energy is but a small proportion of the total energy undergoing transformation, it becomes quite large in amount in the larger-capacity transformers. The larger the transformer the more difficult it becomes to dissipate the heat, for the kilowatt capacity of the transformer increases much more rapidly than the radiating surface.

Transformers are divided into two classes—self-cooled types and artificially-cooled types. The self-cooled types are usually, although not always, immersed in oil. The oil within the windings and core becomes heated and, because of the lesser density of the heated oil, rises to the top of the case, where it becomes cooled. The cooled oil has a greater density than the warm oil

and so flows downward, in close contact with the case, where it is further cooled. After it reaches the bottom of the transformer, it again rises, passing up through the windings. In addition to carrying heat away from the windings and core, the oil is an excellent insulator and dielectric.

In the moderate sizes of transformers, the radiating surface is increased by corrugating the case (Fig. 212). As transformers increase in capacity, it becomes difficult to dissipate the heat by

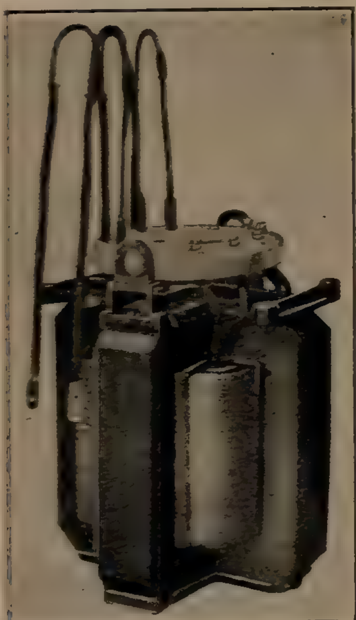


FIG. 211.—Type H transformer removed from tank.

FIG. 212.—Wagner oil-filled, self-cooled, three-phase, distribution-type transformer.

means of the surface of the case alone. One method of increasing the radiating surface is to use exterior tubes running from the top of the case to the bottom (Fig. 213). The hot oil passes out from the tank into the top of the tubes, is gradually cooled, and descends through the tubes to the bottom of the tank, where it again passes up through the windings and core.

Transformers having this tubular construction are limited in size by the side and overhead clearances of the railroads. The tubular principle can be utilized, however, by bolting radiators to the casing to take the place of the tubes, as is shown in Fig. 214. As these radiators are held by bolts, they may be removed during

shipment and bolted in place when the transformer is installed. With large transformers, a layer of gas in contact with the oil is desirable, because of its cushioning effect in case of explosion.

When transformers become warm, the oil and gas expand.

Fig. 213.—Westinghouse 1,000-kv-a., three-phase, 60-cycle, 46,000-23,000-volt tubular-tank transformer.

The gas at the top of the oil is expelled. When the transformer cools, air is drawn into the transformer. Hence, the transformer "breathes." Unless preventive measures are taken, moisture is drawn in during this process; this moisture is readily absorbed by the oil, and the dielectric properties of the oil are correspondingly reduced. Moreover, the oxygen in contact with the oil

oxidizes it, forming a thick "sludge," which adheres to the windings, clogging the oil ducts and frequently resulting in burnouts. Also, oxygen in contact with the oil gives opportunity for fires and explosions in case of internal flash-overs.

To eliminate these difficulties, the transformer tank itself is maintained full of oil by the use of a "conservator" tank on top (Fig. 214), which is partly filled with oil. This reduces the area exposed to the gas. Communication with the external

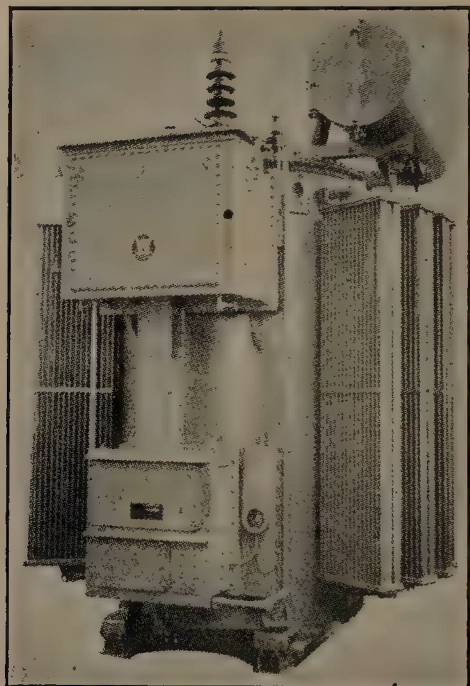


FIG. 214.—12,000 kv-a., single-phase, 60-cycle; 76,200 to 11,500-volt transformer. Provided with 20 per cent. voltage control under load in high-voltage winding. (*Westinghouse Elec. & Mfg. Co.*)

atmosphere is through a "breather." In the Westinghouse "Inertaire"[1] equipment, this breather contains dehydrating and deoxygenating agents. Hence, only dry nitrogen is in contact with the oil, an arrangement which does away with the foregoing difficulties.

The housings in front of the transformer (Fig. 214) contain the tap-changing equipment.[2] By means of this mechanism,

[1] DANN, W. M. and D. R. KELLOGG: "The Inertaire Transformer," *Trans. A. I. E. E.*, Vol. XLIII, 1924, p. 1025.

[2] HILL, L. H.: "Transformer Tap Changing under Load," *Jour. A. I. E. E.*, November, 1927, p. 1214.

the transformer voltage may be changed while the transformer is under load.

Figure 215 shows the core and windings of a high-voltage shell-type transformer removed from its case. When the leads are carried out through bushings in the transformer cover, it is

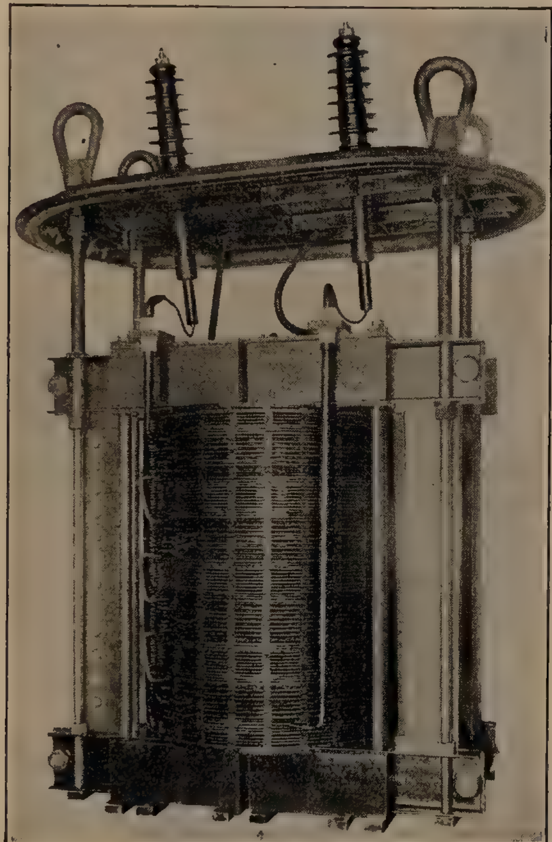


Fig. 215.—Core and windings of General Electric 1,500-kv-a., 60-cycle, 22,000/44,000–2,200/4,400-volt transformer.

necessary to support the core and windings entirely from the cover. The cover with the core and windings can then be lifted as a unit.

There are two different types of artificially cooled transformers, air-cooled and oil-cooled transformers. The air-cooled or air-blast type is ordinarily mounted on a platform under which air pressure is maintained by means of blowers (Fig. 216). The air is forced up through the transformer windings, keeping them at the proper temperature. The advantage of air-cooled

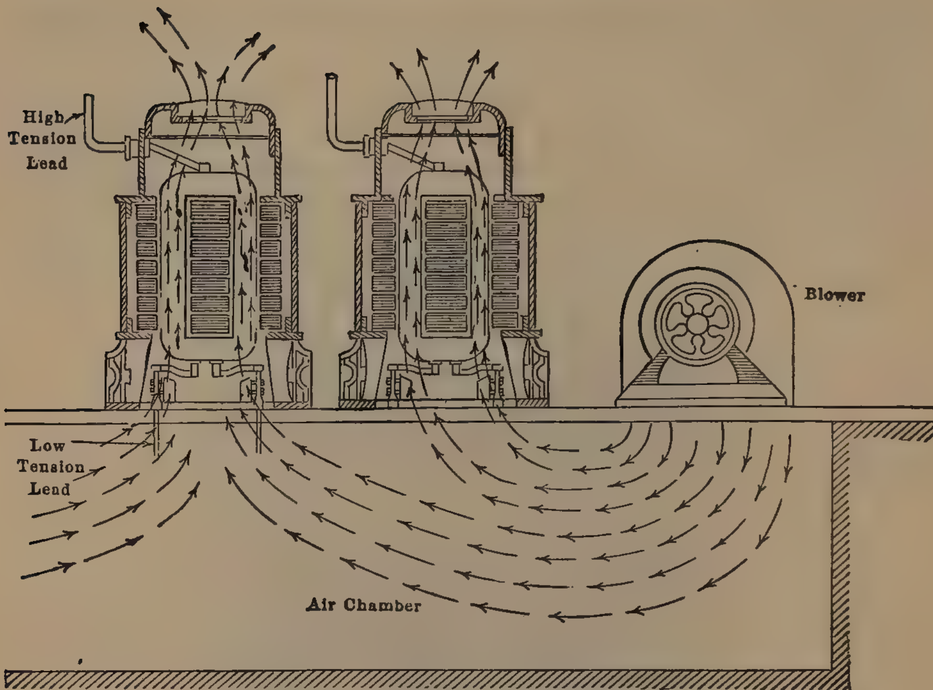


FIG. 216.—Cooling of air-blast transformers.

FIG. 217.—Water-cooled, shell-type transformer removed from its case.

transformers is that fire risk is reduced because the danger of flooding the station with burning oil is eliminated. They are, therefore, used extensively in substations which are located in thickly settled districts. On the other hand, this type of transformer does not have the dielectric strength which the oil adds to the insulation. For this reason, air-cooled transformers are seldom manufactured for potentials in excess of 30,000 volts.

The most common method of artificially cooling the oil type of transformer is to place a copper coil in the top of the transformer tank and circulate cooling water through the coil. This coil is located where it is in contact with the hot oil (Fig. 217). Careful tests should be made at regular intervals for leaks in the cooling coils, as the presence of a very slight amount of water in the oil greatly impairs its insulating and dielectric properties.

118. Three-phase Transformers.—Three-phase transformers have considerably less weight and occupy much less floor space than three single-phase transformers of equal capacity. For this reason, they are commonly used in practice. The principle of the three-phase core-type transformer is illustrated by Fig. 218 (*a*). Three single-phase transformers (secondaries not shown) have each a primary winding upon one leg. These transformers are symmetrically wound, and each winding is connected to one wire of a three-phase system. These cores are placed 120° apart so that the empty legs of the three are in contact. The center leg formed by these three carries the sum of the three fluxes produced by the three-phase currents I_1, I_2, and I_3. As the sum of the three currents at any instant is zero, the sum of the three fluxes must also be zero. No appreciable flux exists in the common leg, and this leg may be eliminated, therefore, without disturbing existing conditions. A more practicable arrangement, from the construction standpoint, is shown in Fig. 218 (*b*). The reluctance of the magnetic circuit for the center coil is less than it is for the two outer coils. This makes the magnetizing current of the middle phase slightly less than that of the two outer phases, but the magnetizing currents are so small that this has no noticeable effect on the operation of the transformer.

Figure 219 shows a three-phase, shell-type transformer. It does not differ from three single-phase, shell-type transformers laid side by side. Owing to the joint use of the magnetic paths

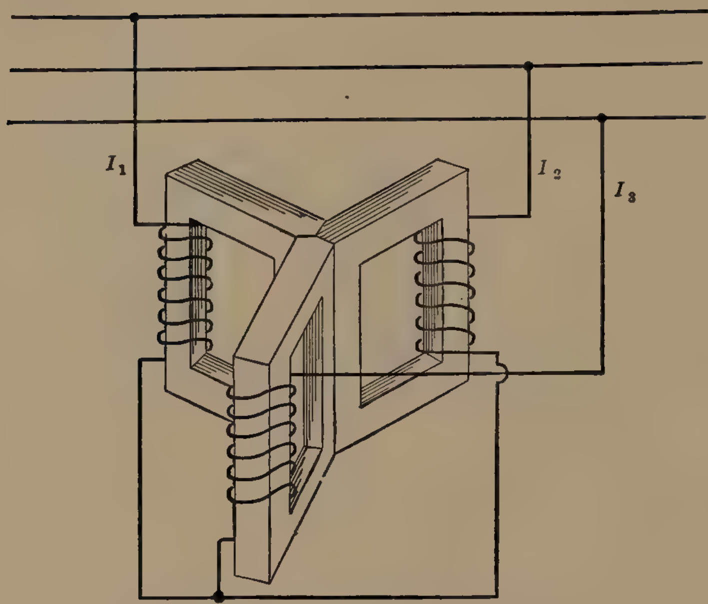


FIG. 218 (*a*).—Principle of three-phase, core-type transformer.

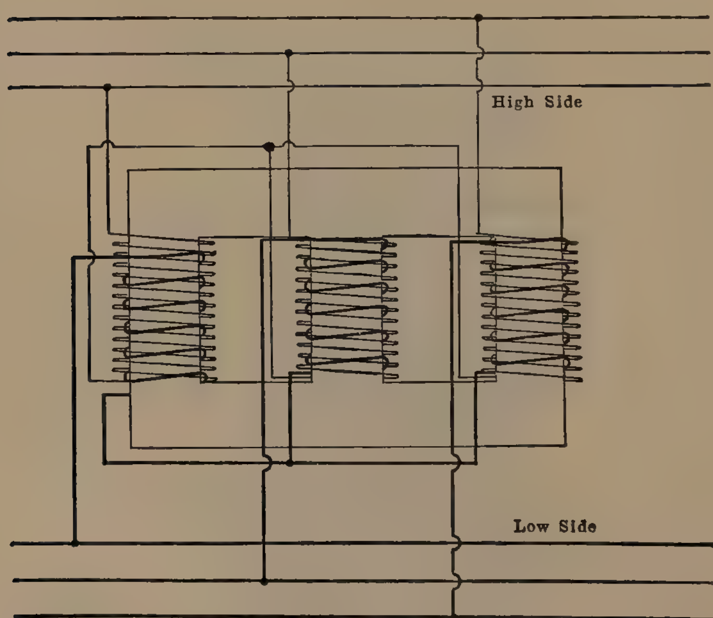


FIG. 218 (*b*).—Practical arrangement of windings on three-phase, core-type transformer, connected Y-Y.

between the coils, there is less iron in this type of transformer than in three equivalent single-phase units. As each phase has a magnetic circuit independent of the others, the three phases are more independent of one another than they are in the core type. Figure 213 shows a complete three-phase transformer with a tubular tank.

The lower cost of three-phase transformers and the smaller space occupied by them is often balanced by the fact that if any one phase becomes disabled, the whole transformer must ordinarily be removed from service. (The shell type may be operated open delta at 58 per cent. of its rating, but this is not always feasible.) If one transformer of a three-phase bank of single-phase transformers becomes disabled, the system may run open delta at reduced capacity, or the transformer may be replaced by a single spare which can be readily substituted.

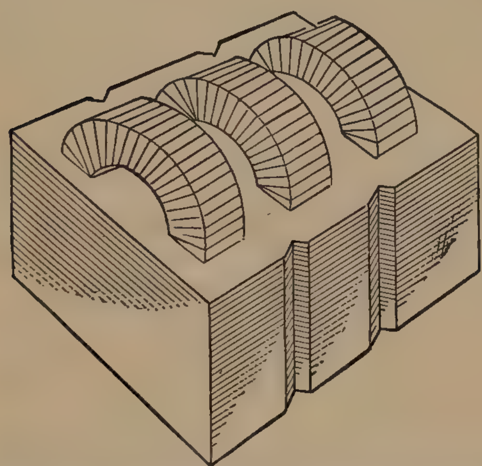


FIG. 219.—Arrangement of coils and laminations in three-phase, shell-type transformer.

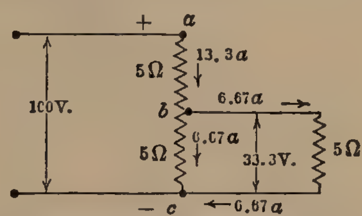


FIG. 220.—Currents in drop-wire system.

119. Auto-transformers.—Figure 220 shows a drop wire, having a a resistance of 10 ohms, connected across a 100-volt supply. The supply may be either direct or alternating current. A load having a resistance of 5 ohms is tapped to the middle of the drop wire and to one side of the line. The parallel resistance of the two 5-ohm coils is 2.5 ohms, which being in series with 5 ohms makes a total resistance of 7.5 ohms across the 100-volt circuit. The total current flowing is 100/7.5 = 13.3 amp. This current of 13.3 amp. divides equally between the two parallel resistances, making 6.67 amp. in each. The voltage across the single 5-ohm resistance is 66.7 volts, and that across the 5-ohm

resistance which constitutes the load is 33.3 volts. In order to obtain 6.67 amp. at 33.3 volts with this system, the line must supply 13.3 amp. at 100 volts. The efficiency of this system is, therefore, very low.

Figure 221 (*a*) shows a transformer whose primary *ac* is connected across 100-volt alternating-current supply. The secondary $b'c'$ has just half the number of turns of the primary *ac*, and, therefore, the voltage across the secondary is 50 volts. This secondary $b'c'$ supplies a 2.5-ohm resistance, so that the secondary current is 20 amp. Instantaneous directions of currents are indicated. Neglecting the magnetizing current, the primary current I_{ac} is 10 amp., flowing *downward* as indicated. It will be noted that the secondary current $I_{c'b'}$ is 20 amp., flowing *upward*.

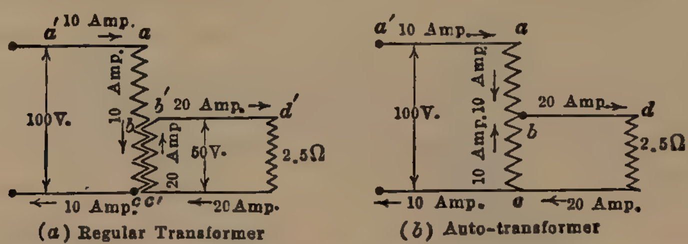


FIG. 221.—Currents and voltages in auto-transformer supplying load at 50 per cent. voltage.

If the secondary winding $b'c'$ be combined with the part *bc* of the primary winding, where *b* is the midpoint of the winding *ac*, no disturbance will occur, as the voltage V_{cb} is equal to the voltage $V_{c'b'}$, and the two are substantially in phase. (The current flows against the e.m.f. in winding *bc* and with the e.m.f. in winding $b'c'$.) Assume that the windings *bc* and $b'c'$ are in contact at every point, giving a single winding, as shown in Fig. 221 (*b*). The current I_{cb} will now be the algebraic difference of the original primary current I_{bc} and the secondary current $I_{c'b'}$, or 10 amp., as shown. Instead, therefore, of having two windings, one of which carries 10 amp. and the other 20 amp., a single winding only is necessary, and its rating need not be greater than 10 amp. The copper represented by the 20-amp. secondary (Fig. 221 (*a*)) may in this case be eliminated, and yet there is sufficient copper to transfer the same power from one circuit to the other. Such a transformer is called an *auto-transformer* or *compensator*.

The primary voltage is E_{ac}, and the winding *ac* receives the power; the secondary voltage is E_{bc}; the ratio of transformation is E_{ac}/E_{bc}; the magnetizing current flows through winding *ac*. The winding *ac*, therefore, could properly be considered as the primary, and the winding *bc* as the secondary. When discussing the *current* and *power* relations within the transformer itself, the treatment is simplified by considering the winding *ab* as the primary and the winding *bc* as the secondary, the magnetizing current being neglected.

The coil *bc* supplies power to the load and is the secondary of a transformer of which *ab* is the primary. Neglecting losses and magnetizing current, both of which are small:

The power delivered to the load is $50 \times 20 = 1{,}000$ watts.
The power in the primary *ab* is $50 \times 10 = 500$ watts.
The power in the secondary *bc* is $50 \times 10 = 500$ watts.

Only 500 watts are transformed, but 1,000 watts pass to the load.

The extra 500 watts are *not transformed* but merely flow *conductively* from the line *a′a* to the line *bd*. In this case, but half the total power is *transformed*.

In the drop wire, the current flowing from *a* to *b* (Fig. 220) undergoes a drop in potential. The power, represented by the product of this drop in potential and the current, goes to heat the wire *ab*. In the auto-transformer, however, the power represented by the current undergoing a drop in potential from *a* to *b* (Fig. 221 (*b*)) is not wasted but is transferred to the magnetic field. This power, transferred to the magnetic field, appears in the winding *bc* where a current of 10 amp. is raised 50 volts in potential. That is, by transformer action, power is transferred from winding *ab* to winding *bc*.

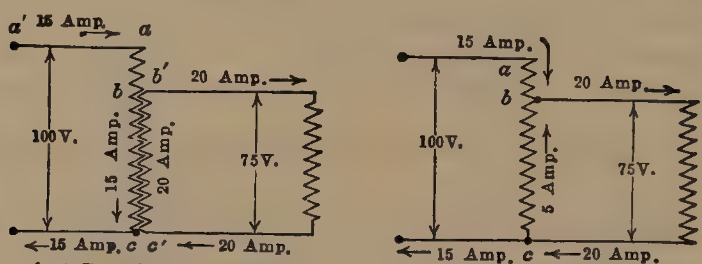
Although diagrammatically the auto-transformer looks like a drop wire, its operation is entirely different. The auto-transformer is superior, as regards both efficiency and voltage regulation.

Figure 222 (*a*) shows a regular transformer, which transforms 1,500 watts from 100 volts and 15 amp. to 75 volts and 20 amp. That is, the voltage is stepped down in the ratio of 4 to 3. The primary current I_{ac} is 15 amp., and the secondary current $I_{c'b'}$ is

20 amp., as shown. When the windings bc and $b'c'$ are combined to make an auto-transformer (Fig. 222 (b)), the net current in bc is but 5 amp. The winding $b'c'$ in (a) may be eliminated entirely, and winding bc in (b) need be only one-third the cross-section of the winding bc in (a). Hence, there is a very considerable saving in copper in the auto-transformer over the regular transformer.

In Fig. 222 (b):

Primary power in $ab = 25 \times 15 = 375$ watts.
Secondary power in $bc = 75 \times 5 = 375$ watts.
Transformed power $= 375$ watts.
Power *conducted* must then be $1{,}500 - 375 = 1{,}125$ watts.



(*a*) Regular Transformer (*b*) Auto-Transformer

FIG. 222.—Currents and voltages in auto-transformer supplying load at 75 per cent. voltage.

Only *one-fourth* the total power involved is now *transformed*, whereas, in Fig. 221, one-half the total power was transformed.

The auto-transformer is sometimes called a *compensator*. It is a type of transformer which transforms a portion of the energy and allows the remainder to flow conductively through its windings. Its action is analogous to the balancer set (see Vol. I, p. 454, Par. 281). The current I_{ab} in dropping through the voltage E_{ab} raises the current I_{cb} to the voltage E_{cb}. Winding ab corresponds to the motor, and winding bc to the generator of Fig. 383, Vol. I (p. 454). In fact, a compensator can be used to obtain the neutral of an alternating-current three-wire system in the same manner as a balancer set is used to obtain the neutral in a direct-current three-wire system. The connections of a compensator used in this manner are shown in Fig. 223. The compensator is superior to the balancer set both in efficiency and in maintenance.

For moderate ratios of transformation, the compensator is much more economical in the use of materials and has a much higher efficiency than a transformer which transforms all the power. With the higher ratios of transformation, more and more of the power is transformed, and less and less conducted. So the auto-transformer is economical only for small ratios of transformation. Also, the low side and the high side are connected together conductively. In commercial systems, therefore, the low side should be grounded at the proper point for reasons of safety, if the high-side voltage is sufficiently high to be dangerous.

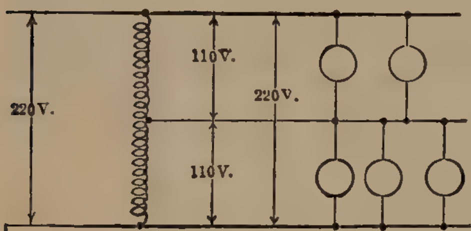


FIG. 223.—Compensator or balance coil used to obtain three-wire lighting system.

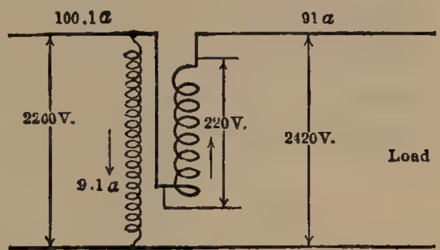


FIG. 224.—Lighting transformer used as booster.

An ordinary lighting transformer can be used as an auto-transformer to change the voltage by a moderate amount. Figure 200 shows a 20-kv-a., 2,200- to 220-volt transformer. The rated primary current

$$I_1 = \frac{20,000}{2,200} = 9.1 \text{ amp.}$$

and the rated secondary current

$$I_2 = \frac{20,000}{220} = 91 \text{ amp.}$$

The high and low sides can carry 9.1 and 91 amp., respectively, without exceeding their ratings. The low side may be connected to raise the voltage, as shown in Fig. 224. Ninety-one amperes can flow to the load without overloading the low-tension coil. This requires 9.1 amp. in the high-side coil, which is now acting as primary. The line current from the supply must be 100.1 amp. If the transformer losses are neglected, the power supplied

$$P_1 = 2,200 \times 100.1 = 220,220 \text{ watts}$$

power delivered

$$P_2 = 2,420 \times 91 = 220,220 \text{ watts}$$

power transformed $= 91 \times 220 = 20,000$ watts.

Assume 97 per cent. efficiency for the transformer. This means that the loss is $0.03 \times 20{,}000 = 600$ watts.

The efficiency of the system is

$$\frac{220{,}220}{220{,}220 + 600} = 99.8 \text{ per cent.}$$

It is to be noted that a device of this type is very similar to the series booster described in Vol. I (p. 358), but that it is much simpler and much more efficient. When an ordinary lighting transformer is used in this manner, the low-side winding should be grounded at one point as the insulation between the low side and core is not designed to withstand full high-side potential.

120. Phasing Transformer Windings.—Both primary and secondary of transformers usually consist of two or more windings, which may be connected in series or in parallel, thus giving the transformer a wider range of voltage and current ratings. If these windings are not connected properly with relation to each

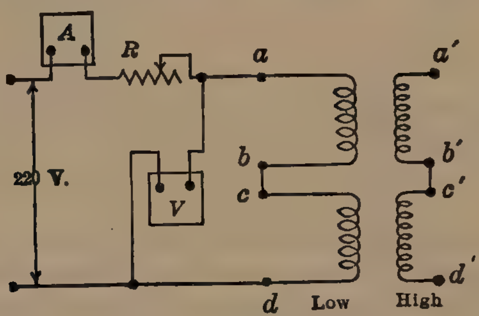


FIG. 225.—Methods of phasing transformer coils.

other, a virtual short circuit results when the transformer is put into operation. There are many methods of phasing such windings. Assume (Fig. 225) that *ab* and *cd* are the two 110-volt windings of a step-up transformer. It is desired to connect *ab* and *cd* in series for 220-volt operation. If 110 volts is available, connect one winding *ab* across this voltage. Connect terminal *c* of *cd* to terminal *b* of *ab*. If the voltmeter across *ad* reads 220 volts, the windings are connected properly for series operation. If the voltmeter reads zero, *a* may be connected to *d* for 110-volt parallel operation. If 220-volt supply only is available, the two windings may be connected in series across this voltage (Fig. 225). If, however, the two windings should be opposition, their impedance is low, so that a virtual short

circuit would result if they were connected directly across this voltage. Hence, a resistance R (or a reactance) must be connected in series to limit the current, if by chance the windings should be in opposition. It follows that when the windings are in opposition, the voltmeter V reads low, and the ammeter A reads high.

To phase the high-side coils $a'b'$ and $c'd'$, use the connections of Fig. 225 and connect a' to d'. If the coils $a'b'$ and $c'd'$ are connected properly for series operation, they are now short circuited. The ammeter A in the primary reads high, and the voltmeter V reads low. If the secondary coils are connected properly for parallel operation, the ammeter A reads low, and the voltmeter V reads high. Also, these secondary coils may be phased in the same manner as the primary, by reversing the transformer.

121. Three-phase Transformer Connections.—There are several methods of connecting three-phase transformer banks, as, for example, Y-Y, Δ-Δ, Δ-Y, Y-Δ, V-V, T-T, etc.

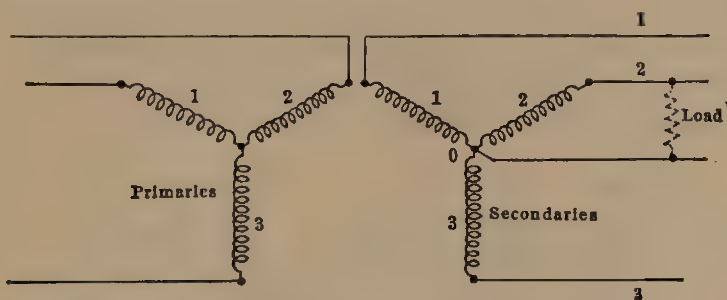


Fig. 226.—Y-Y connection of transformers.

The primaries of single-phase transformers may be connected at will in Y or in delta, as the case may be. But the secondaries must be so connected that the proper phase relations exist. This may be accomplished by the same method that is used for alternator coils (p. 157). The primaries of three-phase transformers, having parts of the magnetic circuit in common, must be phased. Phasing is frequently avoided when primary and secondary leads are brought out of the case symmetrically.

Figure 226 shows a Y-Y connected transformer bank, which may be either a step-up or a step-down bank. Unless the primary neutral is connected to the generator neutral, this con-

nection has the objection of having a "floating" neutral. An extreme case is illustrated by attempting to place a load from wire 2 to the neutral, on the secondary side. This power must be supplied by primary coil 2. This primary coil cannot supply the power because it is in series with the primaries 1 and 3 whose secondaries are open circuited. The two primaries 1 and 3 under

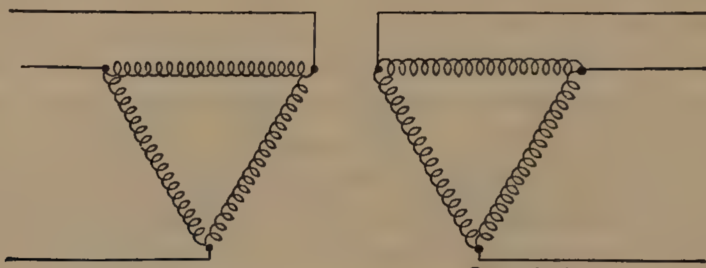


FIG. 227.—Delta-delta connection of transformers.

these conditions act as very high impedances, so that the primary 2 can obtain but very little current through them from the line. Transformer 2, therefore, can supply no appreciable power. In fact, the secondary of 2 may be short circuited, and only a small current will flow. The short circuit merely pulls the primary and secondary neutrals over to wire 2.

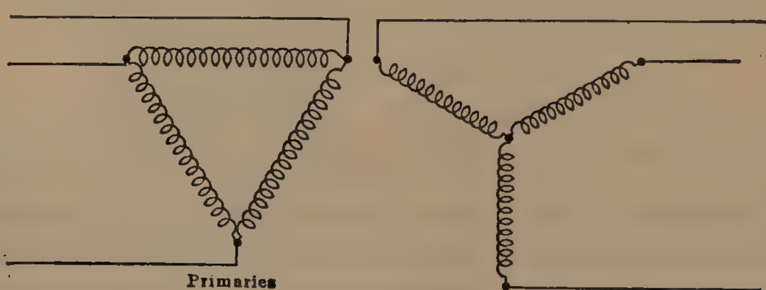


FIG. 228.—Delta-Y connection of transformers.

This difficulty of the "floating" neutral may be obviated by connecting the primary neutral back to the generator so that the primary of transformer 2 can take its power from between its line and the neutral. Another objection to the Y-Y connection is the fact that the secondary coil voltages contain large third harmonics.

The delta-delta bank, shown in Fig. 227, is often used, especially for moderate voltages. Its chief advantage is that, if one transformer becomes disabled, the system may operate in "V" or open delta. In both the Y-Y and the delta-delta connections, the ratios between the primary and secondary line voltages are the same as the individual transformer ratios.

The delta-Y connection, shown in Fig. 228, is a very useful connection for stepping up the voltage. It is not open to the objection of a "floating neutral" and of wave distortion, such as the Y-Y connection involves. Another distinct advantage of the delta-Y connection over the delta-delta connection is that for high voltages the transformers need not be so well insulated. For a 100,000-volt system, the Y-connected transformers need be insulated only for 58,000 ($100,000/\sqrt{3}$) volts, whereas delta-connected transformers must be insulated for 100,000 volts. The Y-delta system is often used for stepping down the voltage. The ratio between line voltages in these two systems is not the individual transformer ratio, for the line voltage on the Y-side is $\sqrt{3}$ times that given by the transformer ratio. A delta-Y bank cannot be paralleled with a Y-Y or a delta-delta bank, even although the voltage ratios are correctly adjusted, as there will be a 30° phase difference between corresponding voltages on the secondary side.

122. V-connection.—It was pointed out in Par. 84 (p. 158) that line voltage must exist between the open ends of the two coils of the delta before the third one is connected. At no load, with only two transformers, three equal three-phase voltages exist around the secondaries, and a three-phase transformation is, therefore, possible with only two transformers. This is called the "V" or open-delta connection (Fig. 229). Under balanced loads, the voltages may become slightly unbalanced. This is not serious in commercial transformers, as their regulation is seldom greater than 2 or 3 per cent.

At first thought, it might appear that the V-connection would have two-thirds the capacity of the delta-connection. Both transformers work at a reduced power factor when connected in V, even though the power factor of the load remains fixed. The kilovolt-ampere capacity of the V-connection, therefore, is less than two-thirds of the kilovolt-ampere capacity of the

delta-connection having individual transformers of equal rating. The ratio of the V-capacity to the delta-capacity is $1/\sqrt{3} = 58$ per cent. rather than $66\frac{2}{3}$ per cent. This can be proved as follows:

Let I be the rated current of each transformer, and E the line voltage. The power, at unity power factor (Fig. 229), is

$$P_1 = \sqrt{3}EI.$$

As the transformer rating is determined by the *current*, the output of three of these transformers in delta would be

$$P_2 = 3EI.$$

Therefore,

$$\frac{P_1}{P_2} = \frac{\sqrt{3}EI}{3EI} = \frac{1}{\sqrt{3}} \text{ or 58 per cent.}$$

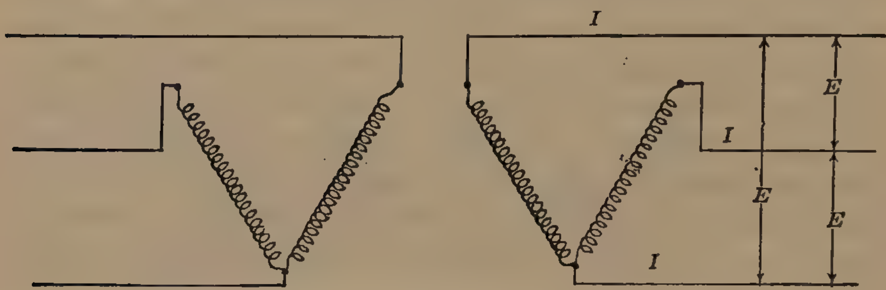


FIG. 229.—V or open-delta connection of transformers.

Oftentimes, in practice, a V-bank of transformers is first installed. The third transformer is added when the increase in load on the system warrants it. The rating of the bank is then increased 73 per cent. with an investment increase of but 50 per cent.

123. Scott or T-connection.—By means of the Scott or T-connection, it is possible to transform not only from three-phase to three-phase by means of two transformers but also from three-phase to two-phase or from two-phase to three-phase. The method of connecting for three-phase to three-phase transformation is shown in Fig. 230 (a) and (b). Two transformers having primaries ad and bc and secondaries $a'd'$ and $b'c'$ are used. The middle point d of the winding bc and d' of the winding $b'c'$ must be accessible. One end d of the primary winding ad is connected

to the middle point d of the primary bc. The respective ends of the three coils are connected to the three-phase supply abc. The transformer bc is called the *main* transformer, and ad the *teaser* transformer.

Figure 230 (c) shows the voltage diagram. The three-phase supply is assumed to be 100 volts across lines, and the transformers have a one-to-one ratio.

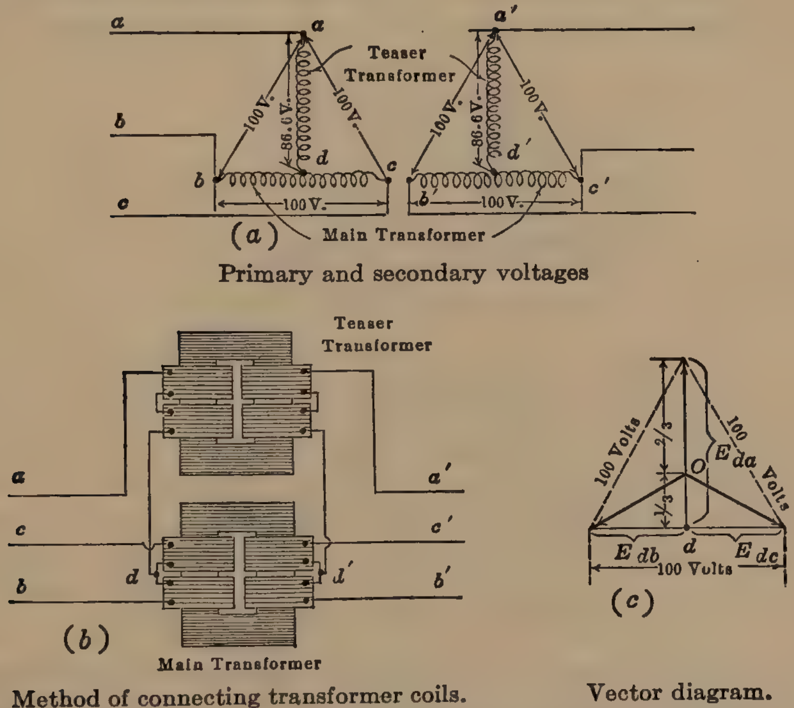


FIG. 230.—T-connected transformers, three-phase to three-phase.

The voltages E_{dc} and E_{db} are each equal to 50 volts and are 180° apart, since coil dc and coil db are both on the *same magnetic circuit*. Each side of the equilateral triangle (Fig. 230 (c)) is equal to 100 volts. The voltage E_{da} is the altitude of the equilateral triangle and is, therefore, equal to $100\sqrt{3}/2$ or 86.6 volts. The same relations hold in the secondary coils, so that $a'b'c'$ is a symmetrical three-phase system. The full capacity of the transformers is not utilized, however. The teaser transformer operates at only 86.6 per cent. of its rated voltage, and in the coils bd and dc the current lags 30° in one and leads 30° in the other at unity power factor. This gives a power factor of 0.866 in the

transformer coils and is, therefore, equivalent to the transformers' operating at only 86.6 per cent. of their rated kilovolt-ampere capacity. If, however, the teaser is designed for 86.6 per cent. voltage, it operates at full capacity, and the capacity of the system is then $\dfrac{100 \times 0.866 + 86.6}{100 + 86.6} = 0.928$ of the total transformer capacity.

If the ends b' and d' of the secondaries be connected, as shown in Fig. 231 (a), a two-phase, three-wire system results. The

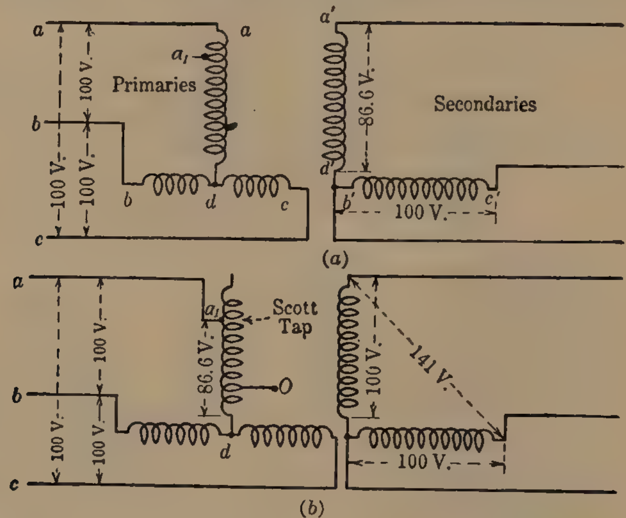


FIG. 231.—Scott or T-connection, three- to two-phase.

voltage $E_{d'a'}$ is equal to only 86.6 volts, whereas the voltage $E_{b'c'}$ equals 100 volts. The resulting two-phase system, therefore, has unequal voltages. This may be corrected, however, if the line a be connected to point a_1 on the primary of the teaser transformer, the point a_1 being such that da_1 represents 86.6 per cent. of the total winding of the teaser transformer, as shown in Fig. 231 (b). This will increase the volts per turn in the ratio of 100 to 86.6 and will raise the secondary voltage a corresponding amount. A symmetrical two-phase, three-wire system, therefore, results. By connecting the middle points of the secondaries together, a symmetrical quarter-phase, four- or five-wire system may be obtained, as shown in Fig. 232.

In any of the foregoing connections, d is not the neutral of the primary, as it is not the center of gravity of the voltages. The

voltages from the point O (Fig. 230 (c) and Fig. 231 (b)) to a, b, and c are all equal. Point O, therefore, is the neutral of the primary system. Point O is two-thirds the way down the teaser transformer winding from a_1 to d (Fig. 231 (b)).

In these connections, the voltages become slightly unbalanced even under balanced loads. This is due to the unsymmetrical phase relations among the voltages and the currents in the individual coils.

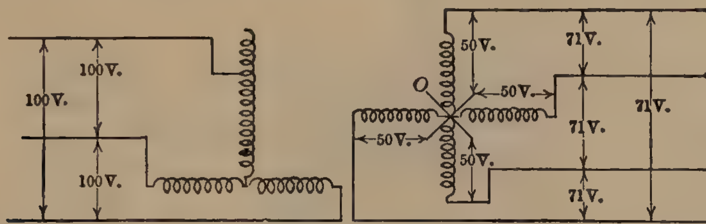


FIG. 232.—T-connected transformers giving quarter-phase, four-wire system with balanced voltages.

124. Constant-current Transformers.—The transformers heretofore considered are constant-potential transformers; that is, the secondary voltage remains substantially constant, and a change of load is accompanied by a corresponding change of current. There are instances where a constant *current* is desired, the most common being series street lighting. It will be recalled that constant direct current is obtained from a series generator. Constant alternating current is ordinarily obtained from a constant current or "tub" transformer.

The construction of the transformer is such that the primary and the secondary can move with respect to each other. The primary coil may be fixed, and the secondary may move; or the secondary coil may be fixed, and the primary may move. Both types are found in practice. Figure 233 shows a transformer in which the primary is stationary and the secondary is movable. The load consists of a number of lamps connected in series. The secondary is suspended from a lever which is counterweighted. A dashpot is provided to prevent rapid fluctuations in the position of the moving coil.

The operation of the transformer is as follows: Assume that the secondary coil is "floating," that is, is free to move either up or down and is delivering a certain current to a series load. The

currents in the primary and secondary flow in opposite directions (Fig. 234). There is *repulsion*, therefore, between the two coils. Assume that the load changes; for example, it decreases. This change of load would be produced by *short cir-*

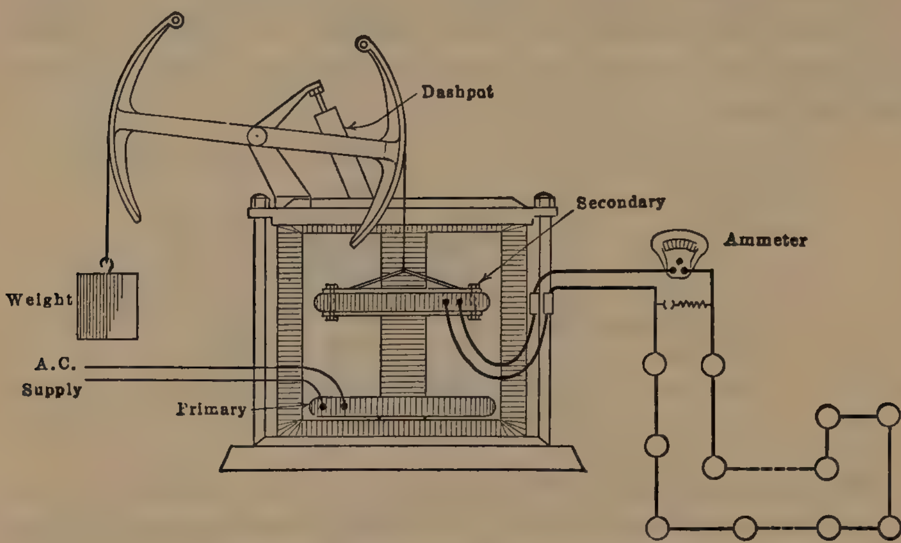


FIG. 233.—Constant-current transformer.

cuiting one or more lamps, causing a *decrease* in the load resistance. Because of the decreased load resistance, first the secondary and then the primary current tends to increase. This increases the repelling force between the two coils, resulting in the secondary's moving farther away from the primary. The leakage flux between the two coils is thus increased, and this reduces the secondary induced volts. The secondary coil will move away from the primary until the secondary current is again at its normal value. The action of such a transformer depends on the change in leakage flux of both primary and secondary, as is shown in Fig. 234. Because of its large proportionate leakage flux, this type of transformer has a very low power factor except at or near its maximum load. This is one objection to its use.

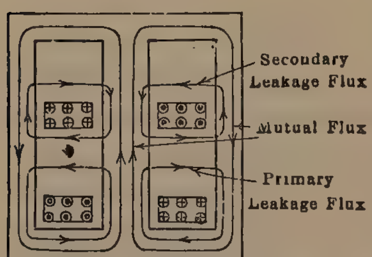


FIG. 234.—Flux paths in constant-current transformer.

Magnetite arcs require a unidirectional current for their proper operation (see Chap. XV, p. 511, Par. 255). In order to obtain this current economically by the use of the constant-current transformer, mercury-arc rectifiers are used in connection with the transformer. Ordinarily, two rectifier tubes are connected in series. The diagram of connections for a single rectifier tube is shown in Fig. 235. The mercury arc has the property of

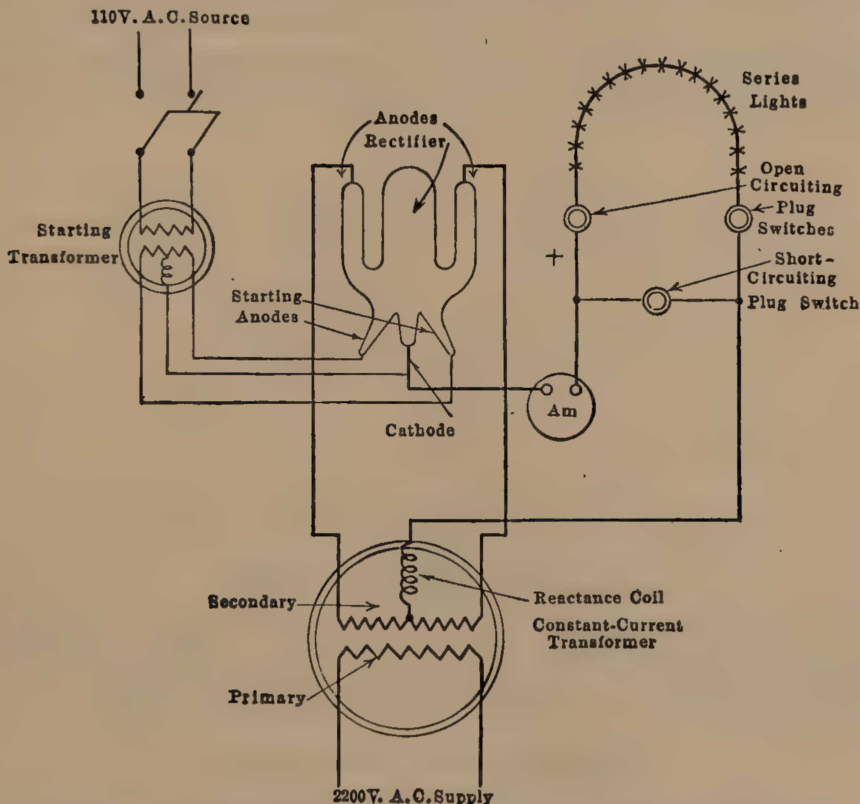


Fig. 235.—Constant-current transformer and mercury-arc rectifier.

allowing current to pass in but one direction. When current tends to pass in the opposite direction, the mercury vapor acts like a valve and prevents its flow. If there were but one circuit through the rectifier, the negative half of the wave would be eliminated, and the result would be a number of disconnected waves, as shown in Fig. 236 (*a*). The rectifier would not operate under these conditions, as the arc would go out between waves and would fail to reestablish itself. By using two anodes,

a succession of connected waves (Fig. 236 (*b*)) is obtained. Even under these conditions, the current becomes zero twice each cycle, so that the arc cannot reestablish itself. By using reactance (Fig. 235), the current is held over each half-cycle, resulting in a rippled, unidirectional current wave (Fig. 236 (*c*)); (also, see Chap. XII, p. 380, Par. 170 (3)).

The starting anode shown in Fig. 235 is an electrode which sends a current at low voltage through the liquid mercury, causing it to vaporize, and in this way establishes the initial arc. The mercury-arc rectifier used under these conditions has a very high efficiency, as the voltage drop across the tube itself

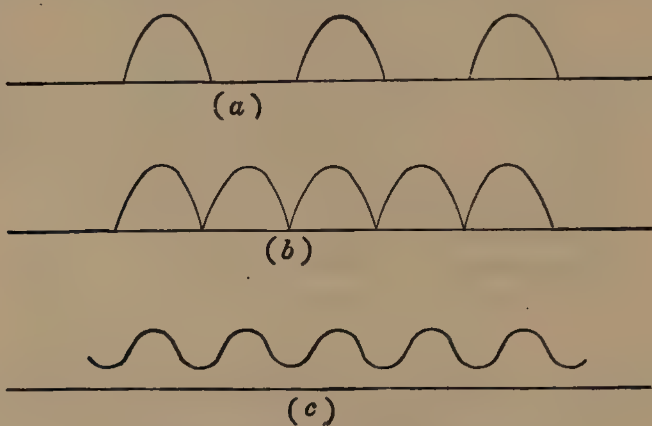


FIG. 236.—Rectified current waves.

is small compared with the circuit voltage. The tubes, however, are fragile, and occasional renewals are necessary.

INSTRUMENT TRANSFORMERS

125. Electrical Measurements at High Voltages.—It is not usually practicable to connect instruments or meters directly to high-voltage circuits. Unless the high-voltage circuit is grounded at the instruments, they may be subjected to high-voltage stresses to ground. This makes it dangerous for anyone to come in contact with the switchboard apparatus. Further, instruments become inaccurate when connected directly to high voltage, because of the electrostatic forces which act on the indicating element. Specially designed instruments may be so constructed that they can be connected directly to high-voltage

circuits, but these instruments are usually expensive and are not suitable for commercial work.

By means of instrument transformers, instruments may be entirely insulated from the high-voltage circuit and yet indicate accurately the current, voltage, power, etc., in the high-voltage circuit. Low-voltage instruments, moreover, having standard current and voltage ranges may be used for all high-voltage circuits, irrespective of the voltage and current ratings of the circuits.

126. Potential Transformers.—Potential transformers do not differ materially from the constant-potential transformers already

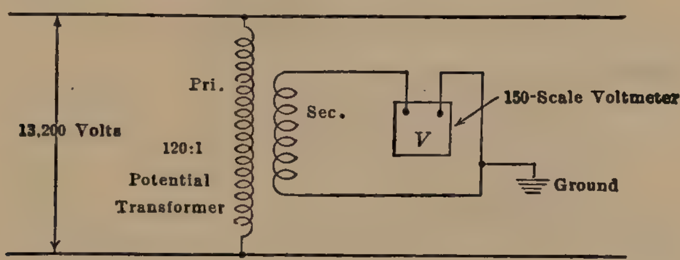


FIG. 237.—Use of potential transformer on 13,200-volt single-phase circuit.

discussed, except that their power rating is small. Below 5,000 volts they are usually air cooled, and above this they are usually oil cooled, the oil being used more for its dielectric qualities than for cooling purposes. As only instruments and sometimes pilot lights are connected to their secondaries, such transformers ordinarily have ratings of from 40 to 200 watts. The low-tension side is almost always wound for 110 volts, and the ratio is then determined by the rating of the high-voltage winding. For example, a 13,200-volt potential transformer would have a ratio of $13{,}200/110 = 120/1$. The ratio of turns may vary a per cent. or so from this value to allow for the transformer impedance drop under load. Figure 237 shows a simple connection for measuring voltage in a 13,200-volt circuit by means of a potential transformer. The secondary should always be grounded at one point to eliminate "static" from the instrument and further to insure safety to the operator. Figure 241 shows a potential transformer used in conjunction with a current transformer for measuring power by means of a wattmeter.

127. Current Transformers.—To avoid connecting alternating-current ammeters and the current coils of other instruments directly in high-voltage lines, current transformers are used. In addition to insulating the instruments from high voltage, they

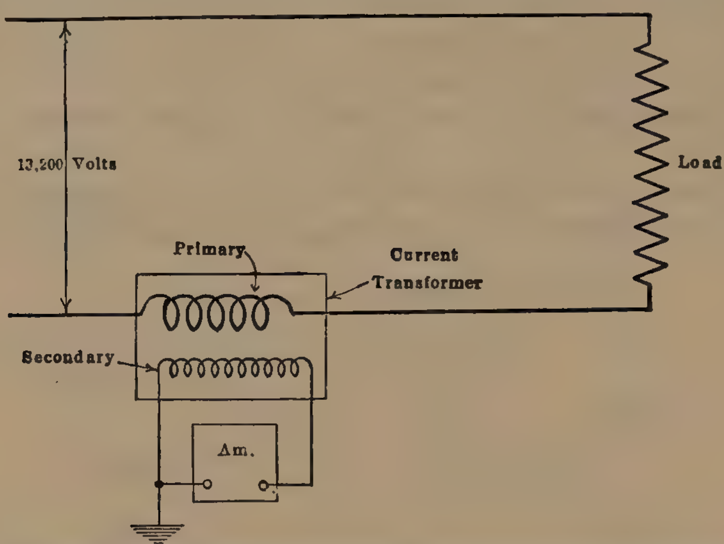


FIG. 238.—Connections for series or current transformer on 13,200-volt circuit.

step down the current in a known ratio. This enables a lower-range ammeter to be used than would ordinarily be required if the instrument were connected directly into the primary line.

The current or series transformer has a primary, usually of few turns, wound on a core and connected in series with the line

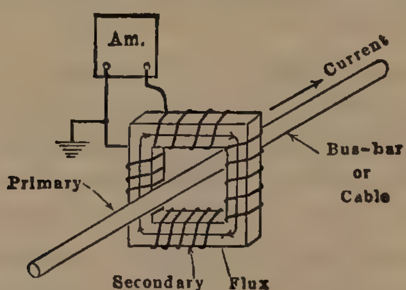
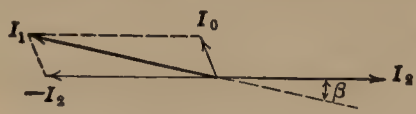


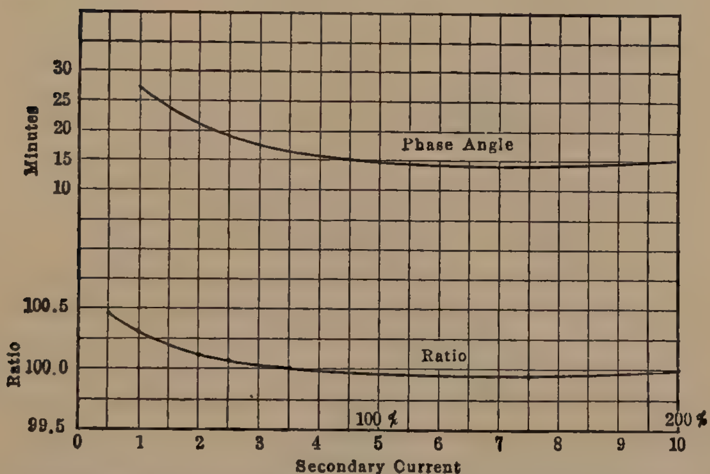
FIG. 239.—Construction of one type of current transformer.

(Fig. 238). When the primary has a large current rating, it may consist of a straight conductor passing through the center of a hollow core, as shown in Fig. 239. The secondary, consisting of several turns, is wound around the laminated core. The ratio

of current transformation is approximately the inverse ratio of turns. For example, if the primary has two turns and the secondary 60 turns, the ratio will be 30/1. The ratio may vary slightly from this value, due to the magnetizing current. In Fig. 240 (a), the primary current I_1 consists of two components, $-I_2$, the component necessary to balance the secondary ampere-turns, and I_0, the magnetizing current. The magnetizing current introduces a slight error in the ratio as well as causing I_2 to depart by



(a) Vector diagram.



(b) Phase angle and ratio curves for typical current transformer.

FIG. 240.—Current-transformer characteristics.

the angle β from the 180° phase relation to I_1. At light loads, the magnetizing current may cause considerable error. Figure 240 (b) shows the variation of phase angle and ratio with load for a typical transformer.

The secondaries of practically all current transformers are rated at 5 amp., regardless of the primary current rating. For example, a 2,000-amp. current transformer has a ratio of 400/1, and a 60-amp. transformer has a ratio of 12/1.

The current transformer differs from the ordinary constant-potential transformer in that its primary current is determined entirely by the load on the system and not by its own secondary

load. If its secondary becomes open circuited, a high voltage will exist across the secondary, because the large ratio of secondary to primary turns causes the transformer to act as a step-up transformer. Also, since the counterampere-turns of the secondary no longer exist, the flux in the core, instead of being due to the *difference* of the primary and secondary ampere-turns, will now be due to the total primary ampere-turns acting alone. This means a very large increase in the flux, causing excessive core loss and heating, as well as a high voltage across the secondary terminals.

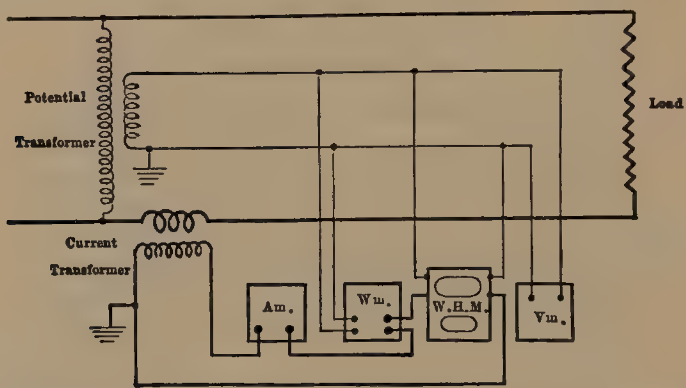


FIG. 241.—Typical connections of instrument transformers and instruments for single-phase measurements.

Therefore, the secondary of a current transformer should not be open-circuited under any circumstances.

Figure 241 shows the method of connecting a typical instrument load, through instrument transformers, to a high-voltage line. The load on the instrument transformers includes an ammeter, a voltmeter, a wattmeter, and a watthour meter. Each secondary is grounded at one point. Correction for ratio of transformation must be applied to all the instrument readings, the wattmeter and watthour meter involving the ratio of both the current and the potential transformers. Usually, in permanent installations, as on switchboards, the instrument scales themselves are so marked as to take into consideration these ratios. The primary power, therefore, may be read directly.

CHAPTER IX

THE INDUCTION MOTOR

128. Principle.—The induction motor is the most widely used type of alternating-current motor. This is due to its ruggedness and simplicity, to the absence of a commutator, and to the fact that its operating characteristics are well adapted to constant-speed work.

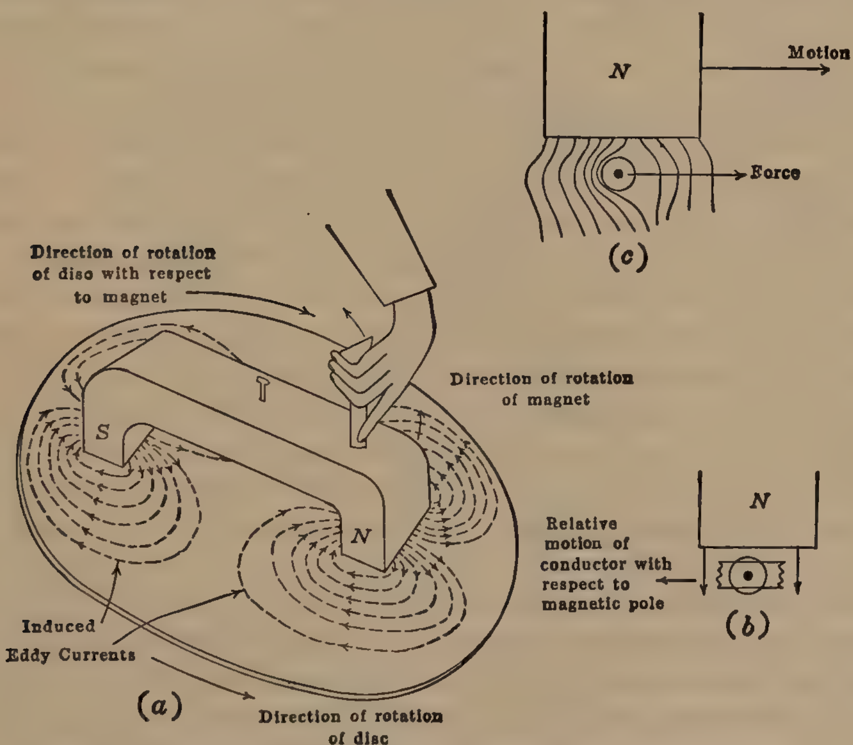


FIG. 242.—Rotation of metal disc produced by rotating magnet.

The principle of the motor may be illustrated as follows: A metal disc (Fig. 242 (*a*)) is free to turn upon a vertical axis. The disc may be of any conducting material, such as iron, copper, or aluminum. A magnet, free to rotate on the same axis as the disc, is placed above the disc, and its ends are bent down so that

its magnetic flux cuts through the disc. When this magnet is rotated, the magnetic lines cut the disc and induce currents in it, as shown in the figure. As these currents find themselves in a magnetic field, they tend to move across this field, just as the currents in the conductors of a direct-current motor tend to move across its magnetic field. By Lenz's law, the direction of the force developed between these currents in the disc and the magnetic field producing them will be such that the disc tends to follow the magnet, as shown in the figure.

To illustrate this more in detail, consider Fig. 242 (*a*), (*b*), and (*c*). In (*a*), the north pole of the rotating magnet is shown as moving in a counterclockwise direction. The conductor beneath the magnet also moves in a counterclockwise direction but more slowly than the magnet. The *relative motion* between the magnet and the conductor is the same, therefore, as if the magnet were *stationary* and the conductor moved in the clockwise direction. This relative motion of the magnet and the conductor is illustrated in Fig. 242 (*b*), where the north pole is shown as being stationary, and the conductor is moving from right to left. Applying Fleming's right-hand rule (see Vol. I, p. 264), the direction of the induced current is toward the observer. The lines of force about the conductor, due to its own current, are, therefore, counterclockwise, and the resultant field is found by combining the conductor field and the field produced by the magnet. The appearance of this resultant field is shown in Fig. 242 (*c*) (also, see Vol. I, p. 365). As the magnetic field is increased in intensity to the left of the conductor and reduced in intensity to the right of the conductor, there is a force developed which urges this conductor from *left* to *right*. That is, the conductor tends to follow the magnet. Actually, the magnet rotates in a counterclockwise direction. The disc, therefore, rotates in the same direction but at a speed less than that of the magnet.

The disc can never attain the speed of the magnet, for, were it to attain this speed, there would be no relative motion of the disc and the magnet and, therefore, no cutting of the disc by the magnetic flux. The disc current would then become zero, and no torque would be developed, a situation which would result in the disc speed's becoming less than that of the magnet. Because the disc cannot attain the speed of the magnet, there must always

exist a *difference* of speed between the two. This difference of speed is called the *revolutions slip.*

It is to be noted that the currents in the disc or armature of this type of motor are *induced* therein, rather than being conducted into the armature, as in the ordinary direct-current motor.

A cylinder may be used instead of the disc, as shown in Fig. 243. In the figure are shown four poles, the magnetic lines of which cut the cylinder. If the frame carrying these poles be revolved

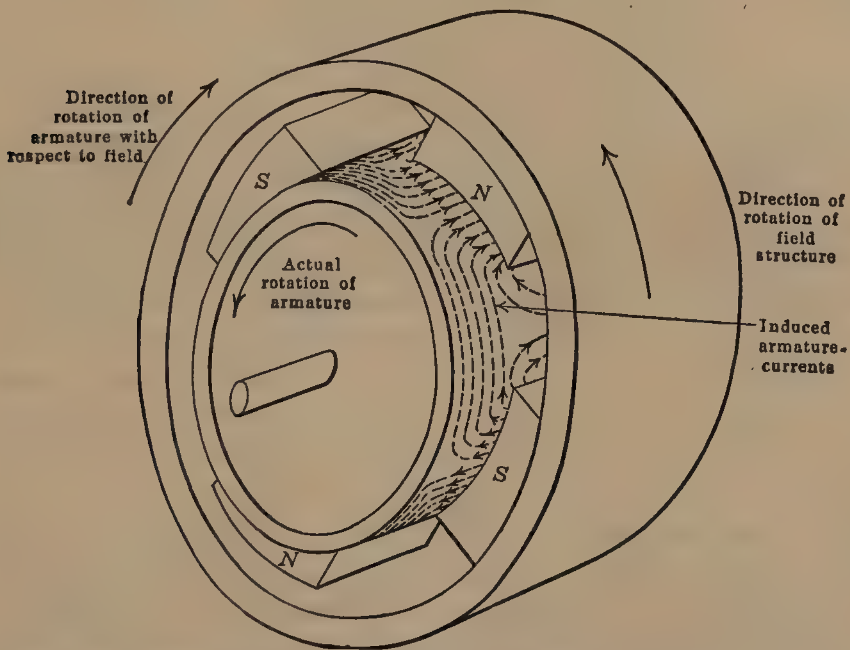


Fig. 243.—Rotation of conducting cyclinder due to induced currents.

by mechanical means, the currents induced in the cylinder will cause the cylinder to rotate in the same direction as that of the rotating frame. This cylinder is more representative of the commercial induction motor than is the disc, although both operate on the same principle.

129. Alternating-current Rotating Field.—The rotating fields described in the previous paragraph were produced by rotating the magnetic poles mechanically. This is practically the same as the rotating poles of an alternator field. Rotating magnetic fields may, however, be produced by sending polyphase currents through polyphase windings, such as alternator windings.

Such rotating fields are produced entirely by electrical means, there being no mechanical rotation of the pole pieces themselves.

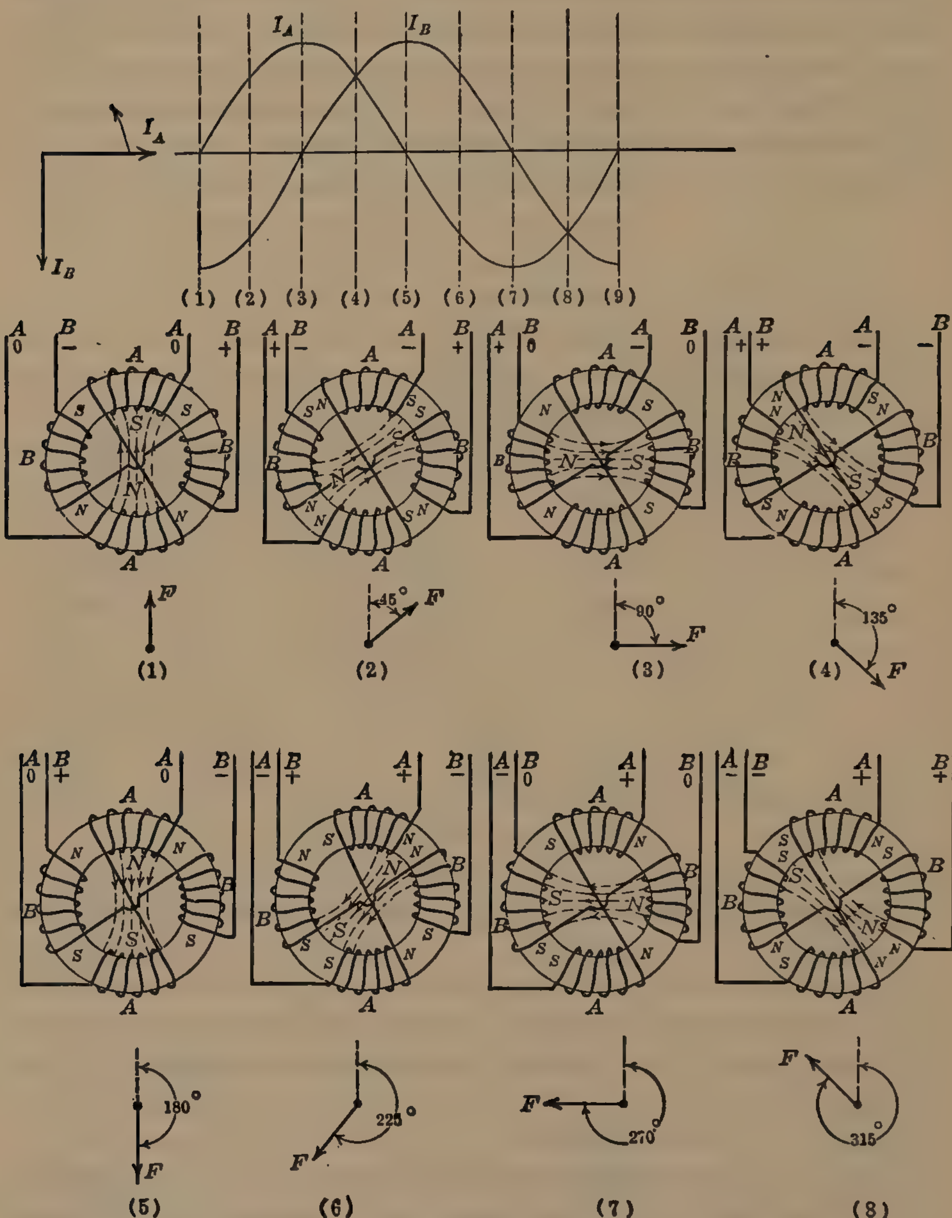


FIG. 244.—Rotating field produced by two-phase currents in gramme-ring winding.

The simplest type of rotating field is that produced by the gramme-ring winding illustrated in Fig. 244. A gramme ring,

wound for two-phase currents, has two separate windings, one for each phase. Each winding consists of two sections located diametrically opposite each other, and each section occupies approximately one-fourth the winding space of the ring. The two windings are called the "*A*-phase" and the "*B*-phase," respectively. Care must be taken to connect the two sections of each winding correctly, the correct method being shown in Fig. 244.

Curves I_A and I_B show the variation with time of the currents in phases *A* and *B*, respectively. As these are two-phase currents, they differ in time phase by 90° or ¼ cycle.

At the instant marked (1), the current in phase *A* is zero, and that in *B* is negative maximum. With the method of connecting the windings, and the direction of the currents as shown, two *S*-poles are formed on the upper ends of the *B*-windings, and two *N*-poles on the lower ends. These four poles combine into two poles, a single *S*-pole and a single *N*-pole, each of these last being twice the magnitude of the individual poles which combined to form them. The resultant field is vertical and is directed upward, as indicated by the arrow *F* beneath diagram (1). In (2), the current in *B* is still negative but of lesser magnitude than in (1). The current in *A* has increased positively until its magnitude is equal to that of *B*. Two *S*-poles and two *N*-poles again combine to form a single *S*-pole and a single *N*-pole, each of double the magnitude of the individual poles forming them. The direction of the resulting field is 45° clockwise from its position in (1). It is to be noted that, while the two currents are passing through 45 electrical time-degrees, the resulting field in the gramme ring advances 45 space-degrees. Diagrams (3), (4), (5), (6), (7), and (8) show at different instants the positions of the gramme-ring field resulting from the combined magnetic effects of phases *A* and *B*. The diagram for (9) would be identical with that for (1). The rotating magnetic field has passed through 360 space-degrees, while the two-phase currents have gone through 360 electrical time-degrees or one cycle. This constitutes a two-pole rotating field, and its speed in revolutions per second is the same as the frequency, or the cycles per second, of the currents. For example, if the currents had a frequency of 60 cycles per second, the field would make 60 r.p.s., or 3,600 r.p.m.

The gramme-ring winding need not consist necessarily of the two separate windings shown in Fig. 244, but may be a mesh-connected winding, as shown in Fig. 245. This is virtually a continuous winding tapped at four equidistant points. Two of the diametrically opposite taps are connected to phase A, and the other two are connected to phase B.

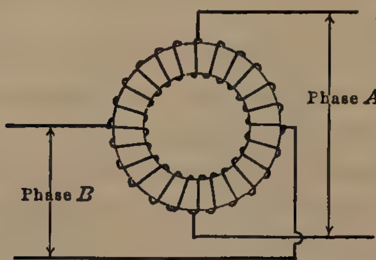


FIG. 245.—Two-pole, mesh-connected, gramme-ring winding for two-phase circuit.

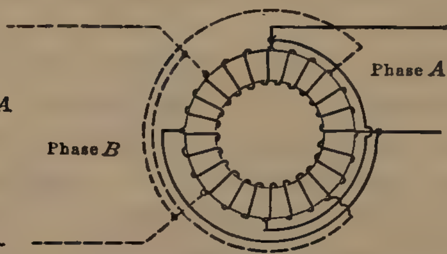


FIG. 246.—Four-pole, mesh-connected, gramme-ring winding for two-phase circuit.

Figure 246 shows a four-pole, mesh-connected winding. This is similar to that of Fig. 245 but is tapped at eight equidistant points. Two diametrically opposite taps are connected to one line of phase A, and the two taps at right angles to these are connected to the other line of phase A. Another similar set of taps, displaced 45° from the A-taps, connect in like manner to phase B. In such a winding, the rotating field completes 1 revolution during 2 complete cycles of the current; therefore, its angular speed is one-half that of the field in the two-pole machine.

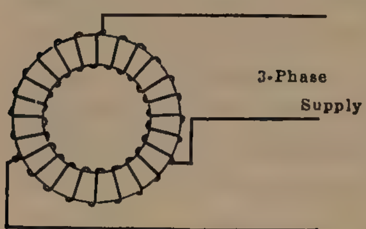


FIG. 247.—Delta-connected, two-pole, gramme-ring winding for three-phase circuit.

Figure 247 shows the manner of connecting a three-phase circuit to a gramme-ring mesh- or delta-connected winding. Each of three equidistant taps is connected to one of the three lines of the three-phase supply. This winding produces a two-pole field whose speed in r.p.s. is the same as the frequency of the supply.

It is to be noted that, in each of the foregoing windings, the angle between the various windings, expressed in electrical space-degrees, is the same as the time-angles between the respective currents in the

windings. (In a two-pole machine, 1 electrical space-degree equals 1 space-degree; in a four-pole machine, 2 electrical space-degrees equal 1 space-degree, etc.)

130. Rotating Fields in Drum-wound Machines.—The commercial polyphase induction motor consists of a fixed member called the *stator*, carrying a polyphase drum winding (like that of an alternator), and a rotating member called the armature or *rotor*. As the stator usually receives the power from the line, it is called the *primary*, and as induced currents flow in the rotor, the rotor is called the *secondary*, just as in the transformer. The motor will operate, however, if power is supplied to the rotor, and the stator acts as secondary. Stator windings are the same as alternator windings for the same number of phases and poles. In fact, the ordinary alternator winding is entirely satisfactory for an induction-motor winding.

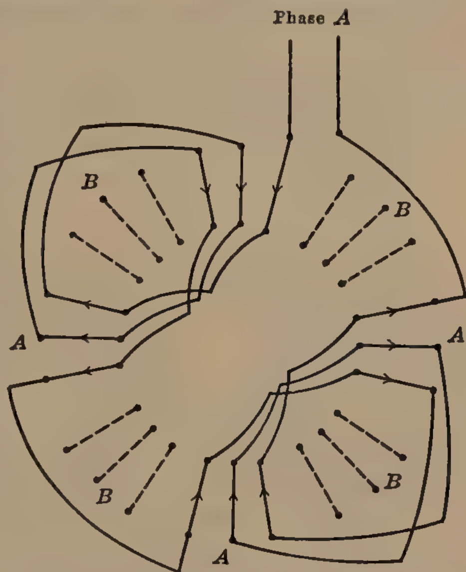


FIG. 248.—Single-layer, four-pole, two-phase induction-motor winding, lap connected.

Figure 248 shows a single-layer, drum winding for the two-phase, four-pole, induction motor stator shown in Fig. 249. In this machine, there are six slots per pole or three slots per pole per phase. Only the one phase, A, is shown connected.

Figure 249 shows a section of this two-phase induction motor, taken perpendicular to its shaft. It is wound with the two-phase, four-pole winding, shown in Fig. 248. The time variation of the two-phase currents I_A and I_B is also shown.

At instant (1), the current I_A is zero, and I_B is negative maximum. By applying the corkscrew rule for determining the relation of magnetic flux to the current producing it, four poles are formed in the stator, two N-poles in the vertical plane and two S-poles in the horizontal plane. At instant (2), the current I_A is positive, and I_B is negative and of the same polarity as in

(1). The resulting N- and S-poles are again determined by the corkscrew rule, and it will be observed that these two poles have advanced 22.5 space degrees in a clockwise direction, whereas the currents have undergone a change corresponding to 45 electrical

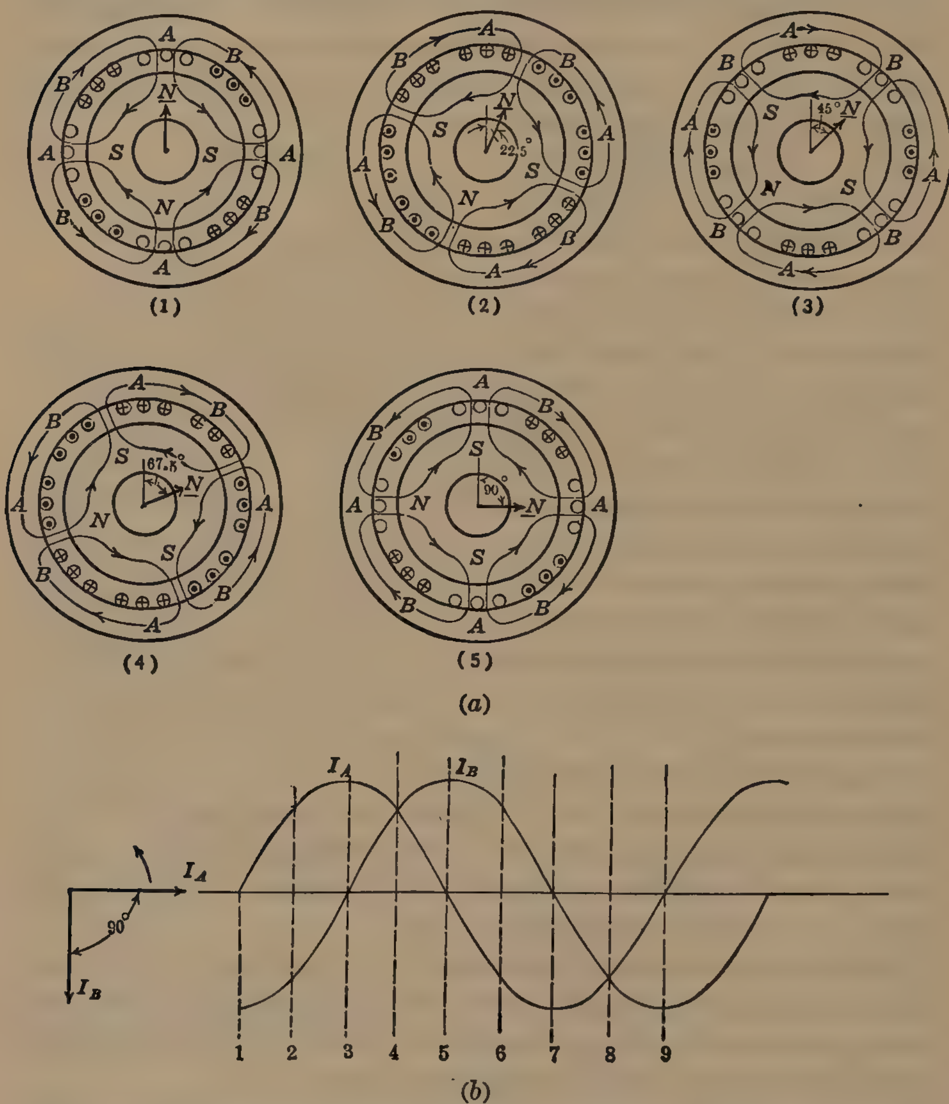


FIG. 249.—Production of rotating field by two-phase currents in four-pole winding.

time-degrees. Positions (3), (4), and (5) are taken at time degrees 90, 135, and 180 of the two currents. Between points (1) and (5), the rotating field has advanced only 90 space-degrees, whereas the currents have passed through 180 time-degrees.

Therefore, the speed of this rotating field in revolutions per second is one-half the frequency of the supply in cycles per second.

The N- and S-poles which are produced by the stator rotate in the air-gap and cut the rotor or armature conductors, inducing currents in them. These currents, reacting with these stator poles, produce rotation of the armature, just as, in Fig. 242, the induced currents in the disc and the flux producing them react and cause the disc to rotate.

Figure 250 shows a single-layer, three-phase, four-pole lap winding adapted to a machine having nine slots per pole or three slots per pole per phase. This winding is very similar to that of Fig. 248 and is used in the induction motor shown in Fig. 251. For simplicity, but one phase, A, is shown connected, the other two phases, B and C, being connected in a manner similar to A.

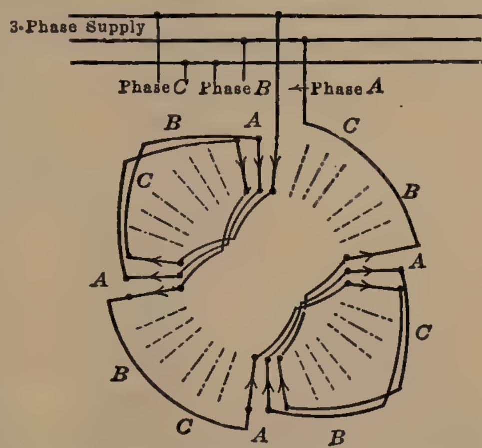


Fig. 250.—Single-layer, three-phase, four-pole, induction-motor winding, lap connected.

Figure 251 shows four successive positions of the rotating field, for corresponding values of the polyphase currents in the stator of this three-phase induction motor. In (1), the current I_A is zero, so that I_B and I_C are opposite and equal. The position of the field is shown at this instant. In (2), the currents I_A and I_C are but half their maximum positive values, and their positions on the stator are such that their phase belts are on each side of the B-belt in which the current is a maximum. The field, therefore, is symmetrical at this position.

It will be noted, also, that the time-angle between successive values of current in 1–2–3 is 30 electrical degrees, whereas the field advances but 15 space-degrees between (1) and (2) and, also, between (2) and (3). Between positions (1) and (4), the currents have advanced 90 electrical time-degrees, but the rotating field has advanced only 45 space-degrees. That is, the advance of the rotating field in space-degrees is equal to one-half the advance of the currents in electrical time-degrees. The speed of such a

field in r.p.s. is equal, therefore, to one-half the circuit frequency in cycles per second.

In general, it may be stated that in order to produce a two-pole rotating field, the angular space-degrees between the phase belts of the winding must be the same as the electrical time-degrees between their respective currents. If the machine has p poles, the angular space-degrees between phase belts is $2/p$ times the electrical time-degrees between their respective currents. For example, in a six-pole, three-phase machine, the successive phase belts start 40° from each other, that is, $\frac{2}{6} \times 120°$ or 40°. In the

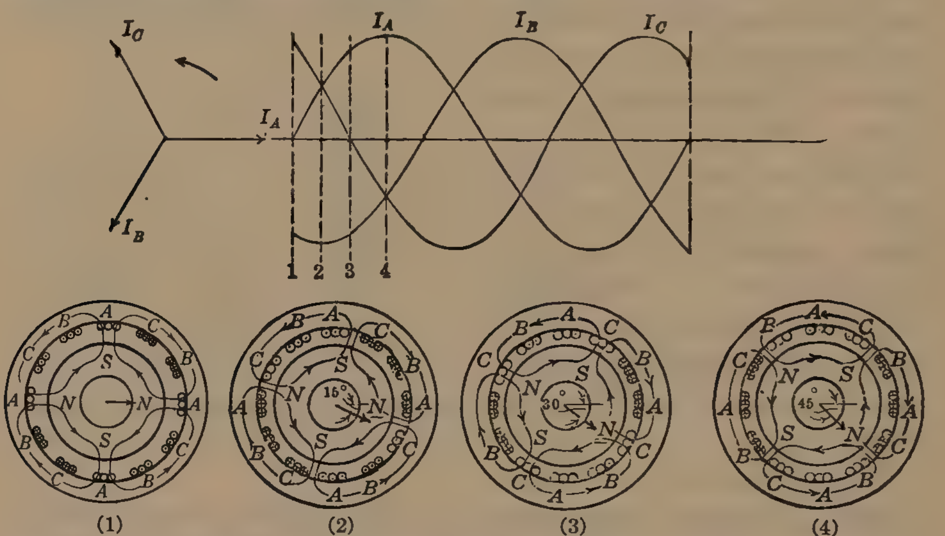


FIG. 251.—Rotating field produced by three-phase currents in four-pole, induction-motor winding.

ordinary drum windings, however (see Chap. VI, pp. 139 and 140, Figs. 131 and 132), the coil sides lap back so that, in the above example, the *reversed* phase belts would be 20° apart. The currents in such adjacent belts are 60° apart, as shown in Fig. 251.

To reverse the direction of rotation of a two-phase, rotating field, reverse the leads of *either* phase; to reverse the direction of rotation of a three-phase rotating field, interchange any two leads.

131. Synchronous Speed; Slip.—It has just been shown that the angular speed of an alternating-current rotating field depends upon two factors—the frequency of the current and the number of poles for which the machine is wound. The relation among speed, frequency, and poles is given by the following equation:

$$N = \frac{f \times 120}{P} \tag{107}$$

where N is the speed of the field in r.p.m.; f, the frequency in cycles per second; and P, the number of poles (see Eq. (2), p. 7). This speed N of the rotating field is called the *synchronous speed* of the motor. The common synchronous speeds for commercial motors at 25 and at 60 cycles per second are as follows:

Poles	R.p.m. $= N$	
	$f = 25$	$f = 60$
2	1,500	3,600
4	750	1,800
6	500	1,200
8	375	900
12	250	600

Slip.—If an armature whose conductors form closed circuits be placed in a rotating field, it will develop torque because of the induced currents, acting in conjunction with the rotating magnetic field.

As has already been pointed out, the armature can never attain the speed of the rotating field, for if it did, the cutting of conductors by flux would cease, there would be no rotor current, and, therefore, no torque.

The difference between the speed of the rotating field and that of the rotor is called the *revolutions slip* of the motor. For example, if the rotor of a four-pole, 60-cycle motor has a speed of 1,730 r.p.m., its revolutions slip is 1,800 − 1,730 = 70 r.p.m., where 1,800 r.p.m. is its synchronous speed.

It is more convenient to express the slip as a fraction of the synchronous speed. Denote the speed of the rotor by N_2 and the synchronous speed by N. Then the slip

$$s = \frac{N - N_2}{N}. \qquad (108)$$

For example, the slip in the above motor is

$$s = \frac{1{,}800 - 1{,}730}{1{,}800} = \frac{70}{1{,}800} = 0.039 \text{ or } 3.9 \text{ per cent.}$$

The rotor speed is

$$N_2 = N(1 - s) \text{ (from Eq. (108))} \qquad (109)$$

The full-load slip in commercial motors varies from 1 to 10 per cent., depending upon the size and the type of motor.

132. Rotor Frequency and Induced E.m.f.—If the rotor of a two-pole, 60-cycle motor is at standstill and voltage is applied to the stator, each rotor conductor will be cut by a north pole sixty times per second and by a south pole sixty times per second, as this is the speed of the rotating field. If the stator be wound for four poles, the speed of the rotating field is halved, but each conductor is then cut by two north and two south poles per revolution of the field and, therefore, by 60 north and 60 south poles per second, the same as in the two-pole motor. Consequently, in each case, the frequency of the rotor currents at standstill ($s = 1.0$) will be the same as the stator frequency. This holds true for any number of poles. At standstill, the motor is a simple static transformer, the stator being the primary, and the rotor being the secondary.

If the rotor of the above 60-cycle motor revolves at half synchronous-speed in the direction of the rotating field ($s = 0.5$), the rotor conductors are cut by just one-half as many north and south poles per second as when standing still, and the frequency of the rotor currents is, therefore, 30 cycles per second.

By taking other rotor speeds, it can be shown that the rotor frequency

$$f_2 = sf \tag{110}$$

where f_2 is the rotor frequency, s the slip, and f the stator frequency. *The rotor frequency is equal to the stator frequency multiplied by the slip.*

Example.—What is the frequency of the currents in the rotor of a 60-cycle, six-pole induction motor, if the rotor speed is 1,164 r.p.m.?

The synchronous speed

$$N = \frac{60 \times 120}{6} = 1{,}200 \text{ r.p.m.} \quad \text{(Eq. (107), p. 274)}$$

The slip

$$s = \frac{1{,}200 - 1{,}164}{1{,}200} = 0.03$$

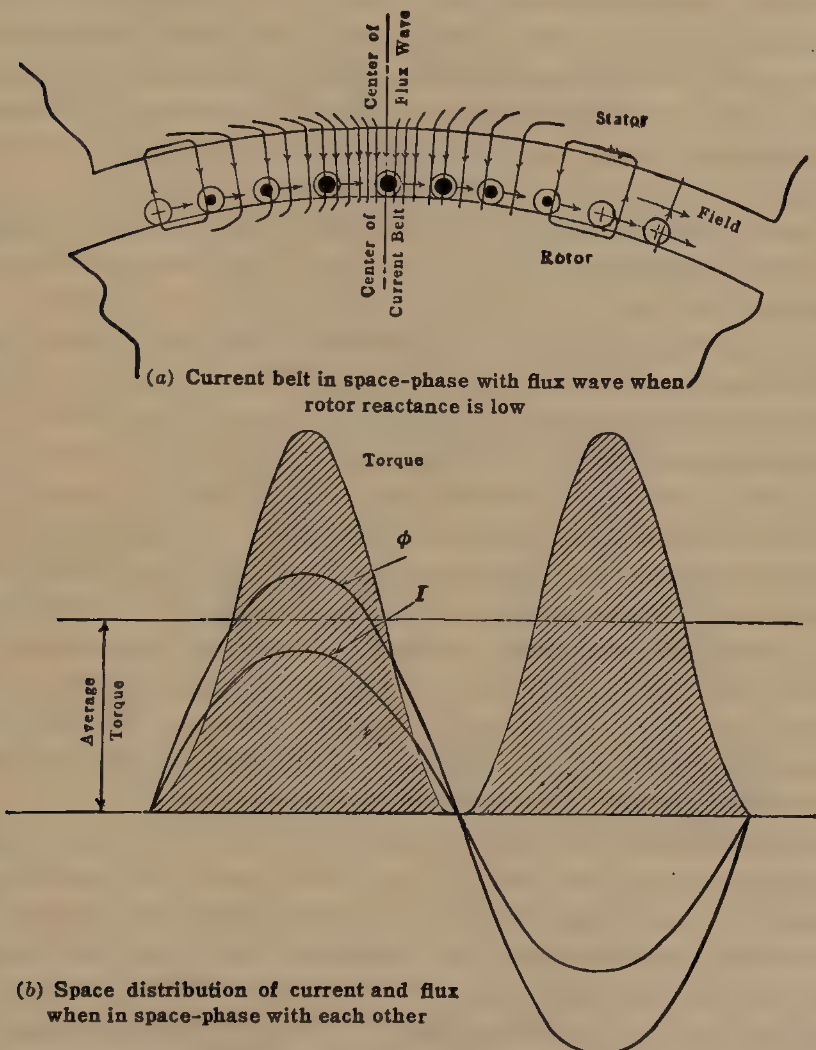
$$f_2 = 0.03 \times 60 = 1.8 \text{ cycles per second.} \quad \textit{Ans.}$$

The rotor frequency has a very important bearing on the operating characteristics of the induction motor.

The induction motor can be used as a frequency changer, provided the rotor is driven mechanically at the proper speed.

Current is taken from the rotor, or secondary, through slip-rings. Under these conditions, some of the power is supplied electrically and some mechanically.

133. Alternating-current Torque.—It has already been pointed out, in connection with the direct-current motor, that the torque



(*a*) Current belt in space-phase with flux wave when rotor reactance is low

(*b*) Space distribution of current and flux when in space-phase with each other

FIG. 252.—Alternating-current torque when current and flux are in space phase.

is proportional to the current and to the density of the magnetic field in which the current finds itself. This same law holds for alternating-current motors, provided the instantaneous values of current and flux are considered.

Figure 252 (*a*) shows the space distribution of flux from one north pole as it glides along the air-gap of an induction motor. This flux is distributed sinusoidally along the air-gap, as is shown by the flux-distribution curve ϕ (Fig. 252 (*b*)).

If the slip be small, the reactance of the rotor conductors is low, because $f_2 = sf$ and $x_2' = 2\pi f_2 L_2$, where f is the stator frequency, x_2' is the rotor reactance at slip s, and L_2 is the rotor inductance. Because of the rotor reactance, the rotor current lags the induced e.m.f. of the rotor by an angle α. At low values of slip, this angle α is very small, since $\tan \alpha = 2\pi f s L_2/R_2$, where R_2 is the rotor resistance.

The induced e.m.f. in any single conductor, l cm. in length, in a field having a density of B gausses, the conductor moving at a velocity of v cm. per second with respect to the field is $e = Blv\ 10^{-8}$ volts, the flux, the conductor and the velocity being mutually perpendicular (see Vol. I, p. 263, Eq. (110)). When, therefore, a conductor is cutting flux at a uniform velocity, the flux being sinusoidally or otherwise distributed in space, the e.m.f. in the conductor is zero when it is moving in a region where B, the flux density, is zero; the e.m.f. is a maximum when the conductor is moving in a region where B, the flux density, is a maximum. As the e.m.f. e is proportional to B at every instant, if v is constant, e will be a maximum when B is a maximum, etc. It may be said, therefore, that e, the e.m.f. per conductor, is in space phase with the flux. It further follows that the wave shape of the e.m.f. *in a single conductor* is the same as the shape of the space-distribution curve of the flux.

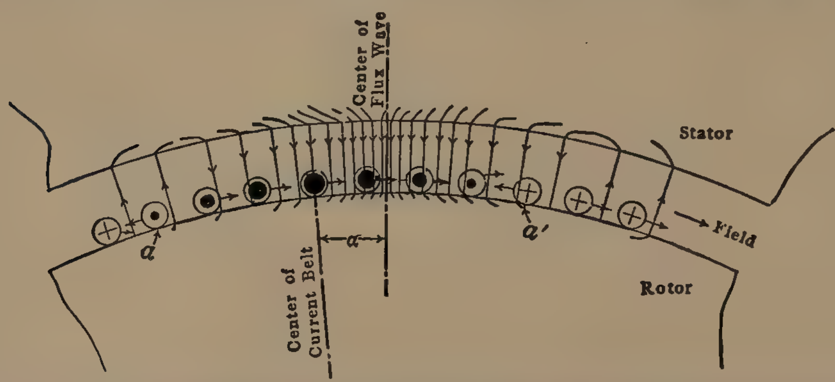
At small values of slip, the angle α, between the induced e.m.f. in each conductor and the current in the conductor, is small, and, therefore, the current in each of the conductors (Fig. 252 (*a*)) is practically in phase with its induced e.m.f. As the induced e.m.f. is a maximum when the conductor is in the field of greatest flux density, the current will be a maximum at practically the same instant. The current is then in time phase with the e.m.f. and, hence, in space phase with the flux. Under these conditions, the current in the particular conductor which is under the center of the pole (Fig. 252 (*a*)) is a maximum, and that in the other conductors is less, decreasing sinusoidally as indicated.

Figure 252 (*b*) shows both the flux distribution in the gap and the current distribution in the conductors of Fig. 252 (*a*), the current in each conductor being proportional to the flux density of that part of the field in which the conductor finds itself. (For simplicity, a smooth current-distribution curve is shown. This would hold true only with a uniform metal sheet about the rotor). The force acting on each conductor is proportional to its current and to the flux density of that part of the field in which the conductor finds itself (see Vol. I, p. 366, Eq. (124)). The force due to each conductor (Fig. 252 (*a*)) is indicated in direction by an arrow attached to that conductor. The torque curve is obtained by taking the product of the current and flux at each point, multiplied by a constant. The torque curve for the conductor belt shown in Fig. 252 (*a*) is given in Fig. 252 (*b*). This curve is obtained by multiplying the current at each point by the flux density at that point. That is, the ordinate of the torque curve, at any point (Fig. 252 (*b*)), is equal to the product of the ordinates of the flux and the current curves at that point, multiplied by a constant. It will be noted that this torque curve is of double frequency, that it is always positive, reaches zero twice every cycle, and is similar to the power curve of p. 23 (Fig. 20).

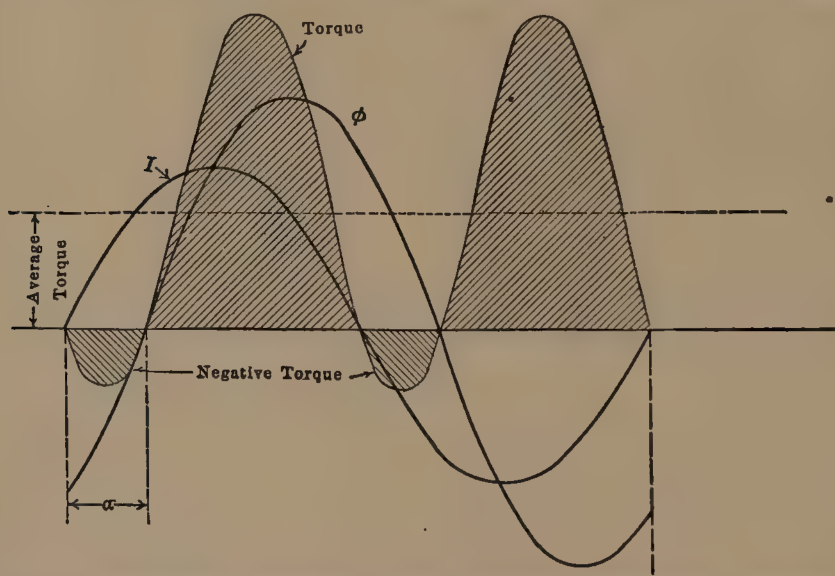
As the value of the slip increases, the reactance of the rotor increases, the reactance being proportional to the rotor frequency and, hence, to the slip, and the angle α by which the current lags its induced e.m.f. increases, since $\tan \alpha = 2\pi f s L_2/R_2$. The current in any conductor will not reach its maximum value until α time-degrees after the induced e.m.f. has reached its maximum value. In the interval between the time when the induced e.m.f. reaches its maximum and the current reaches its maximum, the maximum point of the flux wave has moved along by the conductor by α electrical space-degrees, as shown in Fig. 253 (*a*). As a result, some conductors, as *a* (Fig. 253 (*a*)), find themselves in a reversed field and so exert a torque opposite to that of the other conductors in that belt. Also, conductor *a′* in an adjacent current belt exerts a torque opposite to that of the other conductors in its belt. The torques exerted by conductors *a* and *a′* produce the negative loops of the torque curve (Fig. 253 (*b*)). The torque under these conditions is less than it is in Fig. 252

(b), even with the same value of current and flux. This is due to the negative values of torque, shown in Fig. 253 (b).

Therefore, in order to have maximum torque with fixed values of current and flux, the rotor currents must be in space phase with the flux.



(a) Current belt α space-degrees out of phase with flux wave, when rotor reactance is high.



(b) Space distribution of current and flux, differing in space-phase by angle α.

FIG. 253.—Relation among flux, current, and torque when current belt is not in space phase with flux wave.

The torque

$$T = T_{max} \cos \alpha \qquad (111)$$

where T_{max} is the torque when the current I and the flux ϕ are in space phase, and α is the space-angle between the current I and the flux ϕ.

134. Squirrel-cage Motor.—The squirrel-cage motor is the simplest type of induction motor and the most generally used. The core of the rotor or armature (Fig. 254), like that of the direct-current armature, is usually built up of slotted steel punchings. The winding consists of copper bars placed in slots. These bars have their ends connected together by conducting rings called *end rings*. The bars are usually bolted to the end rings and then welded or brazed. Formerly, solder was used, but considerable trouble was encountered by its melting and being

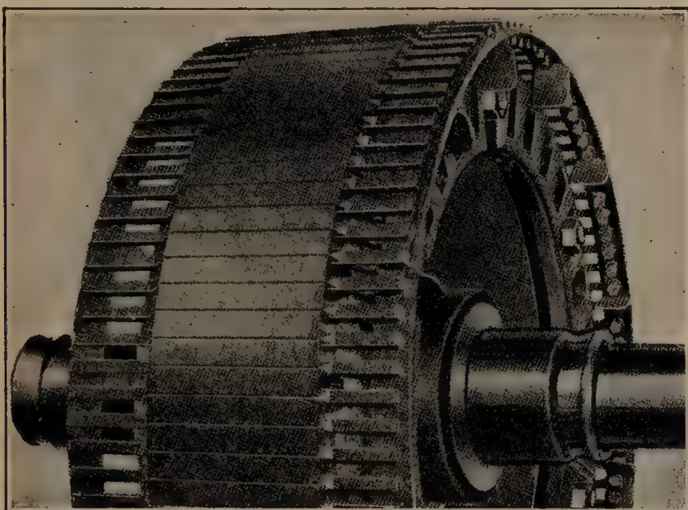


FIG. 254.—Squirrel-cage rotor.

thrown out of the joint by centrifugal action. Another method is to place the rotor in a mould and cast the ends of the bars in a ring of cast copper. The General Electric Company manufactures a rotor in which an aluminum grid is cast integral with the end rings. The methods in which the end rings are cast integral with the bars of the winding are the best from the operating point of view, as the rotor conductors have no opportunity to work loose.

The stator slots in nearly all induction motors of small size (Fig. 255) are of the semiclosed type, like those shown in the rotor of Fig. 256. If open slots are used, magnetic wedges are employed so as to give the effect of semiclosed slots. In the larger sizes of motor, open slots are often used for the stator, as shown in Fig. 256, because of the expense and difficulty of

placing the winding in semiclosed slots, and also because the necessity for semiclosed slots is usually less in the larger motors.

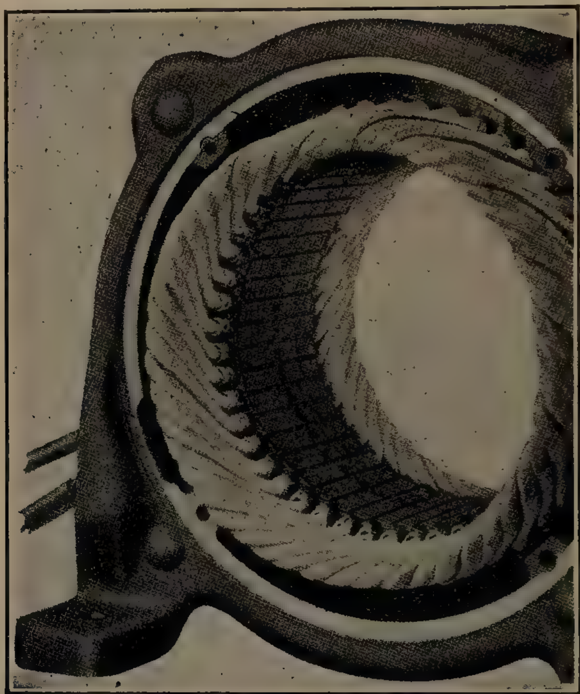


Fig. 255.—Overhung-slot stator, showing how ends of coils are taped into slots.

That is, in the large, higher-speed motors, the pole pitch is large, and, therefore, the number of ampere-conductors per pole is large.

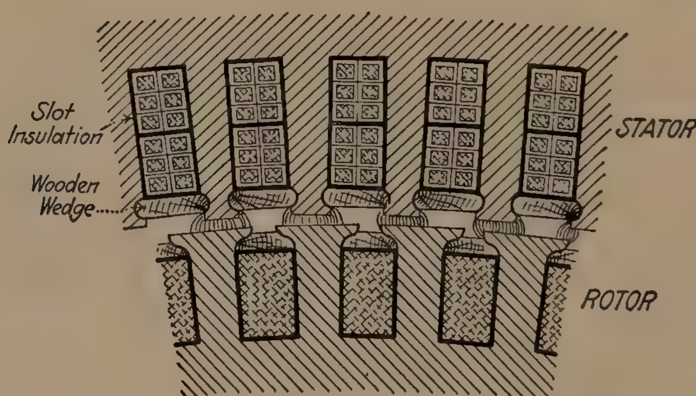


Fig. 256.—Stator and rotor slots of squirrel-cage induction motor.

Consequently, the desired flux density in the gap may be readily obtained without an excessive magnetizing current, even if open slots are used.

In practically all motors of the squirrel-cage type, the slots of the *rotor* are semiclosed, as there is little difficulty encountered in placing the rotor bars in this type of slot.

The advantage of the semiclosed slot is that the effective sectional area of the air-gap is increased, and the magnetizing current is, therefore, reduced. Semiclosed slots reduce the pulsations of flux in the individual teeth and, therefore, reduce the tooth losses, which otherwise might be serious. On the other hand, the semiclosed slot gives a much higher slot inductance than the open slot, and this inductance in the stator and in the rotor lowers the power factor and decreases the starting and the breakdown torques of the motor.

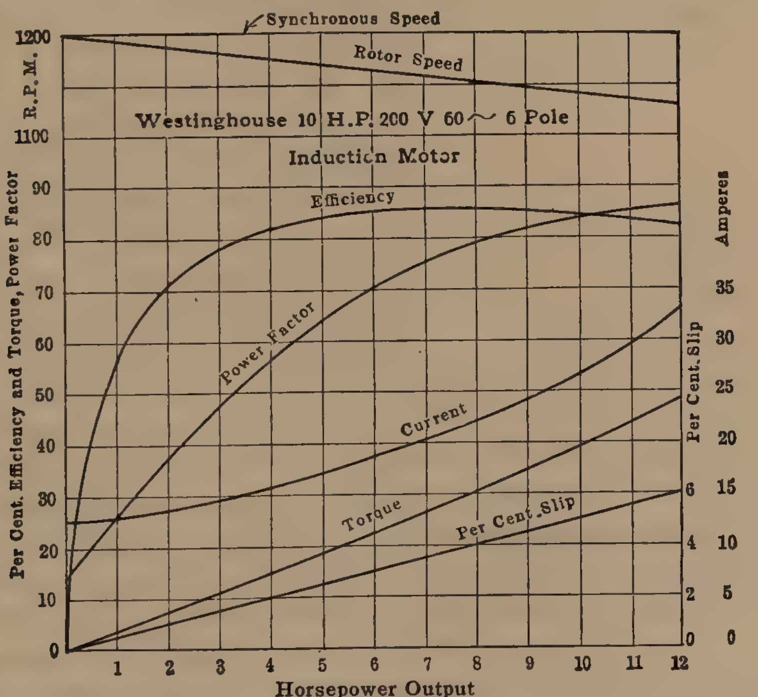


FIG. 257.—Operating characteristics of squirrel-cage induction motor.

135. Operating Characteristics of Squirrel-cage Motor.—The squirrel-cage motor, like the direct-current shunt motor, operates at substantially constant speed. As the rotor cannot reach the speed of the rotating magnetic field, it must at all times operate with a certain amount of slip. At no load, the slip is very small. As load is applied to the rotor, more rotor current is required to develop the necessary torque in order to carry the

increased load. Consequently, the rotating magnetic field must cut the rotor conductors at an increased rate, in order to produce the necessary increase of currrent. The slip of the rotor must, accordingly, increase, so that the rotor speed drops. *The ratio of the slip to the total power delivered to the rotor is proportional to the I^2R loss in the rotor.* As the resistance of the squirrel cage is very low, the I^2R loss is low, and, therefore, the slip for ordinary loads is small. In large motors—50 hp. or greater—the slip is of the order of 1 to 2 per cent. at full load. In the smaller sizes of motor, the slip may be as high as 8 to 10 per cent. at full load.

Figure 257 shows the ordinary characteristic curves of a 10-hp. squirrel-cage motor. It will be noted that the torque, speed, and efficiency curves are very similar to those of a shunt motor. The power factor increases with the load for the following reason:

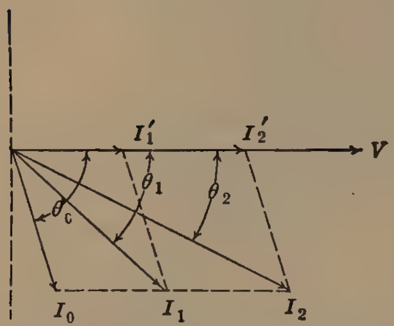


FIG. 258.—Increase of power factor with increase of load.

At no load, the motor takes a current I_0 (Fig. 258). I_0 is mostly magnetizing current, although there is a small energy component necessary to supply the no-load losses. The power factor at no load is $\cos \theta_0$, the value of which may be as low as 0.10 to 0.15. The back electromotive force of the motor remains nearly constant from no load to full load. The flux, therefore, must remain substantially constant, just as it does in the transformer, so that the magnetizing current changes but slightly from no load to full load. As load is applied to the motor, an energy current I_1' is required to carry the load. This current, when combined with I_0, gives the total current I_1 at this load, and the resulting power factor is $\cos \theta_1$. As the load increases, an energy current I'_2 is required. The total current then becomes I_2, and the corresponding power factor becomes $\cos \theta_2$. It will be observed that the power-factor angle decreases, and, therefore, the power factor increases as the load on the motor increases. The increased reactance drops in the stator and in the rotor with increase of load tend to oppose this increase of

power factor and, when the load exceeds a certain value, may even bring about a decrease of power factor. (See circle diagram, Fig. 274, p. 309).

As the power factor increases, a smaller increase of current is required for a given increase of load than would be necessary if the power factor were constant. The current, therefore, increases more slowly than the load, as shown in Fig. 257. At first, the efficiency increases rapidly and reaches a maximum value for the same reason that it does in other electrical apparatus. At all loads, there are certain fixed losses, such as core loss, friction, and windage. In addition, there are the load losses (I^2R)

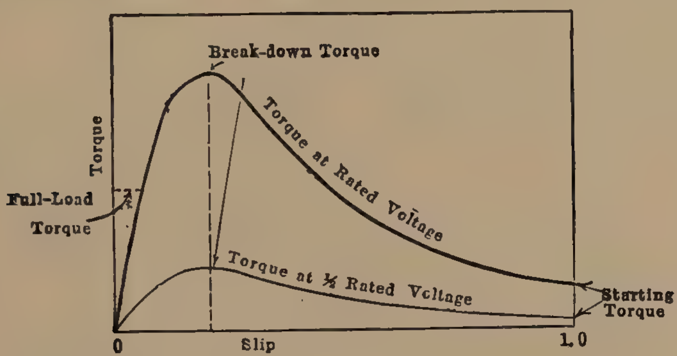


FIG. 259.—Slip-torque curves for squirrel-cage motor.

which increase nearly as the square of the load. At light loads, therefore, the efficiency is low, because the fixed losses are large as compared with the input. As the load increases, the efficiency increases to a maximum, the fixed and variable losses being equal at this point. Beyond this point, the I^2R losses become relatively large, causing the efficiency to decrease.

One disadvantage of the squirrel-cage motor lies in the fact that it takes a very large current at low power factor on starting, and in spite of this large current, it develops but little torque. When the motor is at standstill, the squirrel cage acts as the short-circuited secondary of a transformer, causing the motor to take an excessive current on starting, if full voltage is applied.

Figure 259 shows the variation of torque with slip for two different values of line voltage. It will be noted that for small values of slip up to and beyond full load, which is the ordinary

range of operation, the torque is substantially proportional to the slip. At higher values of slip, however, the torque curve bends over and finally reaches a maximum. This maximum is called the *breakdown torque.* Beyond this maximum point, the torque decreases as the slip increases. For most types of load, this is a point of *instability,* as an increase in load is accompanied by an increase in slip and, therefore, by a decrease in torque. As the motor now develops a decreased torque with an increased load, it must come to a standstill, unless the load is removed. At standstill ($s = 1.0$), the torque is comparatively small.

The underlying cause of this small starting torque is the *reactance of the stator and of the rotor.* The rotor reactance is proportional to the rotor frequency ($x_2' = 2\pi f_2 L_2$). The rotor frequency f_2 is proportional to the *slip.* As the rotor slip increases, the rotor reactance increases proportionately, whereas the resistance does not change materially. The effect of this increased reactance is to produce a greater phase difference between the rotor *currents* and their induced voltages ($\tan \alpha = x_2'/R_2$). As these currents at the same time differ in space phase with the *flux,* less torque per ampere is developed (see Par. 133). In fact, the current and the flux may become so far out of space phase with each other that, even with four or five times the rated current, only a small fraction of the full-load torque is developed. It can be shown that the breakdown torque of an induction motor is decreased by an increase in the rotor reactance ($x_2 = 2\pi f L_2$) where x_2 is the rotor reactance at *standstill.* It is desirable, therefore, that the rotor reactance x_2 and, hence, rotor inductance be as low as possible (see p. 287; Eq. (112)).

It can also be shown that the *torque of an induction motor for a given slip is proportional to the square of the line voltage.* If the line voltage is halved, the *flux* is halved, neglecting the stator impedance drop, and the rotor *current* for a given value of slip is halved. The torque is quartered, therefore, the torque being proportional to the current times the flux, other factors remaining constant. In general, it may be said that the torque for a given slip is proportional to the *square* of the line voltage. For this reason, a 10 per cent. drop in voltage may cause a 19 per cent. reduction in the breakdown and starting torques. The effect of line voltage upon torque is shown in Fig. 259, the torque

at one-half line voltage being one-quarter the torque at full-line voltage for each value of slip.

The stator impedance also reduces the breakdown torque. A high stator impedance means a comparatively large impedance drop in the stator for a given current. This decreases the back e.m.f. E; hence, the air-gap flux becomes less, and, therefore, the value of the rotor current at any given slip is reduced. This results in a reduction of torque for each value of slip.

The effect of each of these various factors upon the breakdown torque is shown in the following equation:

The breakdown torque

$$T_{max} = \frac{KV^2}{r_1 + \sqrt{r_1^2 + (x_1 + x_2)^2}} \tag{112}$$

where K is constant, V is the terminal voltage, r_1 is the stator resistance, x_1 is the stator reactance, and x_2 is the rotor reactance at standstill.

The above equation shows that:

The breakdown torque is proportional to the square of the line voltage.

The breakdown torque is reduced by an increase in the stator resistance and by an increase in the stator and rotor reactances.

The breakdown torque is independent of the rotor resistance.

The stator and rotor reactances are proportional to the frequency and to their respective inductances. It is desirable, therefore, that the stator and the rotor inductances be kept low and that the frequency be not too high.

As the squirrel-cage motor is ordinarily started at low voltage, it develops but little starting torque, because the flux is small and the rotor currents are considerably out of space phase with the flux.

It is desirable that the stator and rotor inductances be as low as possible. This is accomplished by having the slots partially open and thereby reducing the value per ampere of the leakage flux which links the individual conductors. Ordinarily, it is not desirable that the slots be entirely open, as this increases the reluctance of the air-gap, and more magnetizing current is required. This, in turn, reduces the power factor. Also, with open slots the tooth losses may become excessive, particularly in

large motors. The rotor-slot design is actually a compromise among these conflicting factors.

Because of the lower reactance accompanying a lower frequency, a 25-cycle motor will have, in general, greater starting torque and breakdown torque than a 60-cycle motor. On the other hand, the magnetizing current, in general, is higher, because of the higher flux densities employed in the 25-cycle design.

Because of its low rotor resistance, the squirrel-cage motor has excellent operating characteristics for constant-speed work. The slip is small, and the speed regulation is good. In addition, the motor is simple, rugged, and requires but little attention. Some of its fields of application are in machine shops, in woodworking shops, in cement mills, in textile mills; in fact, it is used, in most cases, where the load requires constant speed with but little starting torque.

Below are given some squirrel-cage induction-motor data.

SQUIRREL-CAGE INDUCTION-MOTOR DATA
Three-phase, 40° open, 220, 440, and 550 volts
Westinghouse Electric & Manufacturing Company

Horse-power	Poles	Speed, r.p.m.	Weight of motor alone, pounds	Load			Load		
				One-half	Three-quarters	Full	One-half	Three-quarters	Full
				Efficiency, per cent.			Power-factor, per cent.		
60 cycles									
1	4	1.750	47	75	78	78	66	78	82
3	4	1,750	67	79	80	78	71	82	[illegible]
5	4	1,750	146	83	85	85	74	83	87
7.5	6	1,160	260	83	86	87	73	82	87
10	6	1,160	310	84	87	88	73	82	88
20	6	1,160	480	86	89	89	73	83	88
50	6	1,160	1,170	88	89	89	80	88	90
100	8	870	1,950	90	90	90	80	87	90
25 cycles									
2	2	1,450	150	79	80	81	65	76	83
3	2	1,450	170	81	82	82	75	84	89
5	2	1,450	240	84	85	85	75	84	88
7.5	2	1,450	330	85	86	86	77	87	90
10	2	1,450	470	86	87	87	79	87	91
20	2	1,450	720	85	87	87	80	88	91
50	2	1,450	1,500	89	90	90	90	93	94
100	4	715	2,700	88	89	90	84	91	93

As this type of motor develops very little starting torque, it cannot be used where it must be started under any considerable load. Another disadvantage is that its speed is not adjustable.

136. Starting Squirrel-cage Motors.—Small induction motors up to 5 hp. can usually be connected directly across the line without undue disturbance of the line voltage. Special starting devices should be used for motors of 7.5 hp. and greater.

Figure 260 shows the connections often used for the smaller-sized motors where special starting devices are not required. A double-throw switch, when in the starting position, puts the

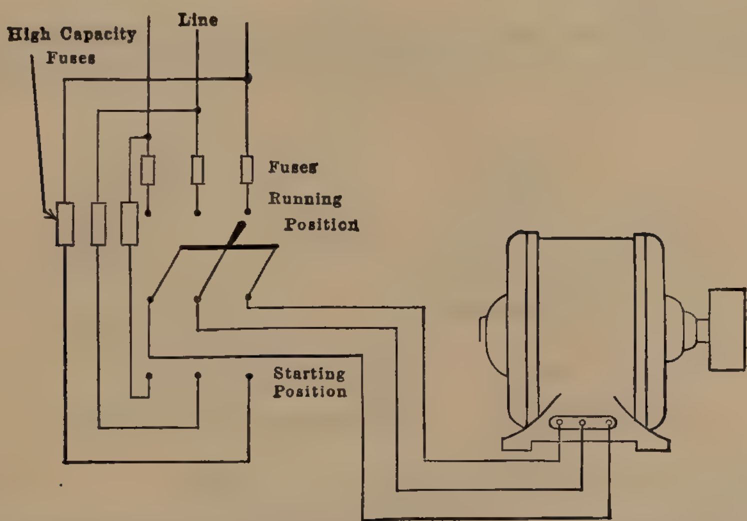


FIG. 260.—Switching connections when motor is connected directly across line at starting.

motor in series with three high-capacity fuses, one in each line. Because of the action of a spring, the switch can make contact on this side only while it is held in position. When the switch is thrown to the running position, the current is supplied through three fuses designed to carry only the safe operating current of the motor. This gives the motor overload protection that would not otherwise be obtained if fuses sufficiently large to carry the starting current were used during normal operation. Resistances (carbon rods or carbon compression rheostats) are sometimes inserted in the starting circuit to limit the starting current. Also, high-capacity fuses at the motor are sometimes omitted, the line fuses giving the required protection at starting.

As the squirrel-cage motor at starting is equivalent to a short-circuited transformer, it is necessary to reduce the starting current in the larger sizes. One simple method (Fig. 261) is to use a delta-connected motor. By means of a triple-pole, double-throw (T.-P. D.-T.) switch, the windings are first connected in Y across the line, thus applying only $1\sqrt{3}$ or 58 per cent. of the normal voltage to each coil. This makes the *line* current one-third the value it would have if the motor were directly across the line. When the motor has attained sufficient speed, the switch is thrown over, connecting the motor in delta across the line.

The most common method of starting the squirrel-cage motor, however, is to use an autostarter or starting compensator,

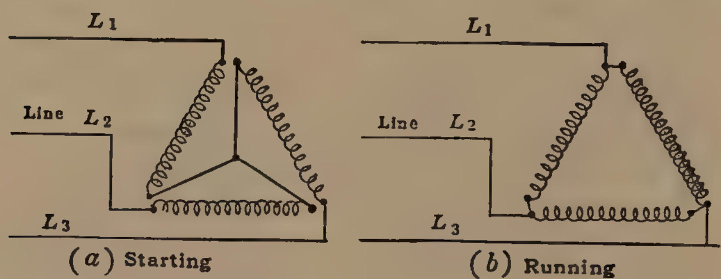
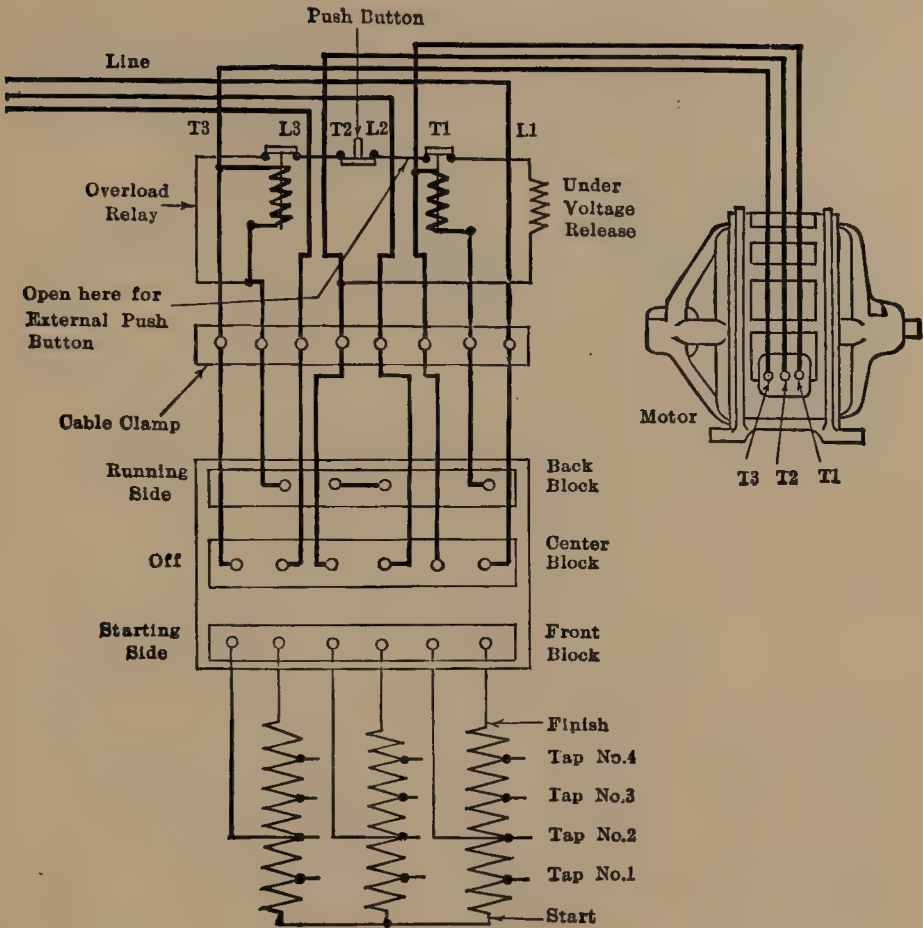
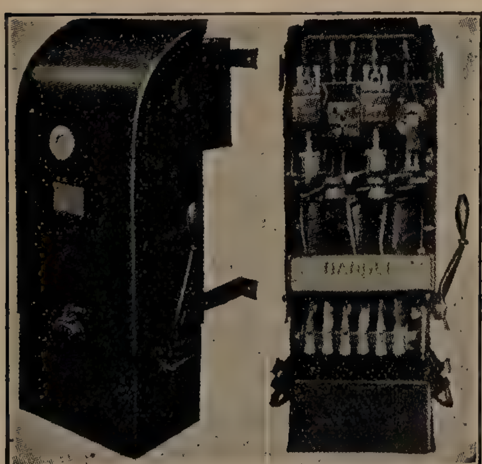


FIG. 261.—The Y-delta method of starting induction motor.

similar to those shown in Figs. 262 and 263. In the General Electric compensator, shown in Fig. 262, the three coils of a three-phase auto-transformer are connected Y. When the switch is in the starting position, the compensator is connected across the line with only the line fuses for protection. Under these conditions, the three motor lines are connected to three taps, one in each phase of the auto-transformer. Hence, the motor voltage is reduced, usually to one-fourth or to one-half its rated value. When the switch is in the running position, the compensator is entirely disconnected from the line, and the motor is connected directly across the line through the running fuses. In Fig. 262, the heavy lines show the path of the current when the compensator is in the running position. It should be remembered that a compensator supplying a motor with half-voltage reduces the line current to one-fourth its normal value. The motor being at half-voltage takes one-half the current that



(a)



(b)

FIG. 262.—Autostarter for squirrel-cage induction motor. (General Electric Co.)

it would take if directly across the line. As this current is supplied by the secondary of a 2 to 1 transformer, the line current is but half the motor current and is, therefore, one-fourth the cur-

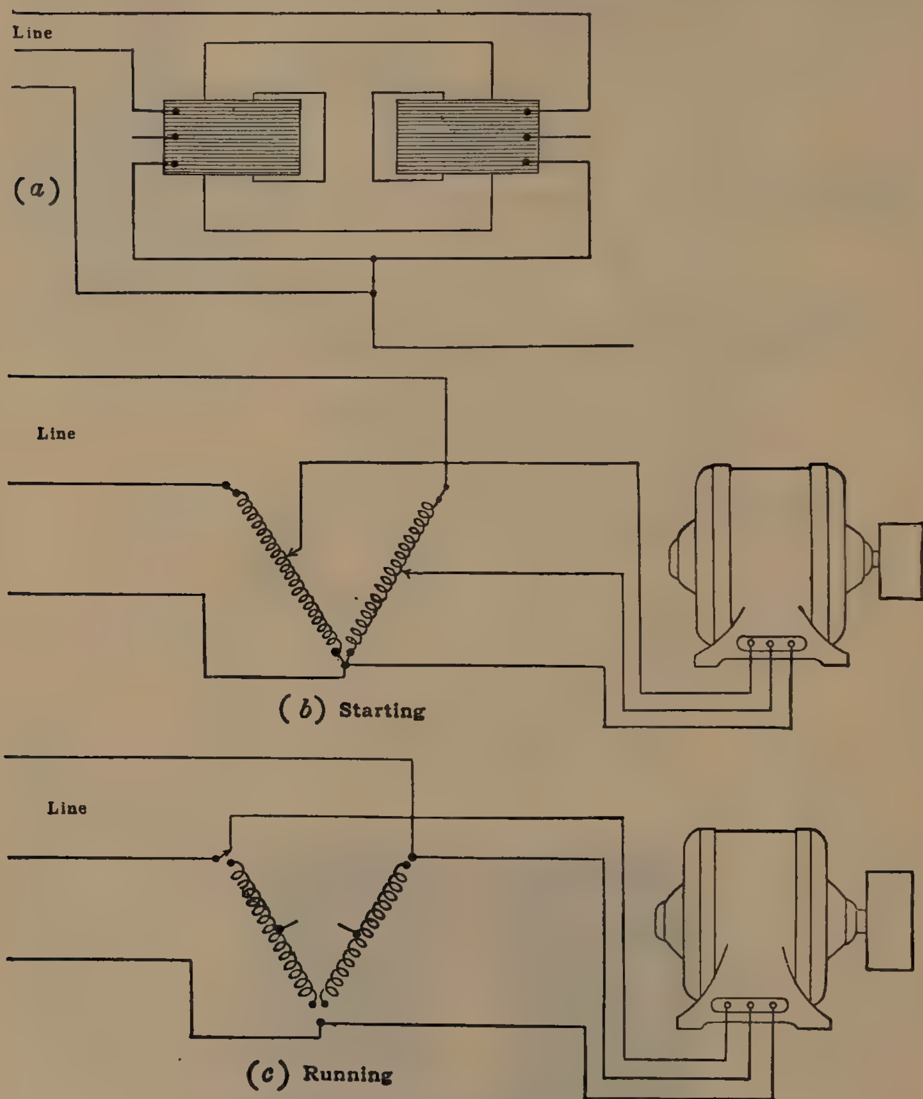


FIG. 263.—V-connected starting compensator.

rent that would have been taken had the motor been directly across the line.

It is not necessary to use a three-coil auto-transformer. In the Westinghouse starting compensator, two coils are mounted on the two outer legs of a transformer core (Fig. 263 (*a*)) very

similar to the core used for three-phase core-type transformers (see p. 244, Fig. 218 (*b*)). On starting, these two coils are connected in V across the line, and two motor taps are taken off, as shown in Fig. 263 (*b*). The motor is thus supplied at a reduced three-phase voltage. When the starting handle is placed in the running position, the two motor taps are connected directly to their corresponding lines (Fig. 263 (*c*)), and, at the same time, the compensator is entirely disconnected from the line. One advantage of this type of starter is that it can be readily used on two-phase as well as on three-phase circuits.

Practically all starting compensators have an under-voltage release, as indicated in Fig. 262. When the line voltage decreases to a low value, a solenoid plunger drops, releasing the starting handle, which springs back to the "off" position.

137. Wound-rotor Induction Motor.—If resistance be introduced in the rotor circuit of an induction motor, the slip for any given value of torque will increase.

A given value of torque requires a definite value of flux and a definite value of current. The flux of the induction motor is practically constant, since the back e.m.f. is practically constant. If resistance be introduced in the rotor circuit, the rotor impedance is increased. (At slips which give the ordinary values of torque, the armature reactance is small as compared with its resistance; hence, the armature impedance is practically all resistance.) If the slip remains constant, the induced e.m.f. of the rotor does not change. The armature current, which is equal to this e.m.f. divided by the rotor impedance, decreases. The torque, therefore, decreases.

To bring the torque back to its original value, the armature current must be increased. To increase the armature current, the armature induced e.m.f. must increase. Since the flux is constant, the increase in the induced e.m.f. may be obtained only by this flux's cutting the rotor conductors at a greater rate. For a given value of torque, therefore, the slip must increase when resistance is introduced in the rotor circuit.

The slip—torque curve will be changed from curve (1) to curve (2) (Fig. 264). It will be noted that full-load torque is obtained at increased values of slip as the rotor resistance is increased. The value of the maximum or breakdown torque will not be

affected, but the point of maximum torque moves toward the point of zero speed ($s = 1.0$). That is, the maximum torque occurs at a greater value of slip. The rotor now runs at reduced speed, but the reduced speed is obtained at the expense of efficiency, for the I^2R losses in the rotor circuit are increased.

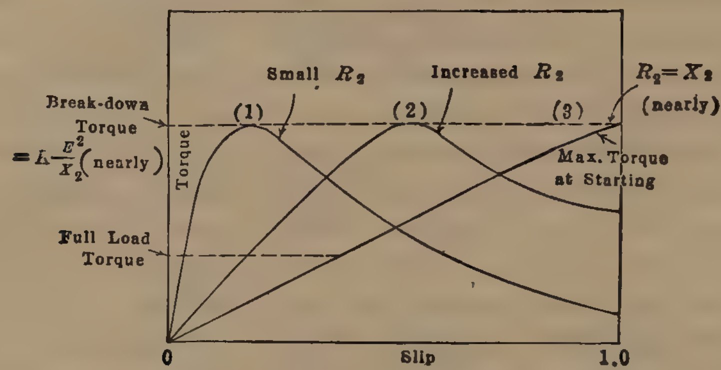


FIG. 264.—Effect on slip—torque curve of inserting resistance in rotor circuit.

It is evident that speed control may be obtained by the introduction of resistance in the rotor circuit. This method of speed control is very similar to the armature resistance method of speed control in the direct-current motor (see Vol. I, p. 396). The lowering of the speed is accompanied by a material lowering

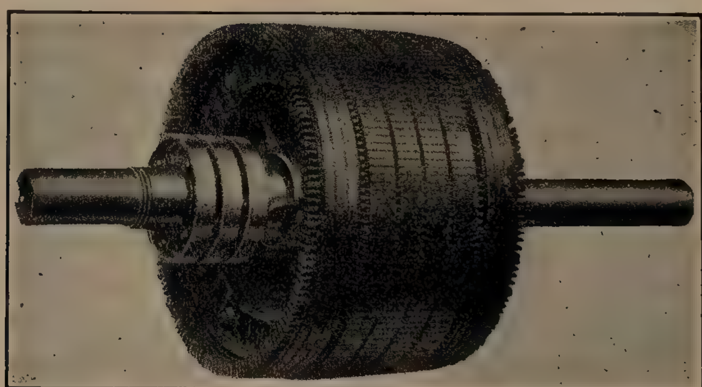


FIG. 265.—Wound rotor of 100-hp., 440-volt induction motor.

of the efficiency and poor speed regulation. The electrical efficiency of the rotor is equal to the ratio of actual speed to synchronous speed. For example, at 25 per cent. slip, the rotor efficiency is 75 per cent. That is, of the power transmitted across the air-gap, 25 per cent. is lost as heat in the rotor resistance.

The other 75 per cent. is converted into mechanical power, although this is not all available at the pulley, because of rotor friction and core losses.

If sufficient resistance be introduced in the rotor circuit, maximum torque may be made to occur at standstill, as shown by curve (3) (Fig. 264). That is, breakdown torque is obtained at starting. In order to obtain breakdown torque at starting, the rotor resistance per phase, R_2, should be approximately equal to the rotor reactance per phase at standstill, x_2.

An adjustable resistance cannot be readily placed in the squirrel-cage rotor, so that three-phase rotors requiring external

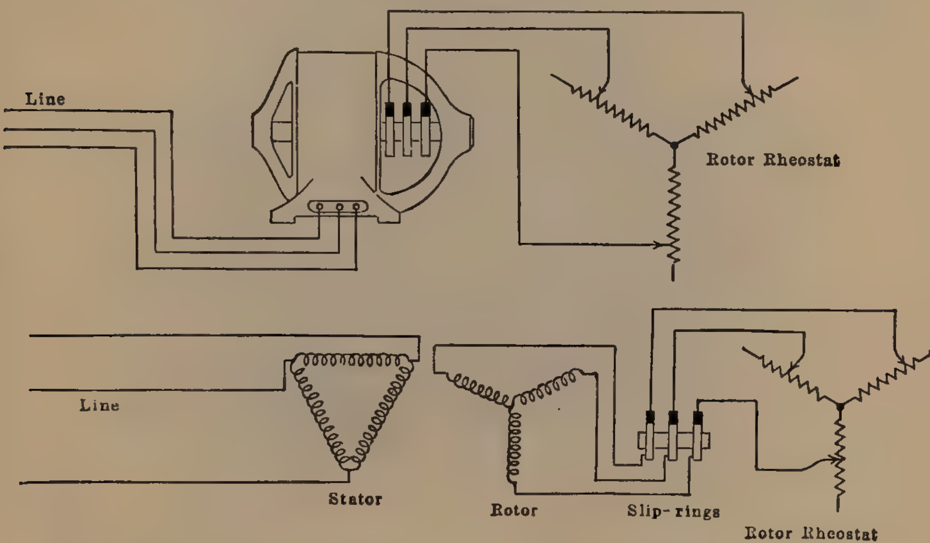


FIG. 266.—Connections for wound-rotor induction motor.

resistance are usually wound either two-phase or three-phase. The two-phase windings may be connected either star or mesh, and the three-phase windings may be connected either Y or delta. Such rotor windings are in every way similar to stator windings. The three ends of the three-phase windings are brought out to three slip-rings, as shown in Figs. 265, 266, and 267. Brushes, bearing on each of these three rings (Fig. 266), connect to Y-connected external resistances, usually through a controller. The entire resistance of each phase is in circuit on starting. This causes the rotor current to be more nearly in space phase with the air-gap flux, so that a large torque is obtained with a moderate value of current. In addition to pro-

ducing a very good starting torque, the starting current of the motor does not greatly exceed the rated current. As the motor comes up to speed, the external resistance is cut out. The motor then operates on curve (1) (Fig. 264).

Even without the controller, the wound-rotor type of motor is more expensive than the squirrel-cage motor, due to the greater cost of winding and connecting the rotor coils. The controller and resistors further add to the cost. In the running position,

Fig. 267.—Slip-ring induction motor, assembled.

this type of motor has a greater slip than the ordinary squirrel-cage motor, because it is not possible to secure the very low resistance obtainable with the squirrel-cage winding. As has been pointed out, such external resistance may be used to obtain speed control at reduced efficiency and with poor speed regulation. Hence, this type of motor has better starting characteristics but poorer running characteristics than the squirrel-cage motor.

Wound-rotor induction motors are used where considerable starting torque is required and frequently where speed adjustment is desired. Common applications of this type of motor are in cranes, elevators, pumps, hoists, railways, calenders, etc.

Another recent use of these wound-rotor induction motors is in the electric propulsion of battleships. The motors are connected directly to the propeller shafts. Two synchronous speeds are obtained by changing the number of poles. Intermediate speeds are obtained by changing the frequency of the generator.

Where a rheostat is used for starting duty only, the rotor conductors may be connected to resistance grids within the rotor itself. Such grids can be short-circuited by copper brushes operated by pushing a rod which protrudes from the center of the rotor shaft. Such a rotor is shown in Fig. 268. This type of rotor cannot be operated with the grids in circuit continuously because of the difficulty of dissipating the heat which is developed within the rotor.

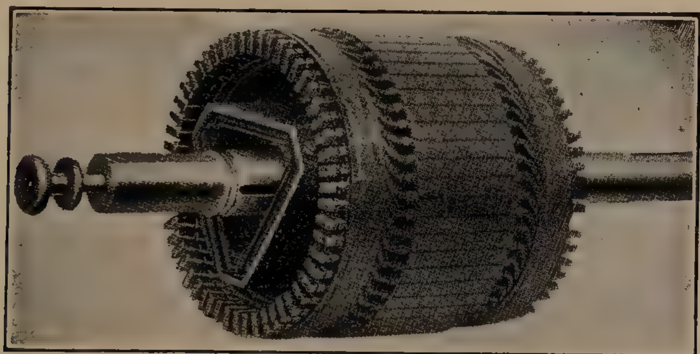


Fig. 268.—Rotor of induction motor having starting resistance within rotor.

138. Induction-motor Air-gap.—The air-gaps of direct-current generators and motors, and of alternators, are much greater than is necessary for mechanical clearance. This is due to the fact that with too short an air-gap, the effect of armature reaction becomes too great, that is, the field is relatively weak as compared with the armature. On the other hand, the air-gap of the induction motor is made just as short as mechanical clearance will permit. The back e.m.f. of the stator varies only a few per cent. from no load to full load. This back e.m.f. is induced by the air-gap flux's cutting the stator conductors. As the speed of the rotating field is constant, the flux in the gap must be substantially constant from no load to full load. In a given motor, the magnetizing current is, therefore, practically constant at all loads. If the length of the air-gap be increased, the reluc-

tance of the magnetic circuit is also increased. As the back e.m.f. does not change except slightly, the flux changes also but slightly. With a fixed flux, therefore, the greater air-gap reluctance will necessitate a greater magnetizing current. This increased magnetizing current lowers the power factor (see Fig. 258, p. 284).

Large slot openings increase the reluctance of the air-gap and so lower the power factor. From the standpoint, therefore, of the magnetizing current, it is desirable to use semiclosed slots or open slots with magnetic wedges. The disadvantage of closing the slot too much is that both the stator and the rotor inductances increase, and the breakdown and starting torques are reduced (see p. 287, Eq. 112). The increase of inductance also tends to lower the power factor.

The small mechanical clearance between the rotor and the stator makes it necessary to have a heavier shaft and heavier and stiffer bearings in the induction motor than are required in other types of rotating machinery of the same speed and size.

139. Speed Control of Induction Motors.—The speed of the rotor of an induction motor is given by

$$N_2 = \frac{f \times 120}{P}(1 - s) \quad \text{(pp. 274 and 275, Eqs. (107) and (109))}$$

where N_2 is the rotor speed in r.p.m., f is the frequency of supply in cycles per second, P is the number of poles, and s is the slip.

Obviously, there are three factors—frequency, slip, and number of poles—which determine the speed of the induction motor. In order to change the speed, it is necessary to change at least one of these factors.

Changing the Slip.—The slip may be changed by introducing resistance into the rotor circuit. This has already been discussed in connection with the wound-rotor type of motor. At a given slip, any value of torque up to the breakdown torque may be obtained by this method. Its disadvantages are lowered efficiency and poor speed regulation.

These disadvantages may be avoided by introducing counter e.m.fs. instead of resistance into the rotor circuit, either at line frequency, which requires that the rotor have a commutator, or by means of an auxiliary commutating machine, which introduces counter e.m.fs. at rotor frequency through slip-rings. This last method necessitates the use of a commutating type of machine

which produces e.m.fs. at rotor, or slip frequency. It must, therefore, be excited by the rotor currents themselves. The Sherbius[1] method of speed control is the most common example of the counter e.m.f. method.

When a current flows against a counter e.m.f. in a rotating machine, mechanical power is developed. This occurs, for example, when the current flows against the counter e.m.f. in a

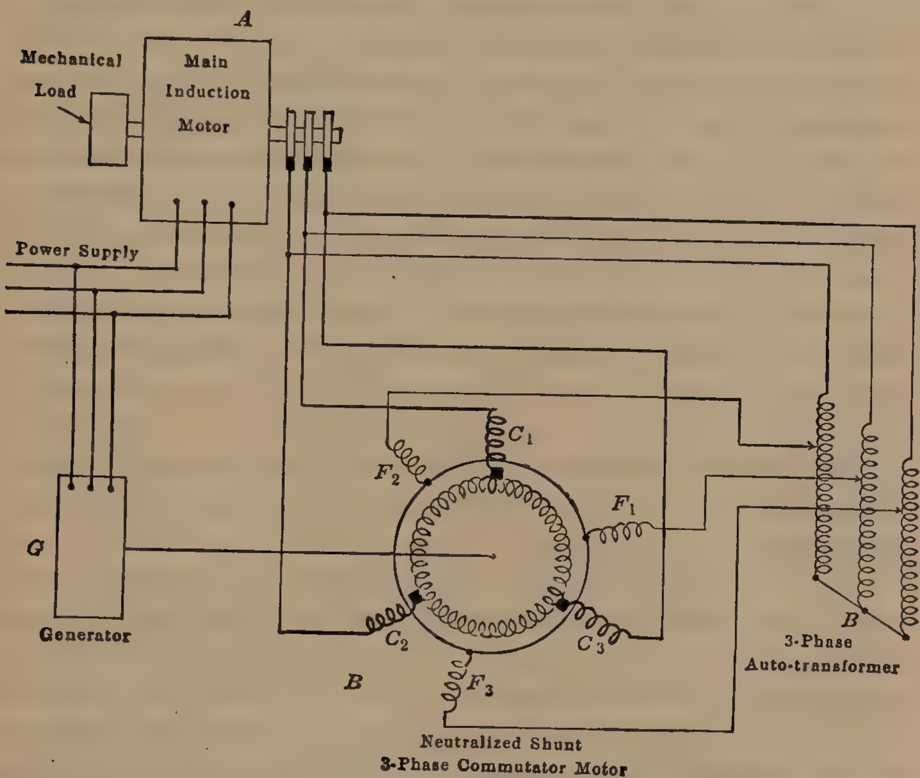


FIG. 269.—Connections for adjusting speed of an induction motor by means of neutralized three-phase commutator motor connected to slip-rings of induction motor.

direct-current motor. The counter-e.m.f. machine accordingly develops mechanical power, which is available for various purposes. Unlike the voltage drop in a resistance, the counter e.m.f. is practically independent of the current (see Counter-

[1] HULL, JOHN I., "Theory of Speed and Power-factor Control of Large Induction Motors by Neutralized Polyphase Alternating-current Commutator Machines," *Trans. A. I. E. E.*, Vol. XXXIX, (1920) Part 2, p. 1135.

Also, PAULY, K. A., "Some Methods of Obtaining Adjustable Speed with Electrically Driven Rolling Mills," *Gen. Elec. Rev.*, p. 422, May, 1921.

electromotive Force Cells, Vol. I, p. 467). A motor employing this method of speed control, therefore, has good speed regulation and efficiency.

Figure 269 shows the connections employed in obtaining speed control by the foregoing method. *A* is the main induction motor having a slip-ring rotor. *B* is a three-phase commutator motor on whose stator are shunt-field windings F_1, F_2, and F_3, spaced 120 electrical space-degrees apart. The rotor or armature of *B* is wound three-phase, and the coils are connected to a commutator in the same manner as they are in a direct-current armature. The three-phase slip-ring currents of the main motor are carried into the armature of *B* through the brushes and commutator, the brushes being spaced 120 electrical space-degrees apart, as shown. Series compensating windings C_1, C_2, and C_3 act along their respective brush axes and assist commutation.

A three-phase auto-transformer *B* is Y-connected across the slip-rings. A tap on each phase of this transformer feeds one of the shunt fields F_1, F_2, F_3, the shunt fields being connected in Y.

A constant-speed generator *G*, usually of the induction type, is mechanically connected to the motor *B* and delivers back into the line the power received from the motor.

A detailed analysis of the operation of this apparatus involves a somewhat complicated vector diagram and is beyond the scope of this book. It can be shown, however, that the commutator motor develops an e.m.f. in each phase which is nearly in phase opposition to its respective phase current. This e.m.f. is practically independent of the slip, if the speed of the commutator motor be held constant by its load, such as the generator *G*. Neglecting impedance drops, therefore, the rotor of the main motor will slip until its e.m.f. is equal to the counter e.m.f. of the commutator motor. As the rotor e.m.f. is proportional to the slip, the rotor slip will be constant at all loads if the impedance drops be neglected.

When a current flows in opposition to an e.m.f. (as in a direct-current motor), it *gives up* energy. The current delivered to motor *B*, therefore, gives up energy, some of which is returned to the line through generator *G*. Ordinarily, in the wound-rotor type of motor, this energy is lost in heating a resistance. The

fact that this counter e.m.f. is constant gives the motor a practically constant-speed characteristic for any one adjustment. Speed adjustments are made by changing the positions of the taps of the autotransformer B. Because of the cost of two extra machines, this method has been but little employed in this country except in the very large units used in steel mills, where the method is now coming into general use.

Change of Frequency.—Commercial power systems operate at constant frequency, and it is impossible to control the speed of induction motors by change of frequency when the motors take their power from such systems. In a few special instances, such as in the electric propulsion of battleships,[1] the motors are the only loads connected to the turbo-alternators. It is possible, therefore, to obtain speed control by changing the speed of the turbines themselves. Even here the range of speed variation is limited, because the efficiency of turbines decreases very rapidly when their speed departs from the speed for which they are designed.

Change of Poles.—By means of a suitable switch, the stator connections may be changed in such a manner that the number of poles is changed. This changes the synchronous speed of the motor and, therefore, the speed of the rotor. If the poles be changed in the ratio of 3 to 2, the winding will probably be designed for two-thirds pitch at the higher speed, making it a full-pitch winding for the lower speed. In such a motor, the best possible design is not usually obtainable at both speeds. That is, desirable characteristics, such as high power factor, etc., are sacrificed at one speed in order that a reasonably good motor may be obtained at the other speed. Sometimes the stator connections are changed from delta to Y at the same time that the pole connections are changed. This changes the voltage per phase and makes possible a better motor at each speed. Because of the complications involved in changing the connections, it is not desirable to obtain more than two speeds by changing the number of poles. To avoid these complicated switching connections, induction motors sometimes have two distinct windings, the two windings being connected for a different number of poles. The 7,500-hp., wound-rotor induction motors used to

[1] *Gen. Elec. Rev.*, April, 1919.

drive the electrically propelled battleship *Tennessee* have this type of winding. One winding is connected for 36 poles, and the other for 24 poles.

In the electrically propelled battleship *New Mexico,* the motors are direct connected to the propeller shafts. The stators can be connected for 24 poles or for 36 poles, giving a speed change of 3 to 2. In wound-rotor types of motors, it is necessary to change the rotor as well as the stator connections. Otherwise, negative torque will be developed by certain of the rotor conductor belts.

Speed Control by Concatenation.—This method requires two motors, at least one of which must have a wound rotor. The speed is changed by changing the slip of one motor, which changes

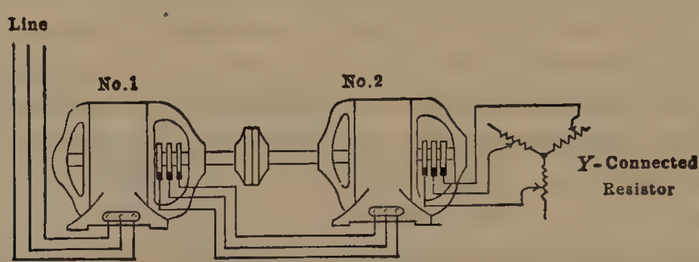


FIG. 270.—Concatenation of induction motors.

the frequency supplied to the other motor. The two rotors are connected rigidly together, as indicated in Fig. 270. Line frequency is supplied to the stator of one motor, as No. 1 (Fig. 270). This first motor should have a one-to-one ratio of transformation between stator and rotor. That is, at standstill, and with the external circuit of the rotor open, the voltage across the rotor slip-rings should be equal to line voltage. Assume that the two motors are similar and that the rotors operate at slightly less than half the synchronous speed of the first motor. The *rotor* frequency of No. 1 motor is slightly greater than half line frequency, as the slip is slightly greater than 50 per cent. (see p. 276, Eq. (110)). The synchronous speed of No. 2 motor, therefore, is practically half that of No. 1 motor. The rotors each operate at a speed which is slightly less than half the synchronous speed of the first motor. The rotors so adjust their speeds that their combined torque is just sufficient to carry the load. It is not necessary that the two motors have the same number of poles. The

various speeds for combinations in which the two motors have a different number of poles may be determined as follows:

Let N be the speed of the combination; f_1 and f_2, the stator frequencies; P_1 and P_2, the number of poles; and s_1 and s_2, the slips. The speed of the first rotor

$$N_1 = \frac{f_1 \times 120}{P_1}(1 - s_1) \quad \text{(p. 275, from Eq. (109)).}$$

The speed of the second rotor

$$N_2 = \frac{f_2 \times 120}{P_2}(1 - s_2) = \frac{s_1 f_1 \times 120}{P_2}(1 - s_2).$$

As the two rotors are rigidly coupled, the speed N_1 equals the speed N_2.

$$\frac{f_1 \times 120}{P_1}(1 - s_1) = \frac{s_1 f_1 \times 120}{P_2}(1 - s_2), \tag{113}$$

from which

$$s_1 = \frac{P_2}{P_1 + P_2 - P_1 s_2}.$$

$P_1 s_2$ is very small in comparison with $P_1 + P_2$, when the combination is operating near its synchronous speed, and it may be neglected. Then

$$s_1 = \frac{P_2}{P_2 + P_1}. \tag{114}$$

If the stator of the second motor is so connected that its rotor tends to turn in a direction opposite to that of the rotor of the first motor, Eq. (113) becomes

$$\frac{f_1 \times 120}{P_1}(1 - s_1) = -\frac{s_1 f_1 \times 120}{P_2}(1 - s_2).$$

Again neglecting the term $P_1 s_2$, the slip becomes

$$s_1 = \frac{P_2}{P_2 - P_1}. \tag{115}$$

The set will not start of itself if connected in concatenation with the rotors tending to turn in opposite directions. It must first be brought up to speed either by an auxiliary motor or by one motor alone, before the second one is connected.

As an example of the speeds obtainable with two motors having different numbers of poles, consider two 60-cycle motors, one having 4 and the other 20 poles. The following synchronous speeds are obtainable:

Four-pole motor alone: 1,800 r.p.m.

Twenty-pole motor alone: 360 r.p.m.

When the 4-pole and 20-pole motors are in concatenation, aiding, the slip of the first motor, from Eq. (114),

$$s_1 = \frac{20}{20 + 4} = \frac{20}{24}.$$

The synchronous speed of the set is

$$N = (1 - s_1)1{,}800 = (\tfrac{4}{24})1{,}800 = 300 \text{ r.p.m.}$$

When the 4-pole and the 20-pole motors are in concatenation, opposing, the slip of the first motor, from Eq. (115),

$$s_1 = \frac{20}{20 - 4} = \frac{20}{16}.$$

The synchronous speed of the set is

$N = (1 - s_1)1{,}800 = (-\tfrac{1}{4})1{,}800 = -450$ r.p.m., or the set now rotates in the opposite direction.

Four different synchronous speeds are obtainable with these two motors, 1,800, 360, 300, -450 r.p.m.

It is to be noted that the synchronous speed resulting from connecting the motors in concatenation *aiding* is equal to that of a 24-pole motor, or a motor whose poles are equal in number to the *sum* of the poles of the two individual motors. When the two motors are connected in *opposition*, the resulting synchronous speed is equal to that of a 16-pole motor, or a motor whose poles are equal in number to the *difference* of the poles of the two individual motors.

It will be recognized that the concatenation method of speed control is very similar to the series-parallel method of speed control for direct-current motors (see Vol. I, p. 400, Par. 252). In concatenation, at starting and for intermediate speeds, resistance is introduced in the rotor circuit of the second motor. When the motors are connected in parallel across the line, resistance is introduced in each rotor circuit and is gradually cut out. Motors operating in concatenation are used abroad, to some extent, particularly in railway work. Because of its rather complicated connections, this system of speed control is not used to any extent in this country.

140. Induction Generator.—If an induction motor be driven above synchronous speed, the slip becomes negative. The rotor conductors then cut the flux of the rotating field in a direction

opposite to that which occurs when the machine operates as a motor. The rotor currents are then reversed with respect to the direction which they had when the machine operated as a motor. By transformer action, these rotor currents induce currents in the stator which are substantially 180° out of phase with the *energy* component of the stator current which existed when the machine operated as a motor.

The induction motor, therefore, can be used as a generator, but it has certain limitations which the synchronous alternator does not possess.

The machine does not have a definite speed for a given frequency, as the synchronous alternator has, but the speed with constant frequency varies with the load. The load is practically proportional to the slip. Because its speed is not in synchronism with line frequency, the machine is often called an *asynchronous* generator. The frequency and voltage of the induction generator are *that of the line to which it is connected,* irrespective of its speed.

An alternator, by itself, cannot deliver power unless its field is excited. The same is true of the induction generator. The alternator usually receives its excitation from a direct-current source or through armature reaction, and the resulting north and south poles rotate in space. The flux in the induction generator is produced by the polyphase exciting currents in the stator windings, and the resulting north and south poles also rotate in the air-gap and at synchronous speed. The currents which excite these north and south poles *come from the line.* The induction generator does not receive its exciting current from a separate source, therefore, but from the same lines that conduct away the energy that it generates. The induction generator cannot generate its own exciting current, but *the exciting current must be supplied by the line.* For this reason, it is necessary to have either a static condenser or synchronous apparatus in parallel with the induction generator for supplying its excitation. A static condenser, however, is seldom practicable.

The induction generator can, moreover, deliver only leading current. If a load requires a lagging current, a *synchronous* machine in parallel with the induction generator must supply the lagging component of this current.

The reason for this is as follows:

Let V (Fig. 271) be the terminal voltage of an induction machine operating as motor. The counter or generated e.m.f. E is approximately 180° from V. Let I_o be the quadrature exciting current lagging 90° behind V, and let I_m' be the motor energy current. The total current taken by the motor is the resultant current I_m, lagging behind V (see p. 284, Fig. 258). When the rotor speed is increased by a sufficient amount, the machine passes from motor to generator action. The magnitude and the phase of the air-gap flux alter by only a slight amount during this transition, just as the flux of a shunt motor does not change in sign and changes in magnitude by only a small amount, if at all, when the machine passes from motor to generator

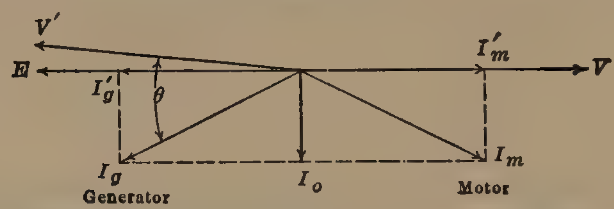


Fig. 271.—Vector diagram of induction motor and induction generator.

action through the speeding up of its armature. The exciting current I_o, therefore, which produces the flux, remains substantially constant in both magnitude and phase, just as the exciting current of a shunt motor does not change much when the machine is speeded up by mechanical means so that it becomes a generator.

As the rotor speeds up, however, it cuts the flux in a direction which is opposite to that occurring when the machine operates as a motor; that is, the slip becomes negative. The induced e.m.f. in the rotor conductors, therefore, reverses in sign, as has already been pointed out. As the rotor reactance is very low at these low values of slip, the currents produced in the rotor are nearly in phase with their induced e.m.f., and they flow in a direction opposite to that which they had as motor currents. These induced currents react on the primary in the same manner that the secondary current of a transformer reacts on the primary. As a result, an energy current I_g', 180° from the motor current I_m', is induced in the primary. The currents in the rotor are nearly in phase with the induced e.m.f. of the rotor. The

induced e.m.f. of the stator E is in phase with the rotor e.m.f. The stator ampere-turns, excluding the effect of the magnetizing current, are equal and opposite to the secondary or rotor ampere-turns. These stator ampere-turns are represented by the vector I_g', in Fig. 271. In the vector diagram, I_g' actually leads E by a small angle which, for simplicity, is neglected in Fig. 271. I_g', therefore, is practically all energy current. For any particular kilowatt load, corresponding to the energy current I_g', the total generator current is I_g, the vector sum of I_g' and I_0. I_g is a *leading* current, as the generator terminal voltage V' is nearly in phase with its induced e.m.f. E. The generator phase angle θ is not determined by the load, therefore, but by the generator itself.

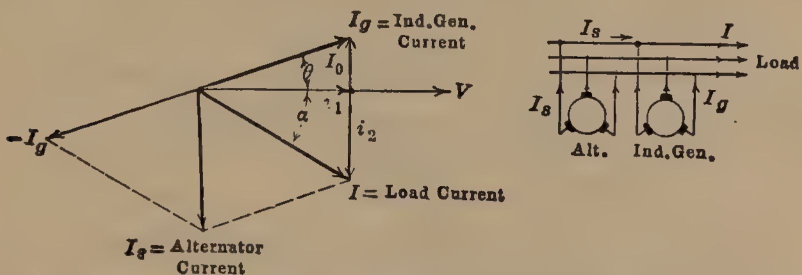


FIG. 272.—Currents supplied by alternator and by induction generator in parallel.

If the load requires a lagging current, this machine cannot supply it. This is illustrated by Fig. 272. A certain load requires a current I, lagging $\alpha°$ behind the terminal voltage V. It is desired to supply as much of this current as possible by means of an induction generator and to allow an alternator to supply the remainder. Resolve the load current I into two components, an energy component i_1 and a lagging quadrature component i_2. The induction generator can by proper speed adjustment supply the energy current i_1. Its leading exciting current I_0 is fixed, however, as has already been demonstrated. I_g, the resultant of i_1 and I_0, therefore, is the total induction-generator current at this load.

Obviously, the alternator must supply that part of the load current which the induction generator cannot supply. That is, the alternator must supply the *difference* between the load current and the induction generator current. To obtain the difference

between two vectors, reverse one and add (p. 14, Par. 8). As I_g is subtracted, it is reversed, and the resulting alternator current is I_s, which is equal in magnitude to the arithmetical sum of i_2 and I_0. It will be observed that the alternator in this case supplies no power. Its entire current is lagging quadrature current and is equal to the *exciting current* of the induction generator plus the *lagging quadrature* current of the load.

If the load were such as to require a leading current, the quadrature component of which was just equal to I_0, theoretically the induction generator could of itself supply the entire load.

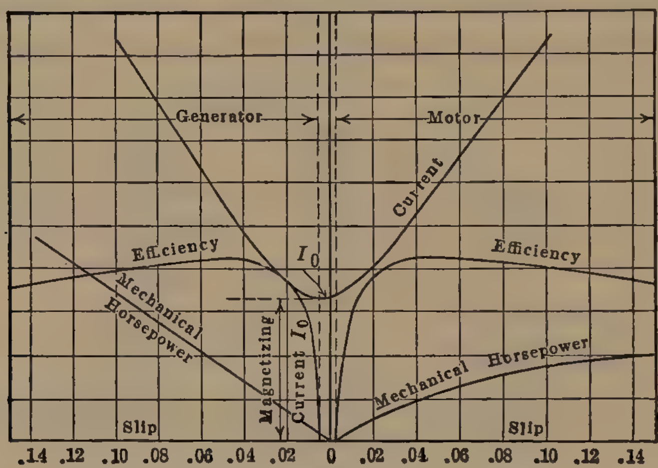


Fig. 273.—Operating characteristics of induction machine as motor and as generator.

Even then it would be necessary to have synchronous apparatus on the system to secure satisfactory operation.

The inability of the induction generator to deliver lagging current is the principal objection to its use. Considerable kilovolt-ampere capacity in synchronous apparatus is required to supply the total quadrature current required. The distinct advantage of the induction generator is the fact that it does not hunt or drop out of synchronism; it is simple and rugged, and, when short circuited, it delivers little or no power, because its excitation at once becomes zero.

The induction generator is also very useful for braking purposes in railway work. If the induction motors be left connected across the line on a down grade, any tendency of the train to drive them above synchronism will be accompanied by generator

action. In addition to braking the train, the generators pump power back into the line and so relieve the main generating station of some of its load. The machine, therefore, requires no complicated control apparatus when used for regenerative braking, such as is required by direct-current motors operating under similar conditions.

Figure 273 shows the variation of current, efficiency, slip, and torque of an induction machine as it passes from motor to generator. It will be noted that the current does not pass through zero, although the power does. The point of minimum current is the exciting current of the machine, shown by I_0. At synchronous speed, the line supplies the core losses, and the friction losses are supplied mechanically. The generator must be driven somewhat above synchronous speed before it supplies its own core losses.

141. Circle Diagram.—The operating characteristics of an induction motor may be determined experimentally without

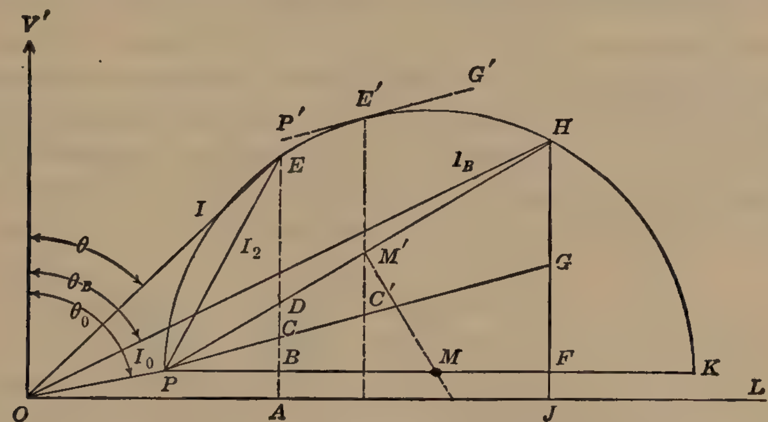


Fig. 274.—Circle diagram for induction motor.

actually loading the motor, just as was done in the case of the alternator and the transformer. It can be shown that, with constant impressed voltage and constant frequency, the locus of the primary current I (Fig. 274), as the load on the induction motor is varied, is an arc of a circle. That is, with change of load, the end E, of the current vector I, moves along the arc of a circle $PEHK$. This diagram is approximate in that it neglects the impedance drop and the copper loss in the stator due to the magnetizing and core-loss currents. To obtain data for the

construction of this diagram, an open-circuit and a short-circuit (or blocked) run are made, as is done with the alternator and the transformer. Using the data obtained from these two tests, the operation of the motor may be determined with a very fair degree of accuracy by the use of such a circle diagram.

The motor is first run at rated voltage without load, and the line voltage V, the line current I_0, and the total watts P_0 are measured. The no-load power-factor angle θ_0 can then be determined ($\cos\theta_0 = P_0/\sqrt{3}VI_0$ for a three-phase motor). The voltage per phase V' is laid off vertically (Fig. 274), and the no-load current I_0 (per phase) is laid off at an angle θ_0 from V' and lagging. The rotor is then blocked. In order that the current may be kept within reasonable limits, the supply voltage per phase, V', is reduced to voltage v', which should be of such value as to give a short-circuit current approximately equal to the rated current. The phase current I', the total power P' and the phase voltage v' are measured under these conditions. Let V' be the rated phase voltage of the machine. $V' = V$ for a delta-connected machine, and $V' = V/\sqrt{3}$ for a Y-connected machine.

The measured current I_B' is increased in the ratio of the rated motor voltage V' (per phase) to the reduced voltage v'. This gives $I_B = OH$, the current per phase which would exist were the rated line voltage V impressed across the motor when blocked. This current lags V' by an angle Θ_B.

$$\cos\Theta_B = \frac{P'}{nI_B'v'}$$

$$I_B = I_B'\frac{V'}{v'}$$

where n is the number of phases.

OL is drawn making an angle of 90° with OV' in a clockwise direction. $I_B = OH$ is laid off, making an angle Θ_B with OV'. Points P and H on the circle are, therefore, determined.

Line PH is drawn. PK is drawn parallel to OL. It is not necessary to know point K in order to construct the diagram.

With PK as a diameter, a semicircle is drawn through points P and H. The center M of this semicircle is found by erecting a perpendicular MM' at the center of PH. The intersection of MM' with PK gives the center M of the circle. With MP as a

radius and M as a center, the semicircle $PEHK$ is drawn. PK is the diameter of the semicircle, and its length in amperes is $PK = \frac{V'}{x_1 + x_2}$, where V' is the phase voltage, and x_1 and x_2 are the respective stator and rotor reactances per phase, referred to the stator.

A perpendicular HJ is then dropped from H to OL. The line HF is divided by G into two segments, such that $HG/GF = I_2^2R_2/I_1^2R_1$, that is, in proportion to the secondary and primary resistances, respectively, as a one-to-one ratio of rotor to stator turns is assumed. Line PG is then drawn.

At any load current I, I_2 $(= PE)$ is the secondary current, being equal to $I - I_0$ vectorially. EA is the energy component of the current I and, therefore, the total power input per phase,

$$P_1 = EA \times V'.$$

The core and friction losses

$$P_c = BA \times V' \text{ per phase.}$$

The primary copper loss $I_1^2R_1 = BC \times V'$ per phase.

The secondary copper loss $I_2^2R_2 = CD \times V'$ per phase.

The output $P = DE \times V'$ per phase.

The efficiency $= DE/AE$.

The torque $T = CE$ (to scale).

The slip, $s = CD/CE$.

The power factor $= \cos\theta = EA/I$.

Draw $P'G'$ parallel to PG and tangent to the circle at E'.

Breakdown torque $T_B = C'E'$ (to scale).

The above diagram is drawn for but one phase of the motor. The values of power, losses, and torque must be multiplied by n if the motor has n phases.

The torque scale may be found as follows:

The torque is equal to a constant times the power, divided by the speed, the value of the constant depending on the units adopted. The power output per phase is $P = V' \times DE$. The rotor speed $N_2 = N(1 - s)$, where N is the synchronous speed in r.p.m.

$$N_2 = N\left(1 - \frac{CD}{CE}\right) = \frac{N(CE - CD)}{CE} = \frac{N \times DE}{CE}. \qquad \text{(I)}$$

The torque developed per phase

$$T' = K\frac{P}{N_2} = K\frac{V' \times DE}{\frac{N \times DE}{CE}} = \frac{K \times V' \times CE}{N} \text{ where } K \text{ is a constant.} \qquad \text{(II)}$$

$V' \times CE$ is the total power per phase delivered to the rotor

The total power delivered to the rotor by n phases

$$P_2 = n \times V' \times CE \text{ watts.}$$

The horsepower output

$$H.P. = \frac{n \times DE \times V'}{746} = \frac{2\pi N_2 T}{33{,}000} \qquad \text{(III)}$$

where T is the *total* torque.

But

$$N_2 = \frac{N \times DE}{CE}. \qquad \text{from (I)}$$

Substituting in (III),

$$\frac{n \times DE \times V'}{746} = \frac{2\pi(N \times DE)T}{CE \times 33{,}000}$$

$$T = 7.04\frac{n \times V' \times CE}{N} \text{ lb.-ft.}$$

$$K = 7.04 \times n. \qquad \text{(II)}$$

As the phases n, the voltage V', and the synchronous speed N are usually fixed, the torque

$$T = K'CE \text{ where } K' = 7.04\frac{n \times V'}{N}. \qquad (116)$$

142. Measurement of Slip.—There are various methods for measuring slip. The slip may be determined by measuring the rotor speed and subtracting this speed from that of the rotating field as determined from the frequency. As the slip is but a very small percentage of either the synchronous speed or the rotor speed and is the difference of two nearly equal quantities, it is not possible to determine it accurately by the measurement of each of these quantities and so finding their difference.

A simple method of measuring slip is shown in Fig. 275. A "target" or disc is fastened to the end of the shaft or to the pulley of the motor. This disc has the same number of black and the same number of white sectors as the motor has poles. This disc is illuminated by an arc lamp which is fed from the

motor mains. When the current in the arc is passing through its zero values, the arc emits but little light. During these periods, therefore, the sectors on the disc are but dimly illuminated. In one-half cycle, the armature of the motor would advance one pole if there were no slip. During this time, each black sector would advance to the position just occupied by the adjacent black sector which preceded it. The same is true of the white sectors. During the period of advancement, the sectors are but faintly visible, because the current in the arc is passing through zero. Each black sector and each white sector is not, therefore, clearly visible until it has reached the position

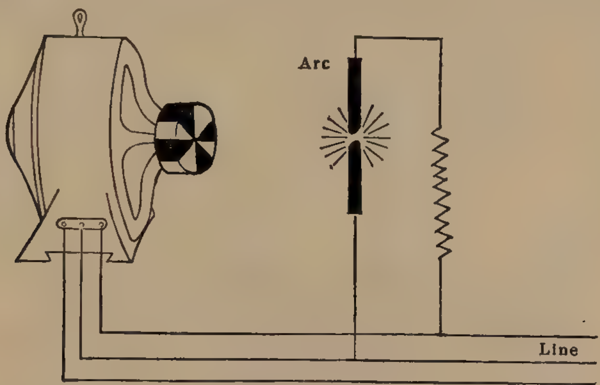


FIG. 275.—Stroboscopic method for measuring slip.

just occupied by the sector of the same color just preceding it. As the disc is intensely illuminated twice every cycle, while the arc current is passing through its maximum values, all the sectors are clearly visible twice every cycle. If the disc, therefore, rotated at synchronous speed, it would *appear* stationary. Due to the fact that each conductor on the rotor does *not* advance one pole each half-cycle, the sectors will not reach the position of the next adjacent sector of the same color but will fall short of this distance, due to the slip. The sectors on the disc will then appear not stationary but will seem to be rotating slowly backward. The number of r.p.m. that they *appear* to rotate is the revolutions slip of the rotor. Figure 275 shows a stroboscope for a four-pole machine. Occasionally, black and white stripes are painted on the face of the pulley (Fig. 275) to serve the same purpose.

A mechanical-electrical method of measuring slip is shown in Fig. 276. Two cylinders of insulating material are driven, one by the induction motor shaft and the other by a small synchronous motor having the same number of poles as the induction motor. Each of these cylinders is fitted with a slip-ring, to which a small contact piece is connected. The synchronous motor always runs at the speed of the rotating field. Every time, therefore, that the induction motor slips one revolution, the contact pieces touch each other, closing the circuit between the two slip-rings. This is indicated by a flash of the light connected in series with the rings through the brushes *b* (Fig. 276).

In the electrical-engineering laboratories at Harvard University, the induction motor and the synchronous motor jointly

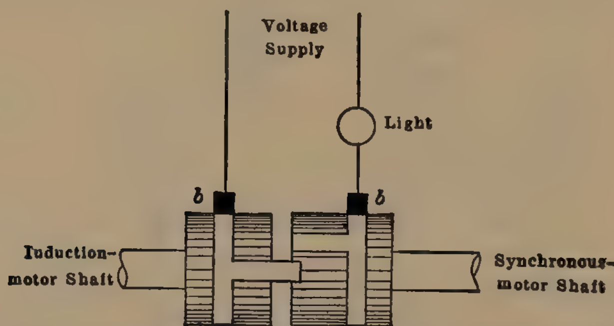


FIG. 276.—Measurement of slip by means of synchronous motor.

drive a differential through gears, a method developed in these laboratories. The speed of the differential is the revolutions slip of the induction motor. If desired, the speed of the differential and, hence, the slip, may be measured with a speed counter with considerable accuracy. By changing gears, the apparatus is adapted to machines having any number of poles.

143. Induction Regulator.—Without auxiliary apparatus, it is practically impossible to maintain the proper voltage at all the distribution points of a system, because, with a fixed voltage at the station bus-bars, the voltage at the ends of short feeders will ordinarily be greater than the voltage at the ends of long feeders. Owing to the ohmic and reactive drops in the lines, the voltage at the load end of the feeder may vary considerably with the load on the feeder. In order to maintain a more constant voltage at the distribution point, without using an excessive amount of

copper, an induction regulator is often connected to each feeder. This maintains the voltage at the distribution point practically constant.

The induction regulator is a transformer having a movable secondary. In this way, it closely resembles the induction motor.

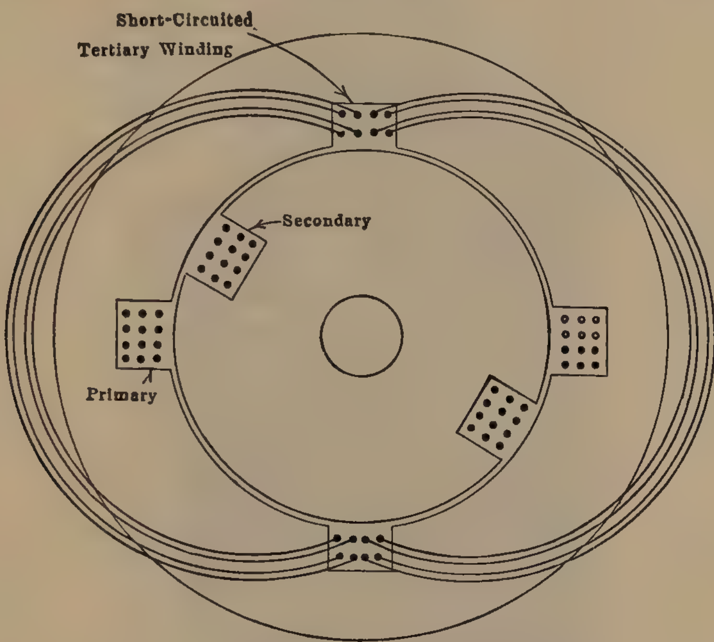


FIG. 277 (a).—Single-phase induction regulator.

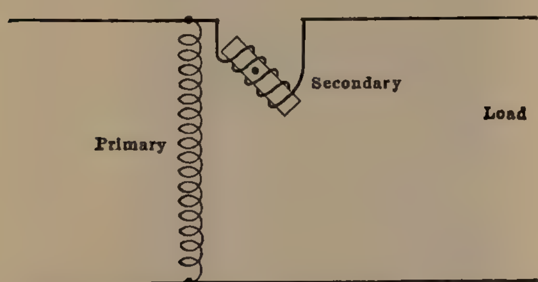


FIG. 277 (b).—Connections of single-phase induction regulator.

The general principle of the single-phase type is shown in Fig. 277. An ordinary winding is placed in the slots on the stator, and a drum winding is placed in the rotor slots. When the secondary is in the plane of the primary, the maximum e.m.f. is induced in the secondary, because the mutual inductance of the

windings is a maximum when the secondary is in this position. When the secondary is at right angles to the primary, the primary flux does not link the secondary, so that the induced e.m.f. in

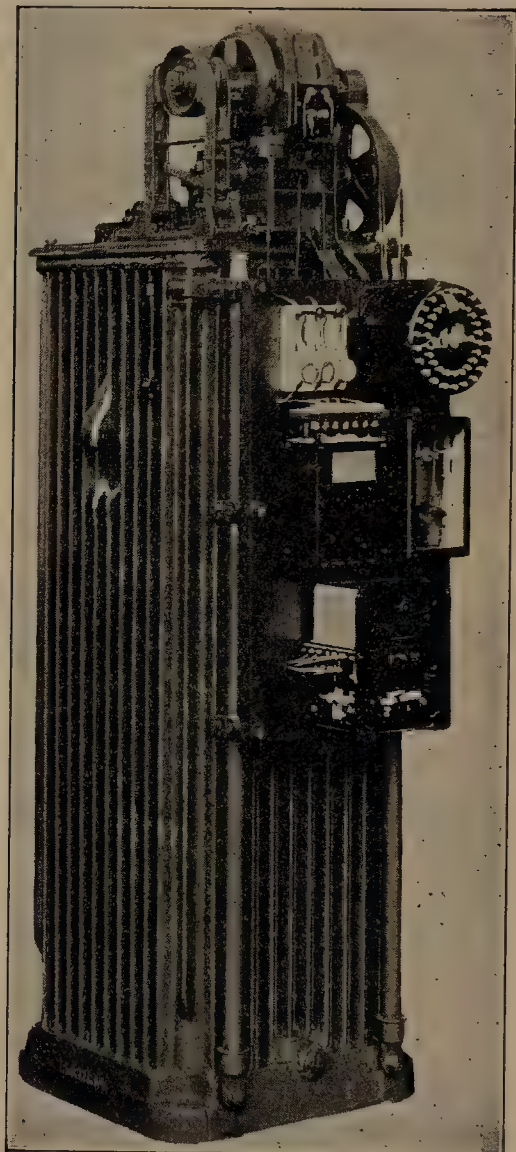


Fig. 278.—General Electric feeder voltage regulator assembled, with panel board.

the secondary is zero. As the mutual inductance of the windings is zero under these conditions, the secondary acts like a choke coil of very high impedance. To prevent this, a short-circuited

tertiary winding is placed on the stator. This acts as a short-circuited transformer secondary and, therefore, reduces the inductance of the regulator secondary to a very small amount. The primary winding is shunted across the line, as shown in Fig. 277 (*b*), and the secondary is connected in series with the line (compare with Fig. 224, Chap. VIII, p. 249). When the secondary is in the plane of the primary in one position, its induced e.m.f. is a maximum, and it is connected to act as a booster. When the secondary is turned 180° from this position, its e.m.f. is also a maximum, but it now bucks the line voltage. Any value of voltage between that corresponding to these two positions is obtainable by varying the position of the secondary.

The secondary is turned by a small motor, controlled by relays (Fig. 278). The relays are actuated by a contact-making voltmeter. If the voltage is too high, one set of contacts causes the motor to turn in such a direction as to make the secondary reduce the line voltage. If the voltage is too low, another set of contacts causes the motor to reverse its direction, and the secondary boosts the line voltage.

The three-phase induction regulator closely resembles the three-phase, wound-rotor induction motor. The three stator windings or primaries are connected across the line in either Y or delta. The three secondaries, which correspond to the three phases of a rotor winding, are insulated from one another, and each is connected in series with one of the three-phase lines. As the stator produces a uniform rotating field, the induced e.m.fs. in the secondaries are constant and are independent of the position of the rotor. Their boosting and bucking effect, however, depends upon the phase relations existing between each induced secondary e.m.f. and its respective line voltage.

The three-phase regulator requires no short-circuited tertiary winding.

CHAPTER X

SINGLE-PHASE MOTORS

144. Series Motor.—It will be remembered that the direction of rotation of either the direct-current shunt motor or the direct-current series motor is the same, irrespective of the polarity of the line voltage. If the line terminals be reversed, both the field current and the armature current are reversed, and the direction of rotation remains unchanged. If such motors be supplied with alternating current, the *net* torque developed acts in one direction only.

With alternating current, the shunt motor develops but little torque. The high inductance of the shunt field causes the field current and, therefore, the main flux to lag nearly 90° in time phase with respect to the line voltage. The armature current cannot lag the line voltage by a large angle if the motor is to operate at a reasonable power factor. There will be considerable phase difference, therefore, between the main flux and the armature current. Consequently, such a motor will develop but little torque per ampere (see Par. 133, p. 277). This particular type of alternating-current shunt motor is, therefore, not practicable.

In the series motor, the armature current and the field current are in phase with each other. The main flux is practically in phase with the field current. The armature current is substantially in phase with the flux, therefore, and the torque curve has no negative loops (see Fig. 252, p. 277). Consequently, the series motor develops approximately the same torque per ampere with alternating current as it does with direct current. Fundamentally, the series motor has possibilities as an alternating-current motor.

For the following reasons, the *ordinary* direct-current series motor does not operate satisfactorily with alternating current:

(*a*) *The alternating-field flux sets up eddy currents in the solid parts of the field structure, such as the yoke, cores, etc., causing excessive heating and a lowering of efficiency.*

In the alternating-current series motor, this difficulty is eliminated by laminating the field structure. Even with laminated field cores, however, losses in the iron occur with alternating current which do not occur with direct current.

(*b*) *There is a relatively large voltage drop across the series fields, due to their high reactance.* This limits the current and, also, reduces the output and power factor to such low values as to make the motor impracticable.

In the alternating-current motor, this difficulty is partially overcome as follows:

A low frequency is used, since reactance X is $2\pi fL$, where f is the frequency, and L the inductance. Even when the field

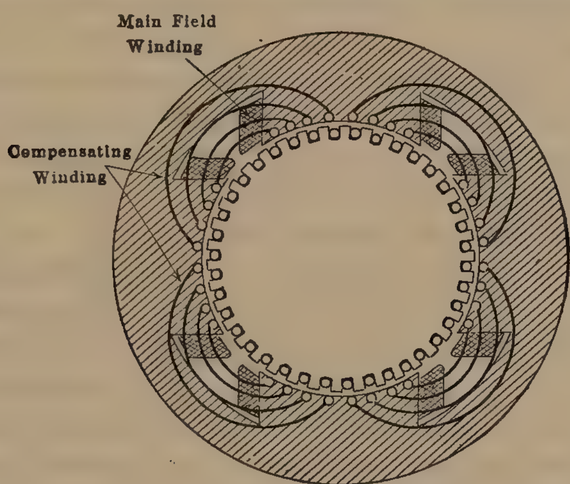


Fig. 279.—Windings of alternating-current series motor.

inductance L is made as low as is practicable, the field reactance X will be considerably too high unless the frequency f is made low. The usual lighting frequency of 60 cycles is much too high, except for motors of fractional horsepower rating. Difficulty is experienced in designing a series motor for a frequency of even 25 cycles. To obtain satisfactory operation, frequencies of $12\frac{1}{2}$ and 16 cycles are commonly used abroad for this type of motor.

The inductance varies as the product of flux and turns. (See Vol. I, p. 209, Eq. 73.) The turns per pole must, therefore, be reduced to a minimum in order to keep the inductance, and, therefore, the reactance, low. To obtain sufficient flux with few

ampere-turns per pole, the reluctance of the magnetic circuit must be reduced to a minimum. This is accomplished by operating the iron at low flux densities and, therefore, at high permeabilities and by using a very short air-gap. Because of the small number of field ampere-turns and the very low flux density, a very short pole of large cross-section is necessary, as indicated in Fig. 279.

(c) *The armature of an alternating-current series motor of a given rating has an unusually large number of conductors.* A motor of fixed horsepower and speed must develop a corresponding torque. The torque developed by a motor is proportional to the product of the field flux and the armature ampere-conductors. If, therefore, the total flux of the alternating-current motor is less than the total flux of a direct-current motor of the same rating, the armature ampere-conductors of the alternating-current motor must be correspondingly increased in order to obtain the required torque. This is one reason why the armature of the alternating-current motor is larger than that of the direct-current motor of the same rating.

(d) *The alternating-current motor has a lesser number of field ampere-turns and a greater number of armature ampere-turns than the corresponding direct-current motor.* That is, the motor has a strong armature and a weak field. This means that the armature reaction is unduly large. The effect of the armature cross-magnetizing turns, therefore, unless compensated, is to produce unusually great field distortion. As this distortion of the field by the cross-magnetizing armature ampere-turns would make commutation practically impossible, this cross-magnetizing action must be neutralized. This accomplished by means of a compensating winding placed between the main poles, as shown in Fig. 279, this winding being embedded in the pole faces (also see Thompson-Ryan winding, Vol. I, p. 323, Fig. 271). In order to reduce the *reactance* of the armature, also, this compensating winding should not only neutralize the cross-magnetizing field of the armature *as a whole* but also should neutralize it *at every point.* Although it is impossible to secure complete neutralization at every point, a close approximation to this is obtained by distributing the compensating winding over the pole faces (Fig. 279) and by making each group of pole-face conduc-

tors carry a current equal and opposite to the current in the group of armature conductors directly under it, as is also indicated in Fig. 279.

The compensating winding may be connected in series with the armature (Fig. 280), in which case the motor is said to be *conductively* compensated. When it is necessary to use the motor on a direct-current system as well as on an alternating-current system, conductive compensation is necessary.

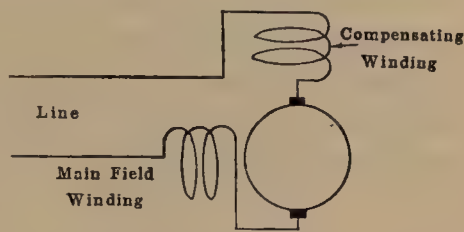


FIG. 280.—Conductively compensated series motor.

If the compensating winding be short circuited on itself (Fig. 281), the winding is linked with the cross-magnetizing flux of the armature and, therefore, becomes the short-circuited secondary of a transformer, the armature ampere-turns being the primary. As the secondary ampere-turns of a transformer are practically opposite in phase and equal in magnitude to the primary ampere-turns if the magnetizing current be small, the ampere-turns of the compensating winding nearly neutralize the ampere-turns of the armature. It is not possible to eliminate entirely the cross-magnetizing flux by this method, any more than it is possible to eliminate the mutual flux in a short-circuited transformer, but it may be reduced to a very small value.

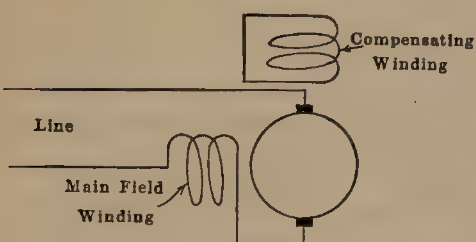


FIG. 281.—Inductively compensated series motor.

The cross-magnetizing flux links the armature turns, causing the armature itself to have large self-inductance. The reactance, therefore, of the armature alone ($X_a = 2\pi fL_a$) is large, a fact which in turn would produce a large reactance (IX_a) drop and so lower the power and the power factor of the motor. The compensating winding, acting like the short-circuited secondary of a transformer, reduces this armature reactance to a small value.

This is analogous to the ordinary transformer, which on open circuit is a very high impedance. When the secondary is short

circuited, the impedance is reduced to a very low value. The necessity for reducing this armature reactance drop to a small value is the principal reason for using a distributed compensating winding rather than a more concentrated one.

(e) *In the alternating-current series motor, a commutating difficulty occurs which is not present in the direct-current motor.*

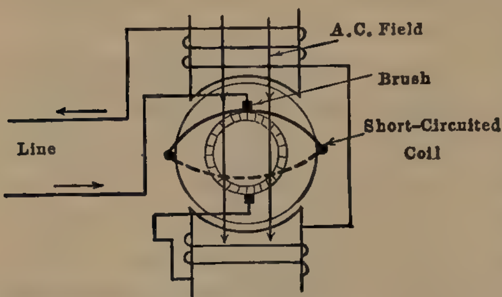


FIG. 282.—Transformer electromotive force in coil undergoing commutation.

Figure 282 shows a coil in the neutral plane undergoing commutation. The coil is, therefore, short circuited by the brushes. The plane of this coil is perpendicular to the direction of the main field, which is alternating, so that the alternating flux of this field links the coil. The short-circuited coil acts as the secondary of a transformer, of which the main field winding is the primary, and therefore has voltage induced in it. As this coil is short circuited by the brushes and has a low impedance, a large current flows. This current causes severe sparking at the brushes. In addition it opposes the main flux and so lowers the torque.

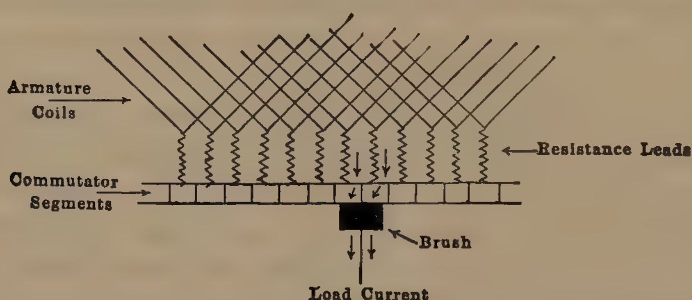


FIG. 283.—Resistance leads inserted to improve commutation.

The induced e.m.f. between commutator segments is reduced by using single-turn coils. To reduce the short-circuit current to a value as low as possible, resistance leads are often inserted between the armature coils and the commutator segments, as shown in Fig. 283. Such leads, by increasing the impedance of the short-circuited coil, reduce the short-circuit current. It will

be noted (Fig. 283) that so far as the *short-circuit* current is concerned, two such leads are in *series*, while so far as the *external* or *load* current is concerned, they are in *parallel*. This makes the resistance of these leads to the short-circuit current four times as great as it is to the load current. Except when starting, such leads are in the circuit but a small part of the time. If the starting period is too long, the leads in circuit at that time may overheat.

Reactances for reducing this transformer current have been suggested in place of resistances, but the difficulty of finding room for such reactances on a rotating armature has prevented their use. The induced e.m.f. per turn in the armature coil under-

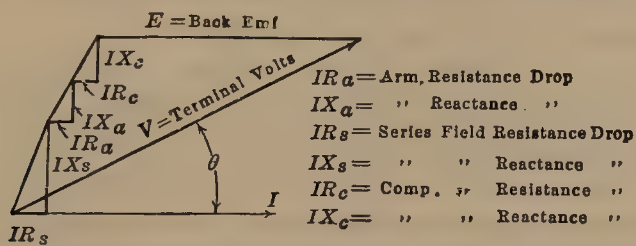


FIG. 284.—Vector diagram for alternating-current series motor.

going commutation is proportional to the flux per pole. In order to keep this voltage within allowable limits, the total flux per pole must be made as small as possible. The number of poles must be increased, therefore, in order that there may be sufficient total flux to develop the required torque. For this reason, an alternating-current series motor ordinarily has more poles than a corresponding direct-current motor.

In order to improve commutation still further, the voltage between commutator bars is kept down to a low value. This requires a large number of commutator segments and a correspondingly large commutator. The voltage between commutator segments is still further reduced by operating the motor at low voltage, usually not over 250 volts.

Figure 284 shows the vector diagram for this type of motor. The resistance drop IR_s, of the main field, is in phase with the current I. The reactance drop IX_s, of the main field, is in quadrature and leading the current I. IR_a and IR_c, the resistance drops of the armature and compensating field, are in phase

with the current. IX_a and IX_c, the reactance drops of the armature and compensating field, are in quadrature with the current and leading. The reactance drop of the series field is much greater than that of either the armature or the compensating field. The armature cross-magnetizing flux is not essential to the operation of the motor and as far as possible can be neutralized by the compensating field ampere-turns. In fact, the motor operation is improved by the reduction of this cross-magnetizing flux. On the other hand, the main flux is essential to the operation of the motor and cannot be reduced in value without reducing the torque per ampere. Hence, it is not practicable to neutralize the main flux, and consequently the series-field reactance drop must be large, even after the turns per pole, etc., have been reduced to a minimum.

When the alternating-field flux is at its maximum value, the armature conductors are cutting the maximum flux, and the back e.m.f. is, therefore, a maximum. When the field flux is at its zero value, the back e.m.f. is zero. The back electromotive force, therefore, is in time phase with the flux and practically in time phase with the current, as shown in Fig. 284.

The terminal voltage V is the vector sum of the back e.m.f. E and the IR and IX voltage drops in the series field, the compensating field, and the armature. The product of the back e.m.f. E and the current I is the power developed in the armature. The power at the pulley is less than this by the amount of the rotational losses. The cosine of the angle θ is the power factor of the motor. In order to have high power factor, the reactance drops must be low and the back e.m.f. high. The reactance drops are lowest and the back e.m.f. is highest at light loads, and, therefore, the power factor of the single-phase series motor is highest at light loads, as shown in Fig. 285. This is the reverse of the power-factor relations which exist in the induction motor and in the transformer.

The single-phase series motor has practically the same operating characteristics as the direct-current series motor. This is illustrated in Fig. 285, which gives the operating characteristics of a typical railway motor. The torque or tractive effort varies nearly as the square of the current, and the speed varies inversely as the current or nearly so.

If conductively compensated, the motor operates satisfactorily with direct current and at increased output and efficiency. When the motor is operated with alternating current, the speed may be efficiently controlled by taps on a transformer. This efficient speed control is not possible with direct current.

The single-phase series motor operates satisfactorily in railway work, notably on the New York, New Haven & Hartford Railroad. From New Haven to Harlem, the locomotives take power at 11,000 volts, 25 cycles, from an overhead trolley wire, by

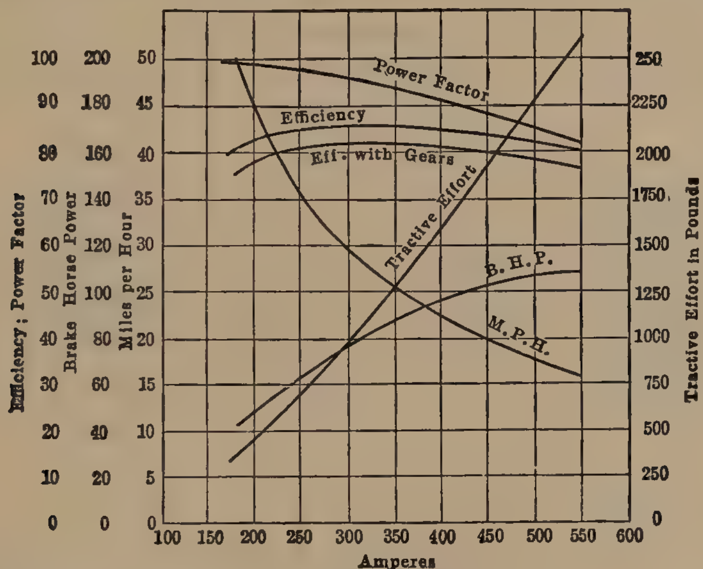


FIG. 285.—Characteristic curves of 430-amp., 235-volt, 25-cycle, single-phase Westinghouse railway motor. Continuous rating, 200 amp., 235 volts.

means of a pantograph trolley. An auto-transformer on the locomotive reduces this voltage to 250 volts, the rated voltage of the series motors. The electric locomotives run from Harlem into the Grand Central Station, New York City, over the New York Central 600-volt, direct-current system. The same motors are used for both direct-current and alternating-current service, the control devices switching over automatically when transition is made from one service to the other. The motors which operate at 250 volts each on alternating current are connected two in series for direct-current operation.

145. Repulsion Motor.—If an ordinary direct-current armature be placed in a single-phase magnetic field and the brushes

be short-circuited, a simple repulsion motor is obtained. In order to develop torque, however, the brush axis must be displaced from the axis of the main field by about 18 or 20 electrical space-degrees, as will be shown.

The principle of operation of such a motor is as follows: Figure 286 (*a*) shows a gramme-ring armature and its commutator. This is the same type of armature as would be used for a direct-current machine. This armature operates in a bipolar magnetic field, the field structure being laminated. The fields are excited by a winding connected directly to a single-phase alternating-current line. At the instant shown, the upper wire is positive, and the current is increasing in a positive direction.

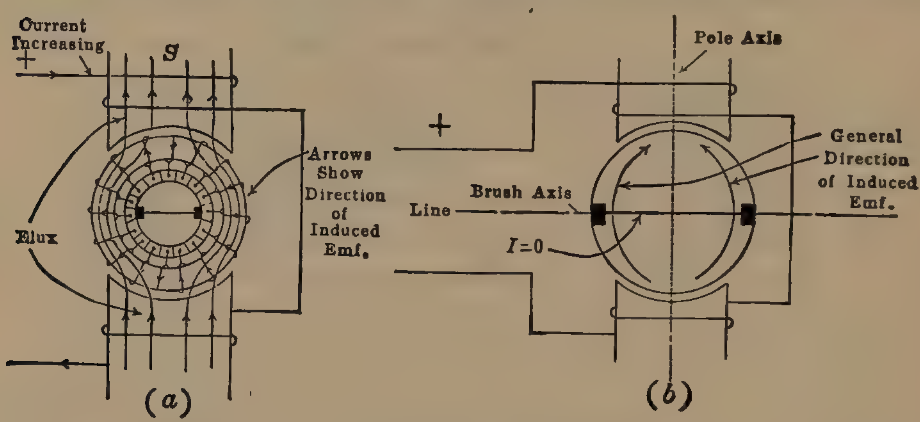


FIG. 286.—Currents and e.m.fs. in the windings of repulsion motor, brushes in geometrical neutral.

The flux which is substantially in phase with this current is also increasing and by the corkscrew rule is directed upward. This flux divides, half going through each side of the ring armature.

It is clear that the winding on each side of the ring armature acts as the secondary of a transformer. The alternating flux produced by the field winding, as primary, therefore, induces an e.m.f. in each half of the armature. By Lenz's law, this induced e.m.f. has such a direction as to oppose the inducing flux. The direction of this induced e.m.f. at the instant indicated in Fig. 286 (*a*) is given by the arrows on the windings. It will be noted, by following through the winding, that the resultant direction of this induced e.m.f. is *upward* in each side of the armature. This is indicated diagrammatically in Fig. 286 (*b*), where the arrows

show the general direction of these induced e.m.fs. through the armature. Were there no brushes, it is evident that no current would flow in the armature winding, as the e.m.f. in one-half of the winding is equal and in phase opposition to that in the other half.

In Fig. 286, the brushes are shown as being in the geometrical neutral and short circuited. Each brush is at the midpoint of its transformer winding. As the total e.m.fs. in each winding are the same and the windings are connected in parallel, each midpoint must be at the same potential. The brushes short circuit two points at the same potential, therefore, and no current flows between brushes.

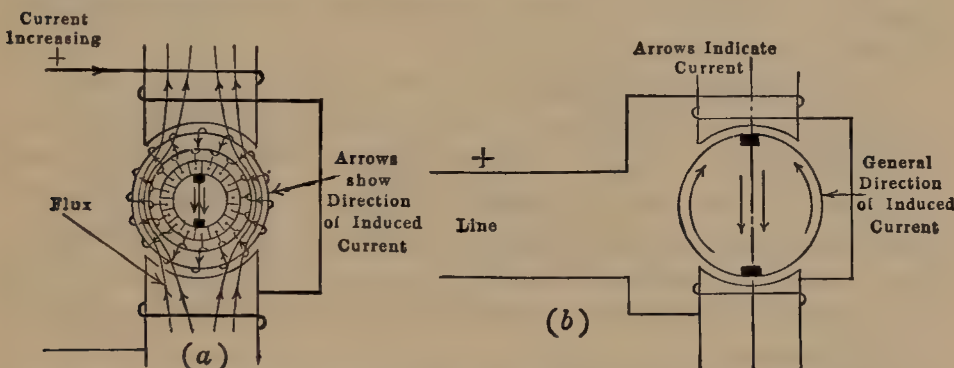


FIG. 287.—Currents in the windings of repulsion motor, brushes along pole axis.

It is clear that *without* brushes there is no armature current, and even *with* brushes there is no armature current, provided the brush axis is at right angles to the pole axis. Under both these conditions, therefore, there is no armature current, and, hence, no torque.

Figure 287 (a) shows the same condition existing in the field and armature as was shown in Fig. 286 (a), except that the brushes now lie along the pole axis. As the general direction of the induced e.m.fs. has not changed, the brushes are now short circuiting the points of the armature winding across which the maximum potential difference exists. Current, therefore, flows between the brushes from both sides of the armature, and in this brush position, the current in the armature is a maximum. But for the following reason the motor develops no torque with the brushes in this position: Two conditions are necessary for

the development of torque. *The angle between the space position of the flux axis and the brush axis must be greater than zero.* For maximum torque, this angle should be 90°. For example, in a direct-current motor with fixed flux and armature current, the maximum torque occurs when the brushes are in the neutral plane, that is, at right angles to the flux. No torque would be developed were the brush axis parallel to that of the flux.

There must be a component of the current in time phase with the flux (see Par. 133, p. 277). If there is 90° time lag between the current and the flux, the current is a maximum at the instant the flux is zero, etc., and the average torque is zero. With flux, armature current, and brush position all fixed, the maximum torque occurs when the flux and armature current are in time phase with each other.

Under the conditions shown in Fig. 287, the brush axis is parallel to the resultant flux. That is, the angle between the flux and the brush axis is zero. A consideration of Fig. 287 (a) shows that the current flows in opposite directions in the two equal conductor belts on each side of the brush axis. Although it can be shown that the armature current is nearly in time phase with the flux, no torque is developed because of the space position of the brushes.

Hence, in this type of motor, no torque is developed when the brush axis is at right angles to the flux, for then there is no current; no torque is developed when the brush axis is parallel to the flux, because the ampere-conductors under each pole develop opposite and equal torques.

It is obvious, however, that if the brushes be placed in some intermediate position, they will be short circuiting points of the winding between which a difference of potential exists, and, therefore, currents will flow in the winding, and, also, the net ampere-conductors under each pole cannot be zero. It can be shown by a close analysis that the armature current is substantially in time phase with the flux. Under these conditions, therefore, the motor develops torque, and, if allowed to do so, the armature will rotate.

Figure 288 (a) shows the brush axis making an angle α with the pole axis. The arrows in this figure show the direction of the armature *current* at the instant when the upper wire is posi-

tive and the current is increasing positively. Figure 288 (*b*) shows diagrammatically the general direction of these currents through the armature and brushes. It will be observed that the current direction in the conductors under each pole is such as to develop torque. Figure 288 (*c*) shows the direction of the induced *e.m.fs.* in the armature, neglecting the distorting effect of the armature m.m.f. on the field flux. The e.m.fs. in each half of the armature act in conjunction, as shown in Fig. 286 (*b*). Assume for the time being that angle β equals angle α (Fig. 288 (*c*)). The *current* paths through the winding are *abcd* and *afed*. In path *abcd*, the e.m.fs. E_{cd} and E_{cb} included in angles α and β,

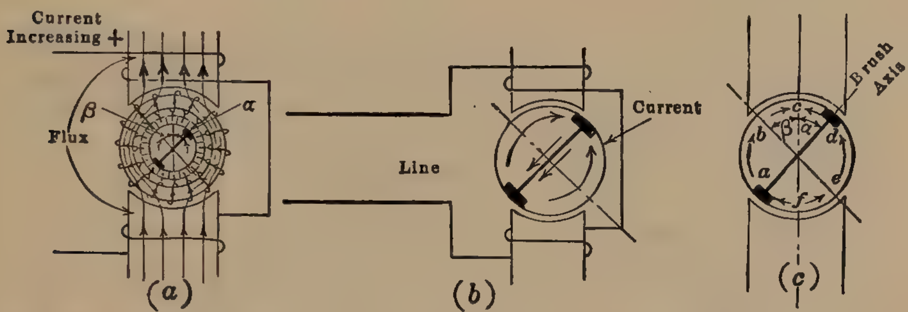


FIG. 288.—Brush position in repulsion motor which gives both current and torque.

respectively, each equal to the brush displacement angle, are equal and act in opposition. They, therefore, cancel each other, leaving E_{ab} as the net e.m.f. through path *abcd*. Likewise, in path *afed*, the e.m.fs. E_{fa} and E_{fe} cancel, leaving E_{ed} as the net e.m.f. through this path. The net e.m.fs. E_{ab} and E_{ed} are effective in sending the current through the armature.

The foregoing is not a rigorous analysis of repulsion motor operation but rather a statement of the general principles on which the operation depends. A rigorous analysis involves vector diagrams of considerable complexity and is beyond the scope of this book.[1]

In this type of motor, the direction of rotation depends upon the brush position. For example, in Fig. 288 (*c*), the direction of rotation may be reversed by moving the brushes so that they

[1] For more detailed analysis of single-phase motors, see R. R., LAWRENCE, "Principles of Alternating Current Machinery," McGraw-Hill Book Company, Inc.

cross the pole axis, the brush axis then making an angle β with the pole axis. Angle β must be less than 90°.

In the foregoing discussion, a gramme-ring winding has been considered, as it is a simple matter to follow the winding, since the conductors do not cross one another, etc. It is well known, however, that a drum winding, for the same number of poles, has the same electrical characteristics as the gramme-ring winding. The preceding analysis applies equally well to a drum-wound armature. Also, the foregoing principles apply to motors of more than two poles. Figure 289 shows the brush positions for a four-pole motor.

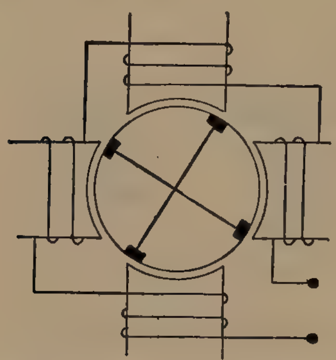


Fig. 289.—Four-pole repulsion motor.

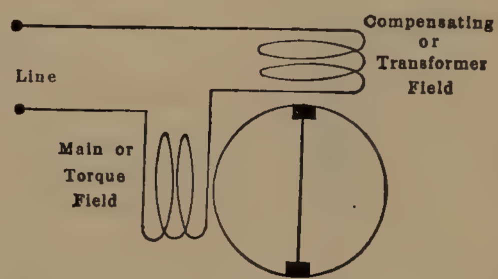


Fig. 290.—Two-pole repulsion motor with compensating or transformer field.

Instead of displacing the brushes from the geometrical neutral so that a potential difference exists between them, which results in a current, giving rise to torque, the same effect may be obtained by using two field windings displaced at right angles to each other, as shown in Fig. 290. A compensating or transformer field, acting along the brush axis, induces e.m.fs., which, in turn, cause currents, shown in Fig. 287, and these currents react with the flux of the main field winding to produce torque. This type of motor should not be confused with the four-pole type of Fig. 289.

Practically all repulsion motors are made with non-salient poles, rather than with the salient poles shown in the diagrammatic illustrations just given. The windings are usually of the distributed type, such as are used for induction motors. The fact that the reluctance to the main-field flux and to the trans-

former-field flux must be kept as low as possible makes it desirable to use non-salient poles and to make the air-gap as short as possible. Otherwise, the magnetizing currents for these fields will be high, lowering the power factor.

Repulsion motors have characteristics similar to those of series motors and have large starting torque. The sparking is very small at synchronous speed (3,600 r.p.m. for a two-pole, 60-cycle motor), but at speeds differing greatly from this, the sparking may be excessive. It will be noted that the motor of Fig. 290 is similar to the inductively compensated series motor of Fig. 281, with the connections of the compensating winding

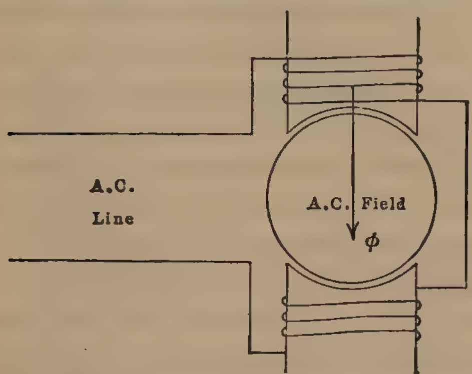


Fig. 291.—Single-phase, alternating field.

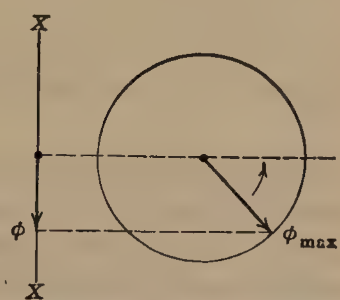


Fig. 292.—Time variation of single-phase alternating field.

and of the armature interchanged. There are several types of repulsion motor on the market which, while differing in detail from the motor just described, involve identical principles.

146. Single-phase Induction Motor.—Figure 291 shows a two-pole motor whose magnetic field is produced by single-phase current flowing in a simple field winding. The current in this field is assumed to vary sinusoidally with time, and if the iron be assumed to operate at moderate flux densities, the flux through the armature will vary practically sinusoidally with time. The variation of this field with time may be represented by the projection of a rotating vector ϕ_{max} upon a vertical axis XX, shown in Fig. 292. The vector ϕ_{max} is equal to the maximum value of the flux, and its speed of rotation in r.p.s. is equal to the line frequency in cycles per second.

It may also be assumed that this single-phase field is made up of two equal and oppositely rotating fields represented by two equal

and oppositely rotating vectors (Fig. 293 (*a*)), the maximum value of each of these fields or vectors being equal to one-half ϕ_{max}. The resultant of two such vectors always lies along the vertical axis and is equal in magnitude at any instant to the field actually existing at that instant. The same thing is represented in Fig. 293 (*b*), which shows the flux distribution curves of two fields ϕ_1 and ϕ_2, each of which is equal to one-half the maximum field. These two fields glide around the air-gap in opposite directions and with equal velocities. Their algebraic sum ϕ at any instant is the value of the resultant field at that instant, and this resultant field is *stationary in space*.

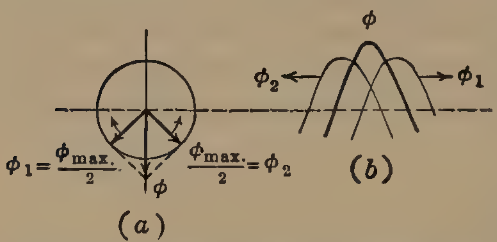


Fig. 293.—Representation of single-phase alternating field by two oppositely rotating fields.

The single-phase field may be considered, therefore, as made up of two equal rotating fields, revolving in opposite directions. (Experiment shows that two such fields actually exist.) Each field acts independently upon the rotor and in the same manner as the rotating field of the polyphase induction motor. One field tends to cause rotation in a clockwise direction, and the other field tends to cause rotation in a counterclockwise direction. Figure 294 shows the slip-torque curve due to each of the two fields. The torques act in opposite directions, as shown. At standstill (slip = 1), the two torques are opposite and equal, and the rotor has no tendency to start. If the rotor in some manner be caused to rotate in the direction in which the torque T_1 is acting, T_1 will immediately exceed the countertorque T_2, and the armature will begin to accelerate in the direction of T_1. As the armature speeds up, T_1 predominates more and more

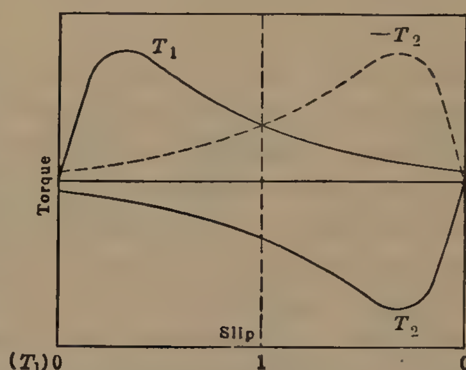


Fig. 294.—Two opposing torques in single-phase induction motor.

over T_2, and the armature approaches synchronous speed without difficulty. The countertorque due to T_2 always exists, however, although it has little effect near the synchronous speed of the field which produces T_1.

When the rotor operates near synchronous speed and rotates in the direction of T_1, its slip is nearly two, as regards T_2. The rotating field which produces T_2, therefore, induces double-frequency currents in the rotor at this speed. These double-frequency currents, however, produce but little torque because of their high frequency. This frequency is practically double the stator frequency. The rotor reactance, therefore, is many times its value at slip frequency. Consequently, these currents are small in magnitude and make a considerable space-angle with the air-gap flux, developing little countertorque (see Par. 133, p. 277).

It is obvious that the single-phase induction motor rotates in the direction in which it is started.

147. Reactions in a Single-phase Induction Motor.—Although the foregoing treatment of the single-phase induction motor gives some idea of its method of operation, it is not a rigorous analysis, nor does it give a physical conception of what actually occurs in the motor.

The reactions occurring in the rotor of a single-phase induction motor are not simple, and several factors must be considered if an exact analysis is to be made. At the instant shown, in Fig. 295, the direction of the main flux ϕ_M due to the stator winding is down into the armature from the north pole N and is increasing positively. In so doing, it links the rotor conductors, and, due to *transformer* action, currents are induced in these rotor conductors. These induced currents in the rotor conductors must flow in such a direction as to *oppose* this flux in the same manner as the secondary ampere-turns of any static transformer oppose the primary ampere-turns. The effect of the rotor conductors is the same as if they were connected as shown in Fig. 295, each conductor being connected with one on the opposite side of the armature to form a closed turn. To oppose the flux ϕ_M, the current must be flowing inward on the right-hand side of the armature and outward on the left-hand side of the armature, as indicated in the figure.

Assume that the armature rotates in a clockwise direction. There will be an e.m.f. induced in the rotor conductors due to their cutting the flux ϕ_M. This induced e.m.f. is called the *speed e.m.f.*, because it is induced entirely by the cutting of the flux ϕ_M due to rotation. Applying Fleming's right-hand rule, this e.m.f. acts inward on the upper half of the armature and outward on the lower half, as shown in Fig. 296. This e.m.f. is alternating and is a maximum when ϕ_M is a maximum. As the rotor conductors are short circuited upon themselves, alternating currents flow in them as a result of this induced e.m.f. The rotor reactance being high as compared with its resistance, these cur-

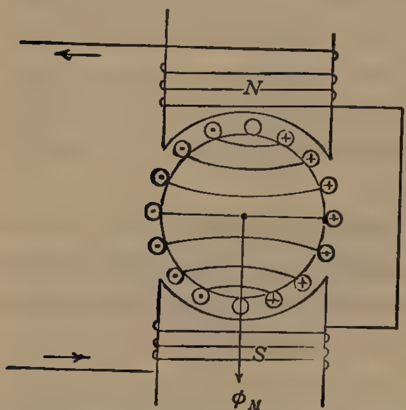


FIG. 295.—Transformer currents in rotor of single-phase induction motor.

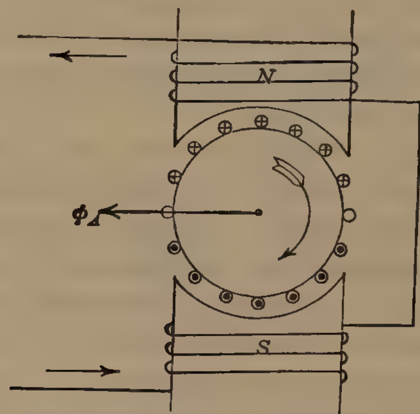


FIG. 296.—Speed current and resulting flux in rotor of single-phase induction motor.

rents lag the induced e.m.f. by very nearly 90°. These currents, moreover, produce a flux ϕ_A, at right angles to ϕ_M, as shown in Fig. 296, just as the ampere-conductors of a direct-current motor produce a field at right angles to the pole axis when the brushes are in the geometrical neutral. In practice, the stator completely surrounds the rotor, the air-gap being uniform. At synchronous speed, ϕ_A is substantially equal to ϕ_M but is 90° from ϕ_M in space.

The speed e.m.f. E_A is obviously a maximum when ϕ_M is a maximum. The current I_A does not reach its maximum until nearly 90° later in time, because the rotor reactance is high as compared with its resistance. In Fig. 295, ϕ_M is shown as having reached its maximum and acting vertically downward. After a quarter-period, ϕ_A reaches its maximum and is acting 90° in space from the flux ϕ_M, as shown in Fig. 296. It will be

recognized that two such fields, acting along axes 90° from each other in space and differing in time phase by an angle of 90°, will produce a rotating magnetic field. This field rotates clockwise in Figs. 295 and 296. As the rotor slip increases, ϕ_A decreases in magnitude because the speed is reduced. The horizontal field, therefore, becomes less than the vertical field, and a so-called *elliptical* field results.

At standstill, ϕ_A is zero, and the rotating field becomes a pulsating field, which has already been described.

It might be supposed that the above rotating field would react on the rotor in the same manner as the rotating field in the polyphase induction motor. Since ϕ_A originates in the armature and also because of its quadrature position, it cannot of itself react on the stator to cause a power current to flow in the stator, and, therefore, it cannot of itself contribute power to the rotor. Due, however, to the resultant rotor currents produced by the combined action of the two fields ϕ_A and ϕ_M, it can be shown that the resulting torque acting on the rotor under the above conditions acts in a clockwise direction and so produces rotation.

148. Operation of Polyphase Motor as Single-phase Motor.— The single-phase induction motor is distinctly inferior to the polyphase motor. For the same weight, its rating is about 50 per cent. of that of the polyphase motor; it has a lower power factor and is less efficient.

If one phase of a polyphase motor be opened, the motor will operate as a single-phase motor, although it will not start under these conditions. The rating and the breakdown torque of a polyphase motor, operating single-phase, are considerably reduced, and if rated polyphase load is applied continuously, the motor may overheat.

Ordinarily, in starting a polyphase motor, all three lines are closed when the compensator is in the starting position, and the motor starts as usual. When the compensator is thrown to the running position, however, a phase may become open through the compensator. This would occur if one of the fuses were blown (Fig. 262, p. 291). The motor then operates single-phase, and the only indication that it may give of this condition is overheating if the load is near the rated value. The best test for an open phase is to insert an ammeter in each line.

149. Starting Single-phase Induction Motors.—As the single-phase induction motor is not self-starting, auxiliary means must be used to supply initial torque. One method is to split the phase by the use of inductanee, resistance, or capacitance.

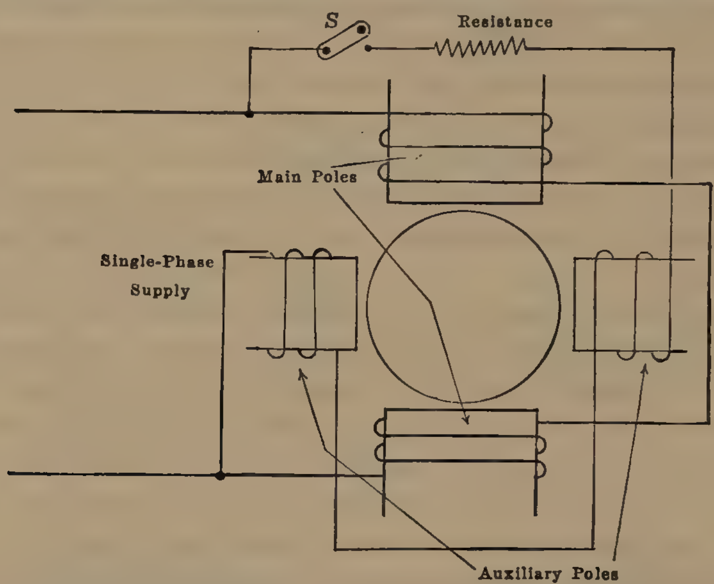


FIG. 297.—Split-phase method of starting single-phase induction motor.

Figure 297 shows one method of splitting the phase, a two-pole motor being shown. The main winding, which is highly inductive, is connected across the line in the usual manner. Between the main poles are auxiliary poles which have a high-resistance

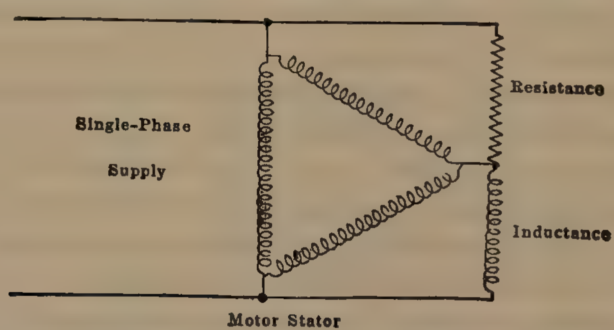


FIG. 298.—Splitting the phase with resistance and inductance.

winding, and this winding is also connected across the line. As the auxiliary winding has a high resistance, its current will be more nearly in phase with the voltage than the current in the main winding. For the best conditions, the two currents should

differ in phase by 90°, but this condition is not readily obtainable and, in fact, is not necessary. These two sets of poles produce a sort of rotating field which starts the motor. When the motor comes up to speed, a centrifugal device in the rotor opens the switch S and disconnects the auxiliary winding.

Another method of splitting the phase is to use a three-phase winding, as shown in Fig. 298, and to connect resistance and inductance as shown. Resistance and capacitance may also be used. Either a delta-connected stator or a Y-connected stator may be used. The resistance and inductance, when connected as shown, displace the phase relations of the currents in the different phases of the stator with respect to one another and so produce a sort of rotating field. All these phase-splitting devices produce an elliptical rotating field. Because of the characteristics of the field combined with the squirrel-cage characteristics of the rotor, the resulting torque is barely sufficient to start the motor, even without load.

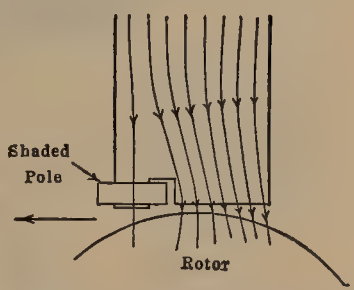


FIG. 299.—Shaded-pole method of starting single-phase induction motor.

The *shaded-pole method* is shown in Fig. 299. A short-circuited coil of low resistance is connected around one pole tip. When the flux is increasing in the pole, a portion of the flux attempts to pass down through this shaded tip. This flux induces a current in the coil which, by Lenz's law, is in such a direction as to oppose the flux entering the coil. Hence, at first, the greater portion of the flux passes down the right-hand side of the pole, as shown in Fig. 299. Ultimately, however, the main flux reaches its maximum value, where its rate of change is zero. The opposing e.m.f. in the shading coil then becomes zero, and later the opposing m.m.f. of the short-circuited coil ceases, the current in this coil lagging its e.m.f. Considerable flux then penetrates the short-circuited coil. After the main flux begins to decrease, the induced current in the shading coil tends to prevent the flux then existing in the shaded portion of the pole tip from decreasing. The flux first reaches its maximum value, therefore, at the right-hand or non-shaded side of the pole and

later reaches its maximum at the left-hand or shaded side. The effect of the shading coil is to retard in time phase a portion of the flux, so that there is a sweeping of the flux across the pole

FIG. 300 (a).—Rotor of Wagner single-phase, type BA motor.

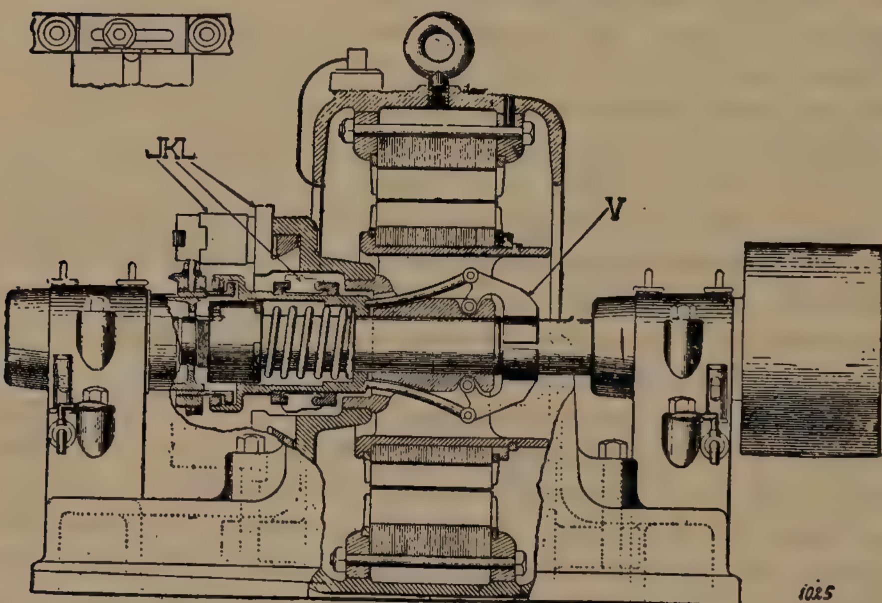


FIG. 300 (b).—Wagner single-phase induction motor.

face from the right-hand to the left-hand side in the direction of the shading coil. This flux cutting the rotor conductors induces currents, which, in turn, produce a torque sufficient to start the motor. The shaded pole is not a common method of starting single-phase induction motors and is used only in motors of very small size.

It will be remembered that this same shaded-pole principle is used as the light-load adjustment in the induction watthour meter (see p. 90).

The preceding metnods of starting the single-phase induction motor produce very weak starting torques, which are insufficient to start the motor except under the lightest loads. The Wagner single-phase induction motor starts as a repulsion motor and has a large starting torque. A cross-section of the motor and a view of the armature are shown in Fig. 300. The armature is similar to the type used in the ordinary direct-current motor, except that the brushes J press on the end of the commutator L rather than radially on its surface. These brushes are short circuited on themselves and are set in a position corresponding to those in Fig. 288 or Fig. 289 (pp. 329 and 330), so that the motor starts as a repulsion motor. It has a large starting torque and comes up to speed rapidly. As it approaches synchronism, a centrifugal device V (Fig. 300 (b)) is thrown outward and pushes the brushes away from the commutator, while, at the same time, a metal ring K presses against the commutator bars on the inside and short circuits them. The motor now operates as a single-phase induction motor.

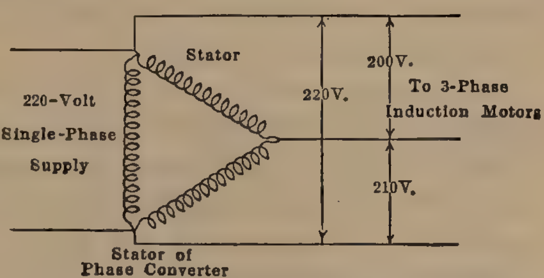


FIG. 301.—Method of obtaining three-phase power from single-phase supply, by means of squirrel-cage induction motor operating as phase converter.

150. Induction Motor as Phase Converter.—If a three-phase induction motor be operated single-phase, as shown in Fig. 301, three-phase voltages exist across its three terminals. The reason for this is as follows:

The back e.m.f. in each phase of a polyphase induction motor is induced by the rotating field cutting the stator conductors. If the stator is wound for two-phase, the induced e.m.fs. at the stator terminals are two-phase; if the stator is wound for three-phase, the induced e.m.fs. at the stator terminals are three-phase. The induced e.m.f. in each phase of a polyphase induction motor is slightly less than the terminal voltage (per phase) by the amount of the stator impedance drop.

It was shown in Par. 147 (p. 333) that, in a single-phase induction motor, a rotating field exists. At small values of slip, this field departs but slightly from a true rotating field such as is produced by polyphase currents in polyphase windings. When a single-phase voltage is applied to one phase of a two-phase or of a three-phase motor, therefore, the rotating field is almost identical with that which exists when polyphase voltages are applied to the terminals. Consequently, if a single-phase voltage be applied to one phase of a two-phase stator, a quadrature e.m.f. exists across the terminals of the other phase. If a single-phase voltage be applied across one phase of a three-phase stator, the voltages across the three terminals will very nearly equal one another and will be approximately 120° apart. As the induced e.m.fs. are less than the applied terminal voltage by the amount of the stator impedance drop, and as the rotating field is somewhat elliptical, the terminal voltages will not be exactly balanced. For example, in Fig. 301, 220-volts, single-phase, are applied across one phase of a three-phase motor, and voltages of approximately 210 and 200 volts are found to exist across the other two phases.

Polyphase induction motors are often used in this manner to produce polyphase voltages from single-phase supply. That is, single-phase voltage is supplied to one phase of the polyphase stator, and polyphase voltages are obtained from the stator terminals. When so used, the motor is called a *phase converter*.

The phase converter is used to some extent in railway electrification. Although the three-phase induction motor is adapted to railway work, there is considerable disadvantage in using the two trolleys which are required if three-phase power is to be supplied to the locomotive. By using a phase converter, the advantages of the three-phase motor for driving may be secured, and at the same time all the advantages of a single trolley are retained. The phase converter receives single-phase power, which is pulsating, and delivers three-phase power, which is substantially steady. This is made possible by the kinetic energy stored in the rotating armature of the phase converter, this energy supplying the power during those times when the single-phase power is negative or is less than the average value of the polyphase power. The armature accelerates and so

stores kinetic energy during the periods when the single-phase power exceeds the average power. The armature slows down and so gives up some of its kinetic energy during the periods when the single-phase power is less than the average power. In practice, the actual speed variations of the armature are slight.

The electric locomotives of the Norfolk and Western Railway are operated by the use of a phase converter. A two-phase converter is used, as only half the power need be converted under

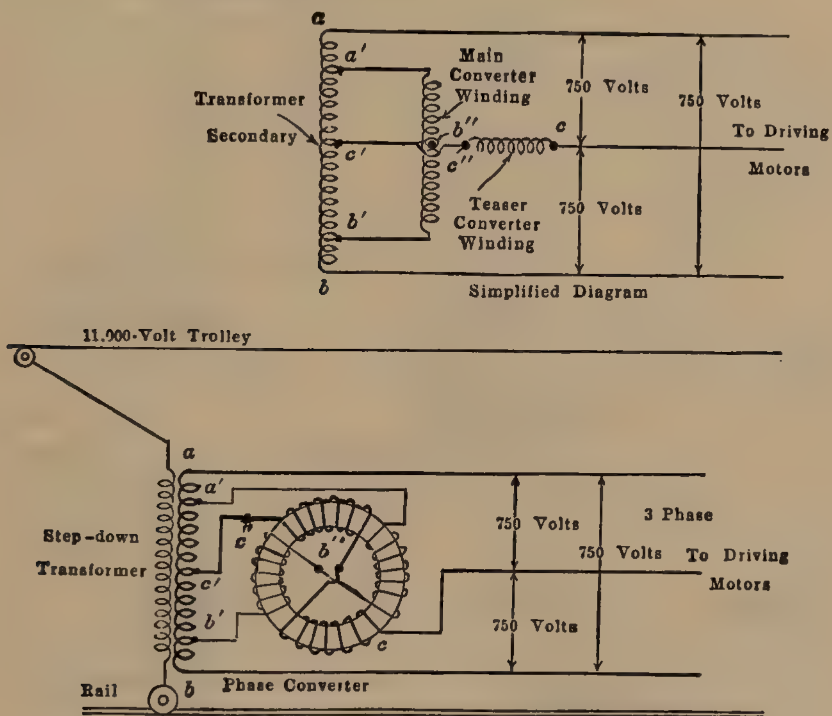


FIG. 302.—Connections of locomotive phase converter.

these conditions, the other half flowing conductively from the transformer secondary to the motors. The power is received single-phase from an 11,000-volt trolley and stepped down by a transformer on the locomotive. Special transformer taps are used to keep the phases balanced. The general diagram of connections is shown in Fig. 302. It will be recognized that the converter and transformer connection is equivalent to a T-connection. This is used in order that three-phase power may be obtained by supplying single-phase power to the two-phase

stator of the converter. The phase ab to the driving motors is supplied directly from the transformer. The winding $a'b''b'$, tapped to winding ab, is the main winding of the phase converter (see Fig. 232, p. 257). The winding $c'c''c$ is the teaser winding tapped to the transformer at c', giving the third wire c of the three-phase system. Ordinarily, the teaser winding would be tapped to point b'', the center of the main converter winding.

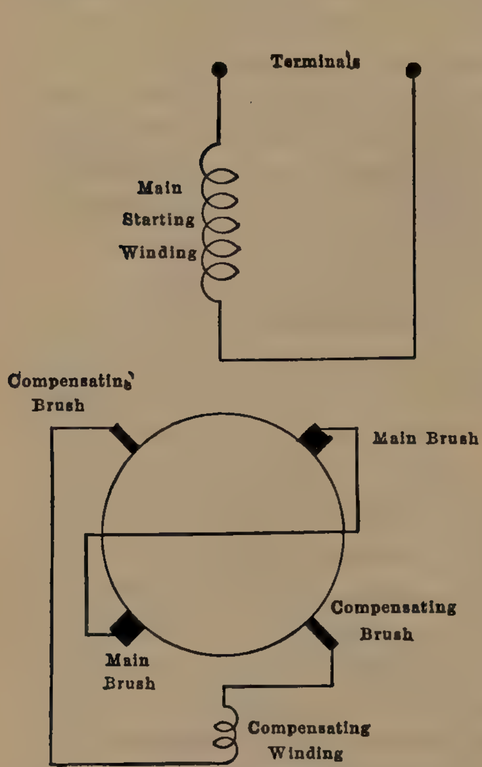


FIG. 303.—General Electric repulsion-induction motor with independent compensating circuit.

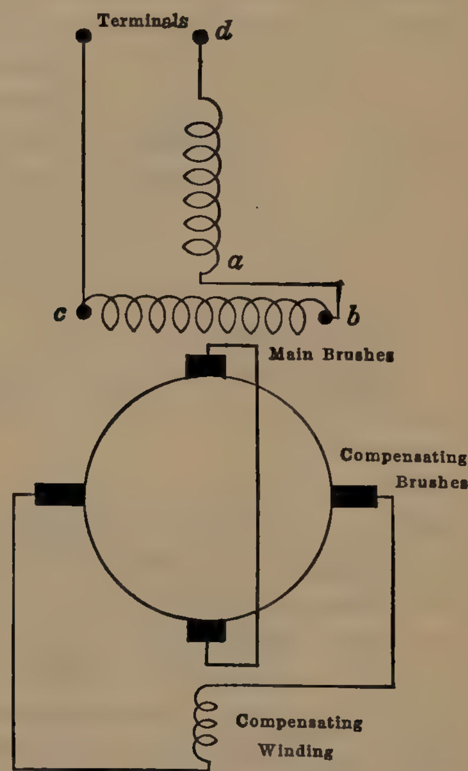


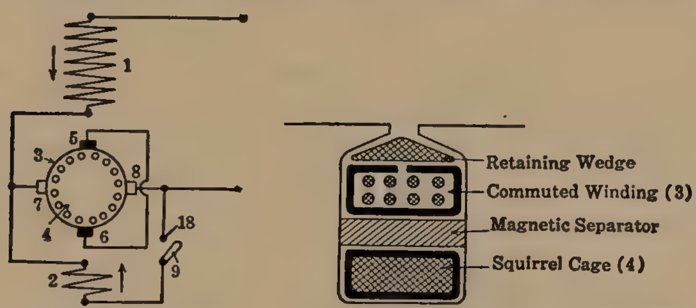
FIG. 304.—General Electric reversible repulsion-induction motor.

For convenience, however, the teaser winding is tapped to point c', the center of the transformer winding, instead. But under balanced conditions, c' and b'' are at practically the same potential, so that, as far as voltages are concerned, connecting the teaser winding to c' is equivalent to connecting it to b''.

151. Repulsion-induction (R I) Motor.—This motor is similar in principle to the simple repulsion motor of Fig. 290 (p. 330),

except that a compensating winding is used, supplied by auxiliary brushes at right angles to the main brushes, as shown in Fig. 303. To reverse the motor, an auxiliary winding *bc* is used (Fig. 304) at right angles to the main winding. The points *b* and *c* are interchanged for reversing. This motor has a starting torque of from 200 to 250 per cent. full-load torque and has a drooping speed characteristic similar to that of a compound, direct-current motor. By using a transformer and by shifting the brushes, considerable speed variation may be obtained.

152. Wagner Type BK, Unity Power-factor Motor.—The Wagner Electric Company has developed a single-phase motor which has a leading power factor up to about half-load, while,



1. Initial field magnetization winding.
2. Auxiliary winding which controls power-factor.

(*a*)

(*b*) One slot of rotor, showing construction.

FIG. 305.—The Wagner Electric Company's unity power-factor, type BK motor.

for greater loads, the power factor is almost unity. The motor is a constant-speed motor, the regulation being from 1.5 to 4 per cent. There are two windings on the motor armature, as is indicated by the slot section in Fig. 305 (*b*). The upper winding is an ordinary drum winding connected to a commutator. Beneath this is a squirrel-cage winding separated from the other by a magnetic separator consisting of a steel wedge.

The diagram of connections is shown in Fig. 305 (*a*); 1 is the main winding, and 2 the compensating winding. When the motor starts, the switch 9 is open, and as the squirrel cage has little effect at starting, due to the screening effect of the magnetic wedges, the motor starts as a series motor. As it approaches synchronism, the squirrel cage has more and more effect. As

synchronism is approached, switch 9 closes by centrifugal action and throws in the compensating winding. The manufacturers claim many advantages for this type of motor, such as high power factor, constant speed; light service required of the commutator because of the assistance of the squirrel cage, suppression of short-circuit coil currents due to the proximity of the squirrel cage, etc.

CHAPTER XI

THE SYNCHRONOUS MOTOR

153. Synchronous Motor.—It will be remembered that the direct-current generator operates satisfactorily as a motor. There is practically no difference, moreover, in the construction of the direct-current generator and the direct-current motor, and there is no substantial difference in the rating of a machine whether it is operated as motor or as generator.

Similarly, an alternator will operate as a motor without any changes being made in its construction. When so operated, the machine is called a *synchronous motor*.

The design of a synchronous motor and of an alternator, each of the same rating and speed, may differ somewhat in details, owing to the desirability of securing the best operating characteristics for each. Except, moreover, in special high-speed, two-pole types, synchronous motors are almost always salient-pole machines, whereas alternators may be either of the salient-pole or of the non-salient-pole type.

154. Principles of Operation.—Figure 306 shows a conductor *a* under a north pole and carrying a current flowing towards the observer. By the well-known law of motor action, a torque develops, tending to drive the conductor from left to right. If the current be alternating, it will reverse its direction for the next half-cycle, and the torque then acts from right to left. The net torque over any given number of complete cycles, therefore, is zero, and no continuous motion can result. This is the condition existing in a synchronous motor when at standstill. The armature conductors carry alternating current, and the poles have

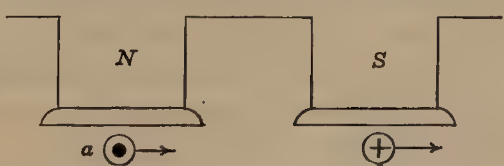


Fig. 306.—Torque developed by synchronous motor.

fixed polarity, being excited with direct current. The synchronous motor, as such, therefore, develops no starting torque.

If, however, conductor a can in some manner be brought under the next pole, which is a south pole, for the half-cycle during which the current is in the reverse direction, the resulting torque will still be from left to right, and a tendency toward continuous motion will result. In a synchronous motor, therefore, a given conductor must move from one pole to the next in each half-cycle, if the machine is to operate continuously. This applies to the rotating-armature type of machine. If the machine is of the rotating-field type, any given conductor must be passed by one pole every half-cycle. In any event, the synchronous motor must operate at constant speed, if the frequency is constant. There may be momentary fluctuations of speed, but if the *average* speed differs by even a small amount from this constant value, the average torque will ultimately become zero and the motor will come to a standstill. The relation of speed, number of poles, and frequency is the same as for the alternator and for the rotating field of the induction motor. That is, the speed $S = 120\ f/P$ r.p.m., where f is the frequency and P the number of poles (see Pars. 3 and 131, pp. 5 and 274).

Example.—A 500-kv-a., 2,300-volt, 10-pole synchronous motor operates on a 60-cycle three-phase system. What is its speed?

$$S = \frac{120 \times 60}{10} = 720 \text{ r.p.m.} \quad \textit{Ans.}$$

155. Effect of Loading Synchronous Motor.—If a load be applied to a direct-current shunt motor, the speed is slightly decreased. This reduces the back e.m.f. $-E$. The line must supply a voltage $+E$, equal and opposite to the back e.m.f. $-E$ and, in addition, must supply the voltage necessary to overcome the IR_a drop in the armature.

That is,

$$V = E + IR_a$$

where V is the fixed terminal voltage, I the armature current, and R_a the armature resistance.

The current

$$I = \frac{V + (-E)}{R_a} = \frac{V - E}{R_a}.$$

When the back e.m.f. $-E$ decreases, more current I flows into the armature. This increased current supplies the extra torque and power required by the increased load.

When load is applied to a synchronous motor, its *average speed* cannot decrease, since the motor *must* operate at constant speed.

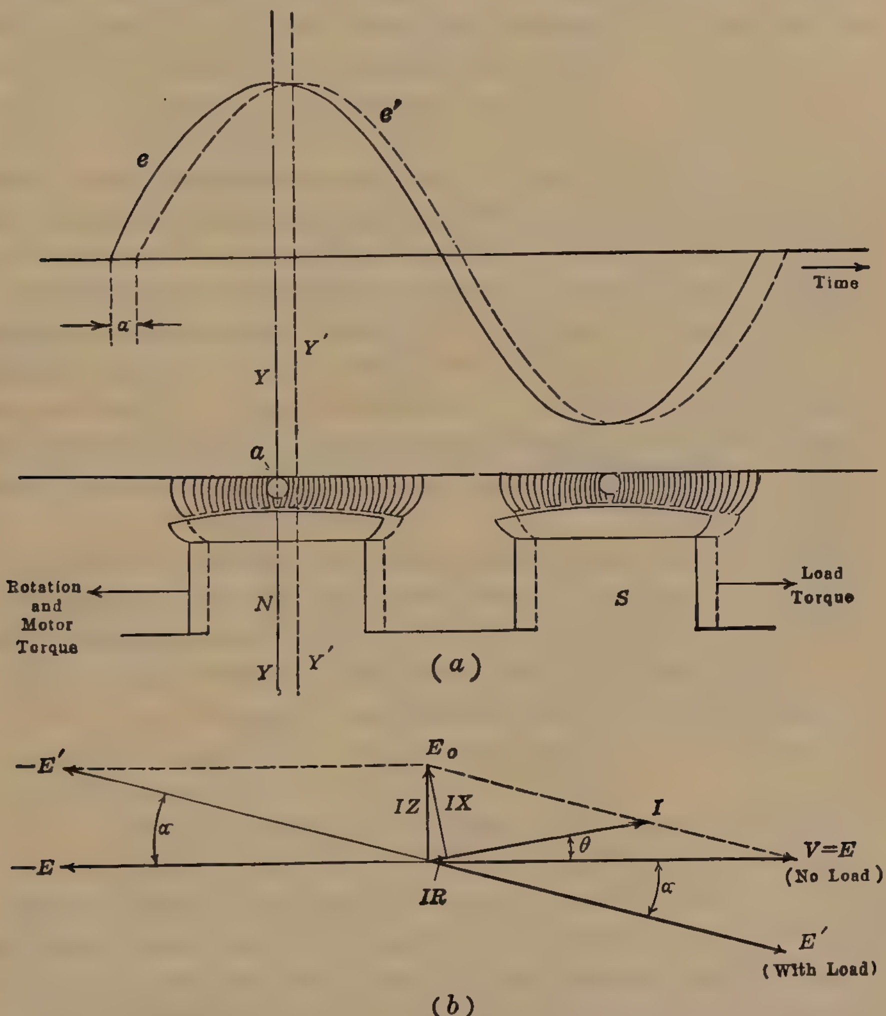


FIG. 307.—Effect of load on phase of induced e.m.f. in synchronous motor.

It cannot draw the increased current from the line in the same manner that the shunt motor does, that is, by operating at decreased speed. Figure 307 (*a*) shows two poles of a rotating-field type of synchronous motor. Neglecting any flux distortion, the e.m.f. induced in conductor *a* is a maximum when

conductor a is opposite the center of a pole. It is zero when the pole reaches such a position that conductor a lies midway between the poles. The value of this e.m.f. e for any position of the pole axis Y-Y is shown by curve e.

Assume that a load is now applied to the motor shaft. This must result in momentary slowing down of the rotor, since it requires time for a motor to take increased power from the line. The rotor, therefore, instead of being in the position shown by the solid lines in Fig. 307 (a), will occupy a given position in space at a later time on account of the effect of the load torque. The relations under this condition are shown by the dotted lines. Because of the application of load, the pole center is now at $Y'Y'$ instead of being at YY. The induced e.m.f. will not reach its maximum value at the same instant, therefore, that it would have reached it had no load been applied. This maximum value now occurs later in time, due to the slight backward angular displacement of the rotor. This is shown by a new curve of induced e.m.f. e', lagging e by an angle α where e is the e.m.f. which would have been induced had no load been applied to the rotor shaft.

This is further illustrated by the use of vectors. Assume that the motor is running without load and that the current is so small that the back e.m.f. $-E$ (Fig. 307 (b)) is sensibly equal to the terminal voltage V and is 180° out of phase with V. (E is the component of the terminal voltage necessary to balance the back e.m.f. $-E$.) The vector sum of V and $-E$ is zero, practically.

Now apply load. The terminal voltage V is assumed to be constant and so is not affected by the load. The induced or back e.m.f. $-E$ will be shifted backward by an angle α because of the backward angular displacement of the rotor caused by the load. Let this new value of back e.m.f. be $-E'$, and let the component of terminal voltage necessary to balance it be E'. The vector sum of V and $-E'$ is no longer zero. A vector *difference exists*, therefore, between V and E'.

In the direct-current motor, the armature current is given by dividing the armature resistance into the difference between the terminal voltage and that component of the terminal voltage E which balances the back e.m.f. $-E$. In the synchronous motor,

the armature current is given by dividing the armature impedance Z into the *vector* difference between the terminal voltage V and the e.m.f. E'.

That is,

$$I = \frac{V - E'}{Z} = \frac{E_o}{Z}. \tag{117}$$

Where E_o is the vector difference of V and E'.

Therefore,

$$E_o = IZ.$$

The above equation for the armature current in the synchronous motor is similar to the equation for the armature current in the direct-current motor (see Vol. I, p. 373, Eq. 127).

As a rule, the reactance of the armature of a synchronous machine is high as compared with its resistance, and the current I lags the voltage E_o which produces it by nearly 90°. This brings the current I very nearly in phase with E' and nearly 180° from the back e.m.f. $-E'$. I, therefore, is largely energy current with respect to $-E'$, which means that it supplies considerable internal *power* to the motor.

The rotor, by *shifting its phase backward when load is applied, causes the motor to take an energy current from the line which supplies the power demanded by the increased load.*

The total power supplied to the motor per phase is

$$P = VI \cos \theta.$$

The total mechanical power developed is

$$P' = E'I \cos (\theta + \alpha).$$

The *net power at the pulley* is less than P' by the amount of the frictional losses and the rotational core losses.

The difference between P and P' is the armature copper loss.

It should be remembered that the *average* motor speed remains constant. The rotor merely takes an angular position slightly back of its no-load position, without altering its average speed. This angular displacement of the rotor may be observed by means of a stroboscope (see p. 313).

156. Effect of Increasing Field Excitation.—When the field of a direct-current shunt motor is strengthened, there is a

temporary increase in the armature induced e.m.f. This decreases the armature current, and the torque is lowered, since the change in armature current is much greater than the corresponding change in the field. As a result, the motor slows down, and its back e.m.f. accordingly decreases. The armature current then increases until it is again of sufficient magnitude to enable the motor to carry the load.

When the field of a synchronous motor is increased, the motor cannot slow down, except momentarily, for it must run at constant average speed. Since its speed is constant, its back e.m.f. must increase when the field is strengthened. It might seem then that the motor would stop, for its induced e.m.f. must

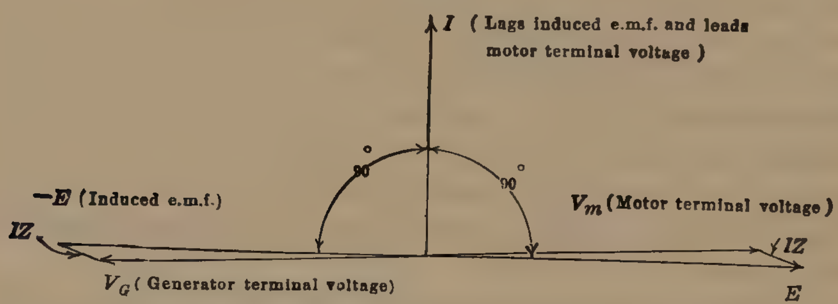


FIG. 308.—Relation of current to voltage in motor and in generator.

apparently become greater than its terminal voltage. In the direct-current motor, an induced e.m.f. exceeding the terminal voltage would mean generator action, with the result that the machine would cease to operate as a motor.

The synchronous motor, however, may operate as a motor, and, at the same time, its back e.m.f. may exceed its terminal voltage in magnitude. Under these conditions, the motor is said to be *overexcited.* Two reactions occur which enable the motor to operate with an overexcited field: (1) The motor takes a *leading* current. A leading current in a motor corresponds to a lagging current in a generator. This is illustrated by Fig. 308. A current I is shown lagging the induced e.m.f. $-E$ by 90°. This current I is *lagging* with respect to the induced e.m.f. $-E$. In a generator, the phase difference between the induced e.m.f. and the terminal voltage is not large (see Fig. 166, p. 173). Hence, current I is also lagging with respect to the generator terminal voltage V_g and is, therefore, a lagging current if the machine is

considered as a generator. Such a current weakens the field through the effect of armature reaction (see Par. 88, p. 163).

When the machine is considered as a motor, the terminal voltage V_m is nearly opposite in phase to the induced e.m.f. $-E$, as shown in Fig. 308.

The current I, therefore, is *leading* with respect to the terminal voltage of the machine when it is considered as motor. It follows, then, that a current which is lagging when a machine is considered from the point of view of a generator is leading when the same machine is considered from the point of view of a motor. *In a generator, a lagging current weakens the field.* Consequently, *in a motor, a leading current must weaken the field.*

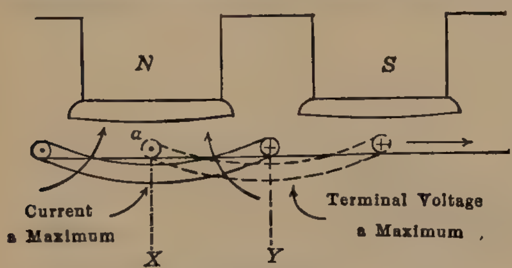


FIG. 309.—Demagnetizing effect of leading current on field of synchronous motor.

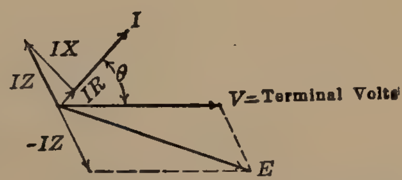


FIG. 310.—Induced armature voltage greater than terminal voltage when synchronous-motor current leads terminal voltage.

This is further illustrated as follows: Figure 309 shows a motor coil moving from left to right. When its axis is in the position Y, shown dotted, the coil sides are under the centers of the poles, and the induced e.m.f. is a maximum. As the terminal voltage is substantially 180°. from the induced e.m.f., it also will be a maximum at this instant, its direction being indicated in the dotted coil. If the current leads this terminal voltage by 90°, it will reach its maximum value ¼ cycle ahead of the voltage or at a time when the axis of the coil is in position X. It will be observed that for this position of the axis, the ampere-turns of the coil act in direct opposition to those of the N-pole. The effect of the leading current in the synchronous motor is, therefore, to weaken the field. In other words, *the armature reaction tends to annul the effect of the increased field current on overexcitation.*

(2) The second effect is illustrated by the vector diagram in Fig. 310. V is the terminal voltage, and I is the armature

current leading V by an angle θ. The resistance drop in the armature is laid off in phase with the current I, and the IX drop in the armature is laid off at right angles to the current I and leading, in the usual manner. The impedance drop IZ is the vector sum of IR and IX. The voltage E, necessary to balance the back e.m.f., is found by subtracting IZ vectorially from V, just as in the shunt motor the component of terminal voltage which is necessary to balance the back e.m.f. is found by subtracting the IR drop from the terminal voltage.

To subtract IZ from V, $-IZ$ is added to V. It will be noted that the e.m.f. E is numerically *greater* than the terminal voltage V. That is, by taking a leading current, the synchronous motor is able to operate with an induced e.m.f. greater numerically

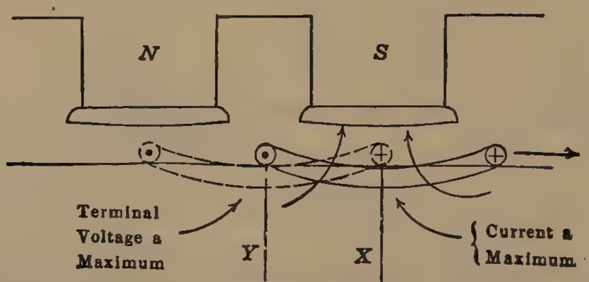


FIG. 311.—Magnetizing effect of lagging current on poles of synchronous motor.

than the terminal voltage. This is analogous to the alternator's delivering leading current with its induced e.m.f. *less* than its terminal voltage. In each case, the flow of power is toward the higher voltage.

157. Effect of Decreasing the Field Excitation.—When the field of a direct-current shunt motor is weakened, the motor speeds up until its back e.m.f. reaches a value which gives the proper armature current for the particular load condition.

When the field of a synchronous motor is weakened, it cannot speed up permanently, for it must run at a constant average speed. It takes a *lagging* current, however. This current has two effects.

Figure 311 shows a coil, dotted, whose axis is in position Y. In this position, the coil sides are opposite the centers of the pole faces, and the back e.m.f. is, therefore, a maximum. The terminal voltage, which is nearly 180° from the back e.m.f., has its

maximum value also for this position of the coil, its direction being indicated in the dotted coil. If the current is lagging the terminal voltage by 90°, it will not reach its maximum value until the coil axis reaches position X. The current under these conditions is in such a direction as to strengthen the S-pole. In a synchronous motor, therefore, a lagging current strengthens the field through the effect of armature reaction. When the field of a synchronous motor is weakened, the motor takes a *lagging current which strengthens the field by armature reaction and tends to annul the effect of the weakening of the field.* A lagging current when a machine operates as a motor is a leading current when the machine is considered operating as a generator. It will be remembered that a leading current in a generator strengthens the field through the effect of armature reaction (see Par. 88, p. 163). That is, a lagging current in a motor has the same effect on the magnetic field as a leading current has in a generator.

A synchronous motor under any given operating conditions requires a certain excitation. If its field is weakened, its excitation becomes inadequate. This deficit is, in part, made up by the motor's taking a lagging current from the line. A lagging current is ordinarily associated with inductance and, therefore, with the excitation of a magnetic field. When the motor takes a lagging current, some of its excitation is, therefore, obtained from the alternating-current line. In this respect, it is similar to an induction motor, except that the induction motor takes *all* its excitation from the alternating-current line. This lagging current required by the synchronous motor to help excite its own field weakens the field of the alternators supplying it, and, as a result, their field excitation must be increased to maintain the line voltage. When, therefore, the field of a synchronous motor is weakened, a part of the excitation which it requires, is supplied indirectly by the fields of the alternators supplying the system.

On the other hand, when a synchronous motor is overexcited, it has a surplus of excitation. It takes a leading current. As a leading current will neutralize a portion of the lagging current of inductive apparatus (see Par. 163, p. 366) connected to the system or else will strengthen the fields of the generators supply-

ing the system, the synchronous motor under these conditions indirectly supplies excitation to other parts of the system.

Figure 312 shows the vector diagram when the motor takes lagging current. The IR and the IX drops are laid off with reference to the current in the usual manner, and the IZ drop obtained. When $-IZ$ is added to V, however, E, which is opposite and equal to the back e.m.f., becomes numerically much less than V. That is, the phase shift of the IZ drop is in such a direction that the machine runs as a motor with a very considerably reduced back e.m.f.

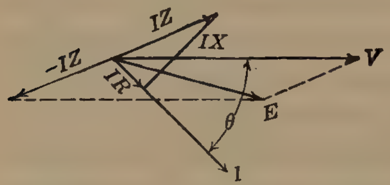


Fig. 312.—Induced armature voltage less than terminal voltage when synchronous-motor current lags terminal voltage.

The synchronous motor with salient poles will usually operate even if the field current is reduced to zero. The alternating current in the stator winding will produce a rotating field, just as in the induction motor. Figure 313 (a) shows such a rotating field for a four-pole machine without a rotor. At the particular instant shown, there are two N-poles vertically opposite, and two S-poles horizontally opposite. If a four-pole, salient-pole rotor

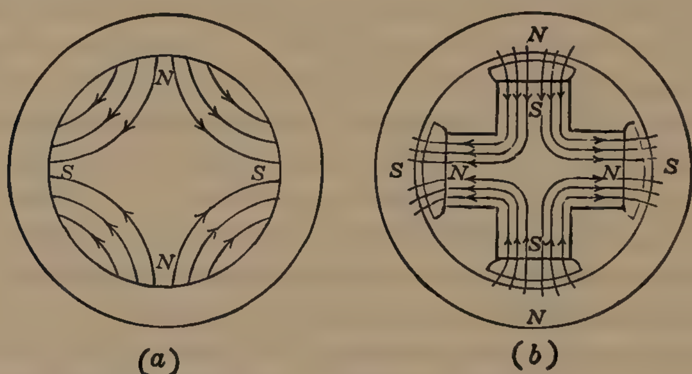


Fig. 313.—Interlocking action of salient poles with rotating magnetic field.

without excitation be placed in this field, the magnetic lines from the stator will attempt to make the rotor take such a position that the magnetic reluctance is a minimum or the flux is a maximum. In order to accomplish this result, the pole pieces of the rotor when running become locked in with the poles produced by the stator winding, as shown in Fig. 313 (b). These rotating

stator poles pull the salient poles of the rotor around with them and in this manner enable the motor to carry a limited load without direct-current excitation. Although the motor may carry a limited load without any direct-current excitation, its power factor will be very low, and the current will be lagging, which is undesirable. It is to be noted that, under these conditions, in the absence of direct-current excitation, the motor takes its entire excitation from the alternating-current lines in the same manner as an induction motor, as has already been described. That is, if sufficient excitation is not supplied by the direct current in the field winding, the motor will take lagging exciting current from the alternating-current line to make up the deficit.

The power factor of the simple *induction motor* for a given load cannot be altered without changing the motor design, and the ordinary induction motor always takes a lagging current. The power factor of the *synchronous motor* can be altered at will, and the current can be changed from lagging to leading by simply changing the field excitation.

158. Synchronous-motor Vector Diagram.—The solutions of the vector diagrams given in Figs. 310 and 312 are accomplished in much the same manner as the solutions of the alternator vector diagrams (pp. 171–181). The trigonometric solution is obtained by projecting the induced e.m.f. E' on the current vector, thus forming a right triangle with E' as the hypotenuse.

Leading Current.—Figure 314 corresponds to Fig. 310 in which the motor current is shown to be leading. The IR and IX voltage vectors are now actual voltage drops, not components of voltage which balance voltage drops (see p. 434). To solve the diagram trigonometrically, the induced e.m.f. vector E', or *od*, and IR are both projected on the current vector, giving *ob* and *ba*. This gives a right triangle *obd*, of which E' is the hypotenuse.

Then

$$E' = \sqrt{(V\cos\theta - IR)^2 + (V\sin\theta + IX)^2}. \qquad (118)$$

Also, E' may be determined by complex notation. That is,

$$\begin{aligned} \dot{E}' &= \dot{V} - \dot{I}\dot{Z} \\ &= V - I(\cos\theta + j\sin\theta)(R + jX). \end{aligned} \qquad (119)$$

Example.—A three-phase, 100-hp., 600-volt, 1,200-r.p.m., Y-connected synchronous motor has an armature resistance of 0.052 ohm per phase and a leakage reactance of 0.42 ohm per phase. At rated load and 0.8 power factor leading current, determine: (*a*) the induced armature e.m.f. per phase E' at rated load; (*b*) the angle α between the current and E'; (*c*) the mechanical power developed within the armature at rated load. The motor has a rated-load efficiency, excluding the field loss, of 0.92 under the foregoing conditions.

(*a*) $$\text{Motor input} = \frac{100 \times 746}{0.92} = 81{,}100 \text{ watts.}$$

Current

$$I = \frac{81{,}100}{\sqrt{3} \times 600 \times 0.80} = 97.6 \text{ amp.}$$

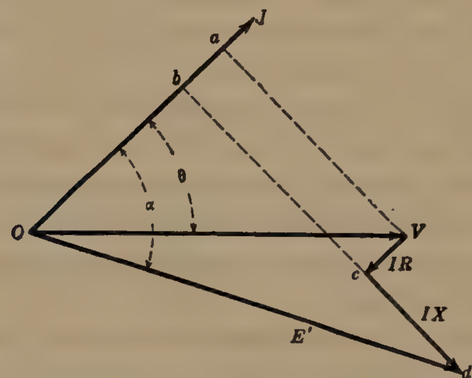


Fig. 314.—Synchronous-motor vector diagram—leading current.

Voltage per phase $= 600/\sqrt{3} = 346$ volts.
Using Eq. (118),

$$E' = \sqrt{[(346 \times 0.80) - (97.6 \times 0.052)]^2 + [(346 \times 0.60) + (97.6 \times 0.42)]^2}$$
$$= 368 \text{ volts.} \quad \textit{Ans.}$$

(*b*) A study of Fig. 314 shows

$$\tan \alpha = \frac{bd}{ob} = \frac{V \sin \theta + IX}{V \cos \theta - IR} = \frac{248.6}{271.7} = 0.915$$
$$\alpha = 42.5°. \quad \textit{Ans.}$$

Using Eq. (119),

$$\dot{E}' = 346 - 97.6(0.80 + j0.60)(0.052 + j0.42)$$
$$= 346 - (-20.5 + j35.9) = 366.5 - j35.9$$
$$|E'| = \sqrt{(366.5)^2 + (35.9)^2} = 368 \text{ volts.} \quad \textit{Ans. (check).}$$

(*c*) The mechanical power developed is equal to the product of the induced e.m.f., the current, and the cosine of the angle between them.

$$P_m = 3 \times 368 \times 97.6 \times \cos 42.5° = 79{,}500 \text{ watts.} \quad \textit{Ans.}$$

This power P_m is also equal to the power input minus the armature resistance loss.

$$P_m = 81{,}100 - 3 \times \overline{97.6}^2 \times 0.052 = 79{,}600 \text{ watts } (\textit{check}).$$

The power developed at the pulley is less than P_m by the rotational losses, friction, windage, and rotational core losses.

Lagging Current.—In Fig. 315, which corresponds to Fig. 312, E' and IR are projected on the current vector.
Then

$$E' = \sqrt{(V \cos \theta - IR)^2 + (V \sin \theta - IX)^2}. \tag{120}$$

The solution by complex notation,

$$\begin{aligned} \dot{E}' &= \dot{V} - \dot{I}\dot{Z} \\ &= V - I(\cos \theta - j \sin \theta)(R + jX). \end{aligned} \tag{121}$$

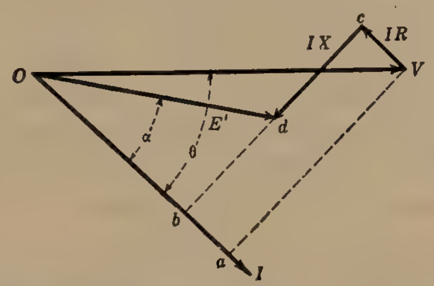


FIG. 315.—Synchronous-motor vector diagram—lagging current.

Example.—Repeat the foregoing problem but with lagging current.
(*a*) Using Eq. (120),

$$E' = \sqrt{[(346 \times 0.80) - (97.6 \times 0.052)]^2 + [(346 \times 0.60) - (97.6 \times 0.42)]^2}$$
$$= 319 \text{ volts. } \textit{Ans.}$$

Using Eq. (121),

$$\begin{aligned} \dot{E}' &= 346 - 97.6(0.80 - j0.60)(0.052 + j0.42) \\ &= 346 - (28.7 + j29.8) = 317.3 - j29.8 \end{aligned}$$

$$|E'| = \sqrt{(317.3)^2 + (29.8)^2} = 319 \text{ volts. } \textit{Ans. (check).}$$

(*b*) $$\text{Tan } \alpha \text{ (Fig. 315)} = \frac{bd}{ob} = \frac{V \sin \theta - IX}{V \cos \theta - IR} = \frac{166.6}{271.7} = 0.613$$
$$= 31.5°. \quad \textit{Ans.}$$

(*c*) $P_m = 3 \times 319 \times 97.6 \times \cos 31.5° = 79{,}600$ watts. *Ans.*
(This is the same value as before, which it should be.)

These values of E' are the induced e.m.fs. in the armature and correspond to E_a (Figs. 168 and 173) pp. 174 and 181. The induced e.m.fs., with no armature current, corresponding to this same excitation and frequency, are found by using the *synchronous* reactance X_s, rather than the leakage reactance X.

159. Synchronous-motor V-curves.—If the power P delivered to a three-phase synchronous motor be kept constant and the field current I_f varied, the power factor of the motor will change. The power for a three-phase motor is

$$P = \sqrt{3}VI \cos \theta$$

where V is the terminal voltage, I the line current, and $\cos \theta$ the power factor of the motor. As both P and V are constant, any decrease in the power factor ($\cos \theta$) must be accompanied by a corresponding increase in the current I. Likewise, any increase in the power factor must be accompanied by a decrease in the current I.

A change in the field current at constant load, therefore, changes the line or armature current I. In order to determine the relation between the field current and the armature current and also the characteristics of a synchronous motor as regards its ability to correct the power factor of a system, the so-called *V-curves* of the motor are obtained. These V-curves show the relation which exists between the armature current and the field current for different constant-power inputs. Several curves are usually obtained, each curve representing a constant value of power input.

The connections for making such a test are shown in Fig. 316. The field current is varied by means of the field rheostat. For each value of field current, as read on the direct-current ammeter, the corresponding value of the alternating line current is noted. The electrical power delivered to the motor is kept constant by adjusting the load applied to the motor shaft. A polyphase wattmeter is desirable for this experiment, as it eliminates the adding or subtracting of individual instrument readings, which is necessary when two single wattmeters are used.

Figure 317 shows a set of typical V-curves. The curve AB is obtained when the motor is running at very light load. At very low values of field current, the armature current is large and is lagging. As the field current is increased, the power factor increases, and the armature current decreases until it reaches its minimum value I_1. If the field current be still further increased, the armature current begins to increase and becomes leading. In other words, the motor passes from *underexcitation* to *over-*

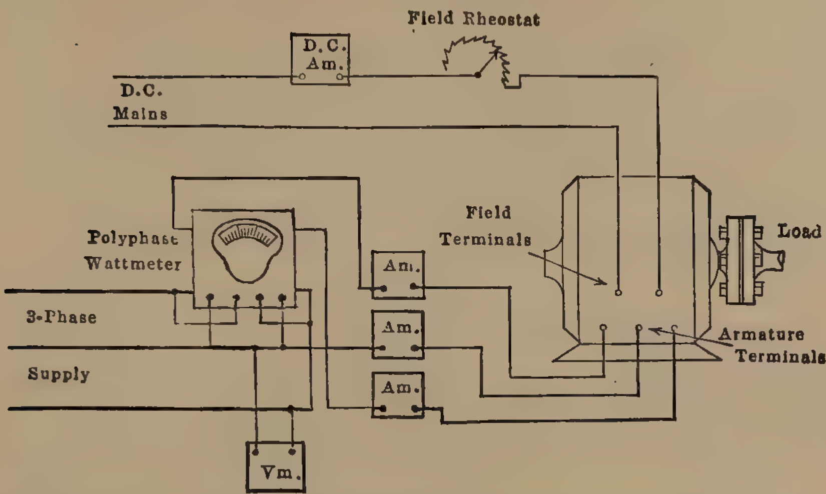


FIG. 316.—Connections for obtaining V-curves of synchronous motor.

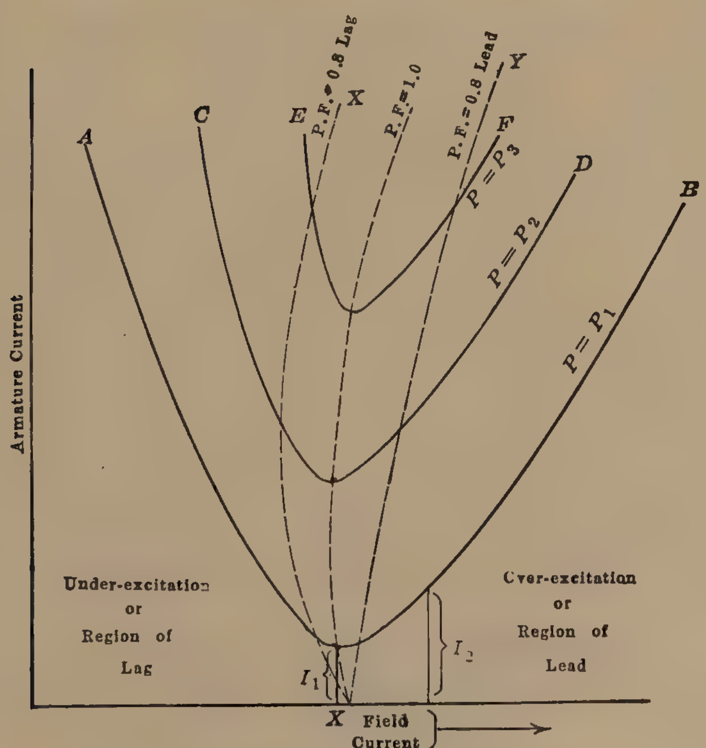


FIG. 317.—V-curves of synchronous motor.

excitation when the field current is increased from a low to a high value.

The current I_1 is the value of the current at unity power factor. This is illustrated in Fig. 318. Let I_2 be the value of line current for some power factor, $\cos \theta_2$. The power (for one phase) is

$$P_1 = V'I_2 \cos \theta_2$$

where V' is the phase voltage.

But

$$(I_2 \cos \theta_2) = I_1 \tag{122}$$

for all values of θ_2.

In other words, for constant power P_1, I_1 is always the energy component of the current regardless of the power factor. The current vector, therefore, will always terminate on the line XX perpendicular to V'. The current is a minimum at I_1, where the current is in phase with V'. The power factor is then unity. The excitation corresponding to the armature current I_1 is called the *normal excitation* of the motor for any given load. For an excitation less than the normal value, the motor takes a *lagging* current and is said to be *underexcited;* for values of the excitation greater than the normal value, the motor takes a *leading* current and is said to be *overexcited.*

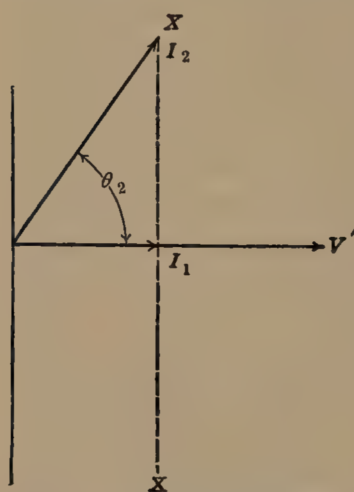


FIG. 318.—Vector diagram showing current variation in synchronous motor with constant power input.

By aid of the V-curves, the power factor for any other value of line current and given input may be obtained. For example, assume that it is desired to obtain the power factor for some value of leading current I_2 (Fig. 317). From Fig. 318, the power-factor $\cos \theta_2 = I_1/I_2$. The power-factor for any current I, therefore, may be found by dividing the current I into the minimum or normal value of the line or armature current I_1 for the given input P_1. The power represented by curve AB is obviously

$$P_1 = \sqrt{3}VI_1$$

for a three-phase motor having a *line* voltage V.

CD (Fig. 317) is a V-curve taken for a value of power P_2, which is obviously greater than P_1. *EF* is a third curve taken for a still greater value of power P_3. A curve drawn through the lowest points of the V-curves is a unity power-factor curve. Curves XX and XY, drawn through the V-curves at the proper points, are 0.8 power-factor curves, XX being for *lagging* current and XY for *leading* current. Curves for other power factors may also be found in a similar manner. These curves are called *compounding curves*.

It should be noted that the normal field current varies with the value of power input to the motor.

160. Amortisseur or Damper Windings.—Figure 319 shows the rotating field structure of a synchronous motor, around which a squirrel-cage winding is built. The conductors of the squirrel cage are embedded in the pole faces of the rotor. This winding serves two purposes: It assists the motor in starting, and it damps out any tendency of the rotor to oscillate or "hunt."

Such windings are called *amortisseur* or *damper* windings or simply *dampers*. If the motor is connected to a system which receives its power from a reciprocating engine unit, there may be pulsations in the supply frequency caused by the variable driving torque of the engine. The synchronous motor is very sensitive to phase changes, as has already been shown, and small changes in the phase of the supply voltage may produce considerable changes in the energy current which the motor takes from the line. This produces pulsations in the motor torque. If these pulsations have a frequency nearly equal to the natural frequency of oscillation of the rotor, they may cause it to oscillate periodically about its normal position. That is, the rotor alternately accelerates and retards, although the average speed does not change. This is called *hunting* (see Par. 102, p. 208). These oscillations may become so great as to cause the motor to fall out of synchronism.

Hunting may also be caused by system disturbances, such as switching, short circuits, etc., and also by sudden changes of load on the motor shaft. Hunting due to such causes usually dies out at a rapid rate, but the first oscillations may be sufficiently great to cause the motor to fall out of synchronism.

The action of the damper winding involves the principle of both the induction motor and the induction generator. So long as the rotor is rotating at synchronous speed, the rotating field of the armature or stator does not cut the dampers, and they have no effect. That is, the armature m.m.f. rotates synchronously with the field, and there is no relative motion between the field flux and the dampers. Assume that the rotor slows down

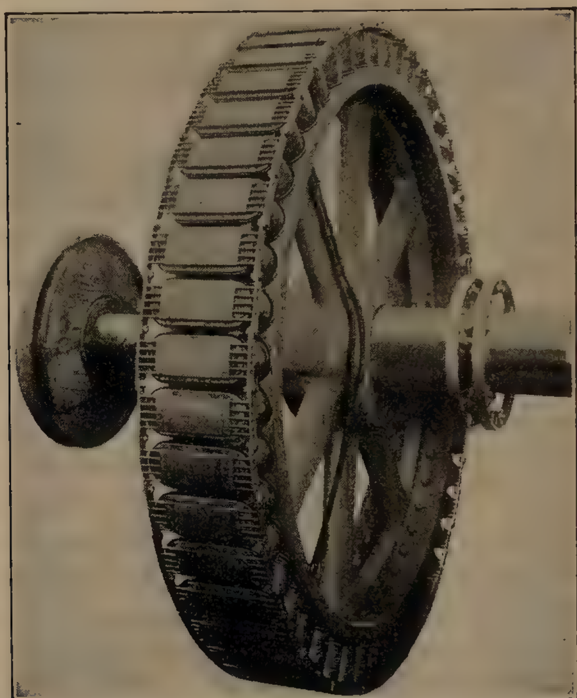


FIG. 319.—Rotor of 600-volt, 60-cycle, 48-pole synchronous motor showing amortisseur winding.

momentarily. For an instant, the rotating field due to the armature m.m.f. is rotating more rapidly than the field structure. This is equivalent to the rotor's slipping temporarily, and currents are induced in the dampers. This is induction-motor action, and the currents in the dampers are in such a direction that they tend to pull the rotor back again towards synchronism.

Again, if the field poles, for some reason, swing ahead of their normal position, the dampers cut the rotating field in the opposite direction, or the slip becomes negative, temporarily. Induction-generator action follows, putting a load on the rotor and tending to slow it down. These dampers always tend to pull

the motor back, therefore, into synchronism and thus prevent hunting. Such windings are often used on alternators, particularly of the engine-driven type, to prevent hunting.

161. Starting the Synchronous Motor.—As has been pointed out, the *synchronous* motor is not self-starting. It must first be brought nearly or actually to synchronous speed before it can operate. There are several methods of accomplishing this.

The direct-current exciter for the motor is frequently connected directly to the motor shaft. If a direct-current source of power is available, the exciter may be operated as a motor and thus bring the synchronous motor up to speed. The field of the synchronous motor is then excited, and the motor synchronized, just as with an alternator.

If an exciter or sufficient direct-current power is not available, a small induction motor, geared or direct connected to the synchronous-motor shaft, may be used for bringing it up to speed. If the induction motor is direct connected, its synchronous speed must have a higher value than that of the synchronous motor, in order to compensate for the slip of the starting motor. Such starting motors are often disconnected mechanically after the synchronous motor has been connected to the line. The disadvantage of using an induction motor is the additional motor, the gears where used, etc. This method of starting is practically not used at the present time.

The synchronous motor is often used to drive a direct-current generator. If sufficient direct-current power is available, the generator may be used as a motor to bring the synchronous motor up to speed. After the motor is synchronized, the field of the direct-current machine is strengthened, and it then acts as a generator, taking mechanical power from the synchronous motor.

The synchronous motor may start as an induction motor. First, the field circuit is opened. A polyphase alternating voltage is then impressed on its stator, and a rotating field is, therefore, set up about the rotor. As a rule, it is desirable to use a compensator so that reduced voltage is applied to the stator windings. The rotating field sets up currents in the pole faces of the rotor and in the amortisseur winding as well, if such exists. This is obviously induction-motor action. As the paths of the pole-face currents and of the currents in the dampers have considerable

inductance (see Par. 133, p. 277), only a comparatively weak starting torque can be obtained. On starting, the rotor currents may be large, and the rotor frequency is that of the stator. The rotor reactance, which is proportional to the rotor inductance and to the frequency, is large. This causes the rotor currents to lag the induced e.m.fs. by a considerable angle, and, hence, the rotor currents make considerable space-angle with the flux (see Fig. 253, p. 280. The motor develops little torque, therefore, even with considerable line current. The motor under these conditions is very similar to the squirrel-cage induction motor, which has a very small starting torque.

The starting torque, however, though small, is usually sufficient to start the machine, which then accelerates until it is at or near synchronism. Before the compensator is thrown into the running position, the field switch is usually closed, so as to minimize disturbances to the system. If the rotor is slipping slightly, it will usually pull into synchronism when the field switch is closed, the field poles locking in with the poles produced by the armature m.m.f. (Fig. 313, p. 354).

The motor may pull into synchronism before the field circuit is closed. The flux (see Fig. 313) sweeping by the salient poles shows less and less tendency to leave them as the rotor approaches synchronism, owing to hysteresis. That is, the flux tends to persist in the poles after the magnetizing force is decreased (see Vol. I, p. 206, Fig. 168). This action may be strong enough to pull the rotor into synchronism before the field circuit is closed.

When the field circuit is closed, it may excite the motor poles so that their polarity is opposite to that produced by the revolving field, *i.e.*, by the armature reaction (Fig. 313). The rotor is then thrown back one pole, or, in other words, it slips a pole. This may cause considerable disturbance to the system, and for this reason the field is usually closed when the compensator is in the starting position. This difficulty may be avoided by applying a weak direct-current field to the motor as it approaches synchronism. This causes the armature reaction to act in conjunction with the direct-current field windings, and the poles then come into synchronism with the same polarity as will be produced by the direct-current excitation. After the motor has

pulled into synchronism, it is necessary merely to strengthen the direct-current field to the desired value. The starting compensator may then be thrown quickly into the running position.

When voltage is first applied to the synchronous motor, there may be a very high voltage induced in the field winding. The stator acts as the primary of a transformer, the primary having comparatively few turns. The flux produced by the stator or primary cuts the field winding at synchronous speed, and as the field has a very large number of turns, a very high e.m.f. is induced in the field. This e.m.f. may be sufficiently high to puncture the field winding. The field winding, therefore, should be insulated for voltages considerably in excess of that which normal operation requires. The field is sometimes short circuited or is shunted by a resistance when starting, in order to decrease this high voltage. The induced e.m.f. in the field decreases as the rotor comes up to speed, until at synchronism it becomes zero.

162. Starting Synchronous Motor under Load.—Under the usual starting conditions, the synchronous motor develops but little torque. Hence, it cannot start under load, except, perhaps, with very small loads, such as idle shafting. To meet this difficulty, the General Electric Company has developed the super-synchronous motor the external armature of which rotates, as well as the usual rotating field structure (Fig. 320). Assume that load is connected to the rotor. As usual, reduced voltage is applied to the armature but through slip-rings. The field structure or rotor is at first prevented from turning. Hence, the stator must rotate in a direction opposite the normal direction of rotation of the field structure. The armature comes up to speed and is brought into synchronism, the field being closed in the ordinary manner. A brake is then slowly applied to the rotating armature. Because of the ordinary reaction between armature and field, the field structure must rotate in a direction opposite to that of the armature, and its speed must be such that the relative speeds of field and armature are always equal to the synchronous speed of the machine. The armature is ultimately brought to a standstill, and the brake locked. The field structure is now running in the usual manner at its synchronous speed. It is obvious that when the rotor is being brought up to speed under

these conditions, the motor may develop the same values of torque up to the pull-out torque which it develops under ordinary running conditions.

The Westinghouse Electric & Manufacturing Company has met this problem of starting under load by incorporating a magnetic clutch in a standard synchronous motor. When starting the motor, the clutch cannot be applied, so the rotor is free and starts readily. When the motor has reached synchro-

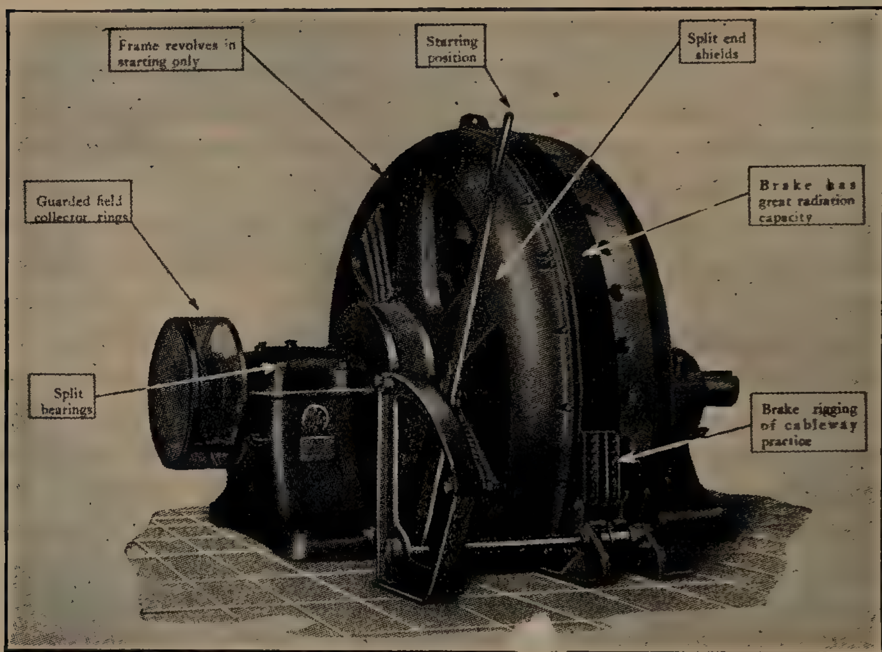


FIG. 320.—General Electric super-synchronous motor.

nous speed, and the field is closed, the clutch is brought into operation by applying direct-current excitation. This brings two friction surfaces into engagement. The pressure between them can be gradually increased by increasing the excitation of a magnetizing coil, and the load can thus be accelerated to synchronous speed without shock or jar.

163. Synchronous Condenser as Corrector of Power Factor.—The fact that the power factor of the synchronous motor may be varied at will makes it useful in many installations, particularly in those which operate at low power factor. It will be recalled that a low power factor means larger generators, more

transmission copper, poorer regulation, and reduced efficiency. Factories and mills using induction-motor drive often have an overall power factor as low as 0.5, which is very undesirable. If it is possible to use a synchronous-motor drive in any part of the installation, the motor may be operated overexcited and, therefore, will take a leading current. This leading current neutralizes some of the lagging current of the system and so improves the system power factor.

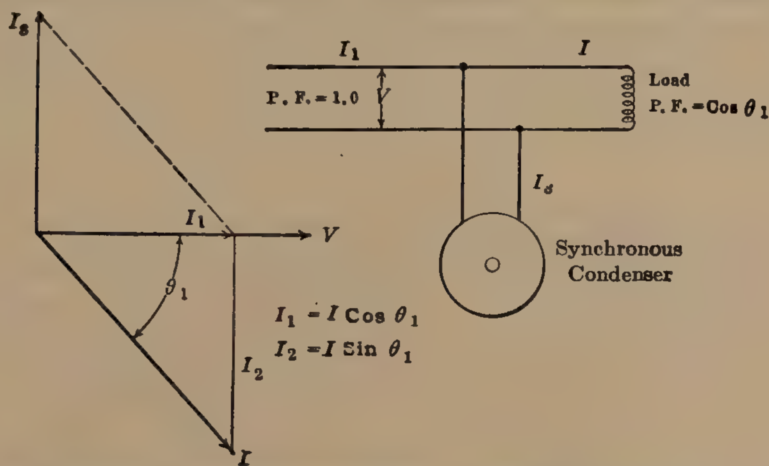


FIG. 321.—Raising power factor to unity by means of synchronous condenser.

This is illustrated in Fig. 321 for single-phase or for one of the phases of a polyphase system. Let V be the voltage of the system, and let the total current be I, lagging the voltage V by an angle θ_1. It is desired to obtain the size of synchronous motor necessary to raise the system power factor to unity. The synchronous motor is to run without load.

Resolve the current I into two components, an energy component $I_1 = I \cos \theta_1$ and a quadrature component $I_2 = I \sin \theta_1$. The energy current of the synchronous motor is small compared with its quadrature current, when the motor is operating without load, and is added at right angles to the quadrature current. In determining the total current taken by the synchronous motor, therefore, this energy current may be neglected. For unity power factor, the motor current will then be substantially equal to the quadrature current I_2 but leading. The rating of the synchronous motor, therefore, is $VI_s = VI_2$ volt-amp. per phase.

If it be desired to raise the power factor to some value less than unity, a smaller synchronous motor can be used. In practice, it usually does not pay to raise the power factor above 0.9 or 0.95, as little is gained by any increase above these values. These last few per cent. of improvement in the power factor, moreover, require a much greater proportionate increase in motor capacity.

In Fig. 322, the load power factor is $\cos\theta_1$. The load on the synchronous motor is assumed to be zero, and its losses are neglected. The load current I is resolved into two components

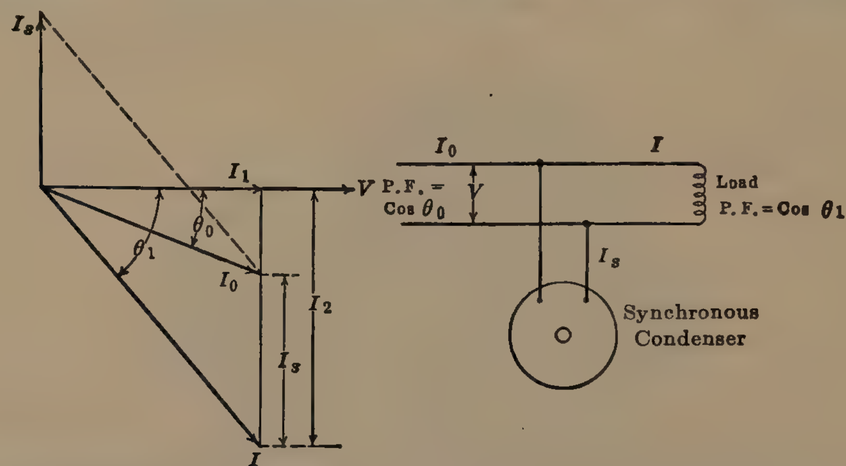


FIG. 322.—Raising power factor to cos θ_0 by means of synchronous condenser.

I_1 and I_2 as before. It is desired to determine the size of synchronous motor necessary to raise the power factor to $\cos\theta_0$.

The resultant current I_0 is laid off $\theta_0°$ behind V but terminating on line I-I_1, since the power and, hence, the energy current I_1 is fixed. The synchronous-motor current I_s has the value (Fig. 322),

$$I_s = I\sin\theta_1 - I_0\sin\theta_0 = I_2 - I_0\sin\theta_0. \qquad (123)$$

It will be noted that the resultant current I_0 is the vector sum of the load current I and the motor current I_s.

When a synchronous motor is operated without load for the purpose of merely correcting power factor, it is called a *synchronous condenser*. Such a synchronous condenser should not be employed unless its investment charges and cost of operation are considerably less than the increased charges occasioned by the low power factor. Other considerations, such as voltage

control, however, are important. When a user of electric power buys on either a kilowatt-hour or a kilowatt basis, a low power factor is not detrimental to him, except, possibly, to increase slightly the cost of his mains. This low power factor is, however, detrimental to the power company, which must install larger generators, conductors, transformers, etc. For this reason, many power contracts now penalize low power factor.

164. Synchronous Motor as Corrector of Power Factor. The synchronous motor may correct the power factor of a system and, at the same time, deliver mechanical power.

Assume, in Fig. 323, that a certain system takes I amp. at a voltage V and that the current I lags V by $\theta_1°$. It is desired to raise the power factor of the system to unity by means of a synchronous motor, while at the same time the motor is to supply mechanical power requiring VI_1' watts from the line.

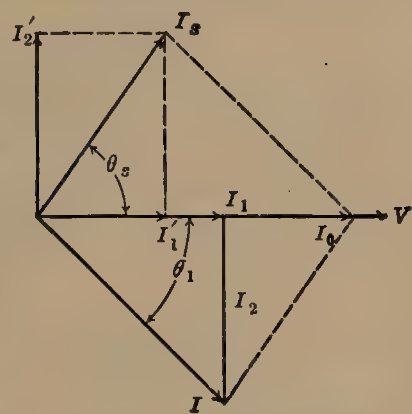


FIG. 323.—Raising power factor to unity by means of loaded synchronous motor.

The synchronous motor must first take a quadrature leading current I'_2 in order to counteract the lagging quadrature current I_2 of the load.

$$I_2' = I_2 = I \sin \theta_1.$$

In addition, the synchronous motor must take an energy current I_1' to supply its losses and also the power required by its load. The total synchronous motor current

$$I_s = \sqrt{(I_1')^2 + (I_2')^2} \tag{124}$$

and the power factor of the synchronous motor

$$\cos \theta_s = \frac{I_1'}{I_s}. \tag{125}$$

Example.—A certain machine shop takes 200 kw., at 0.6 power factor, from a 600-volt, three-phase, 60-cycle system. It is desired to raise the power factor of the entire system to 0.9 by means of a synchronous motor, which at the same time is to drive a direct-current shunt generator requiring that the synchronous motor take 80 kw. from the line. What should be the rating of the synchronous motor in volts and amperes?

The vector diagram is shown in Fig. 324. Assume that the system is Y-connected. The problem will be worked for one phase only.

The voltage to neutral

$$V = \frac{600}{\sqrt{3}} = 346 \text{ volts.}$$

The current per phase

$$I = \frac{200{,}000}{\sqrt{3} \times 600 \times 0.60} = 321 \text{ amp.}$$

The energy current of the load, $I_1 = I \cos\theta = I \times 0.6 = 192.6$ amp.
The quadrature current of the load, $I_2 = I \sin\theta = I \times 0.8 = 256.8$ amp.
At 0.9 power factor, the resultant power factor angle $\theta_0 = 25.8°$.

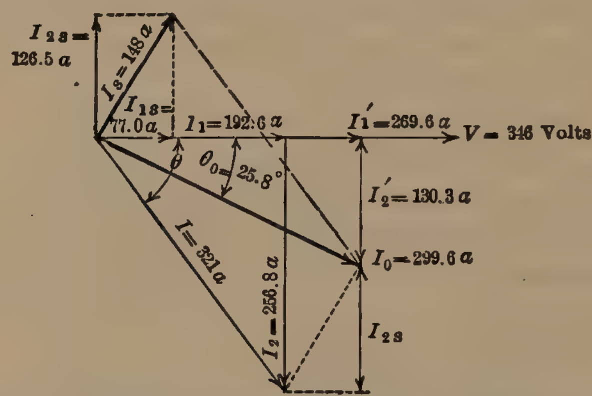


FIG. 324.—Vector diagram for synchronous motor which raises power factor to $\cos\theta_0$ and at same time supplies power.

The energy current of the synchronous motor

$$I_{1s} = \frac{80{,}000}{\sqrt{3}\ \ 600} = 77.0 \text{ amp.}$$

Total energy current $I_1' = I_1 + I_{1s} = 192.6 + 77.0 = 269.6$ amp.
The quadrature current of the system,

$$I_2' = 269.6 \tan 25.8° = 269.6 \times 0.4834 = 130.3 \text{ amp.}$$

The quadrature current of the synchronous motor

$$I_{2s} = I_2 - I_2' = 256.8 - 130.3 = 126.5 \text{ amp.}$$

The total synchronous-motor current

$$I_s = \sqrt{(I_{1s})^2 + (I_{2s})^2} = \sqrt{(77.0)^2 + (126.5)^2} = \sqrt{21{,}930} = 148 \text{ amp.}$$

The synchronous motor will then be rated at 600 volts, 148 amp. or will have a rating of 154 kv-a. *Ans.*

The resultant current I_0 the vector sum of I and I_s is shown in Fig. 324.

165. Synchronous Motor as Regulator of Voltage.—Figure 325 shows one phase of a power system, which may be either a single-phase or a polyphase system. A constant voltage V_g is

supplied to the system by a generator or by a power plant. At the receiving end of the line is a synchronous motor whose terminal voltage is V_m. Between V_g and V_m are both resistance and reactance in series. These may be the usual resistance and reactance of a transmission line, or they may exist in an impedance coil, having a resistance R and reactance X, inserted between the supply mains and the motor terminals.

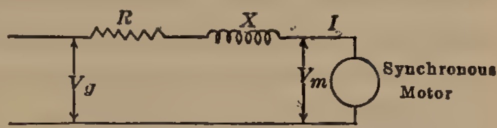
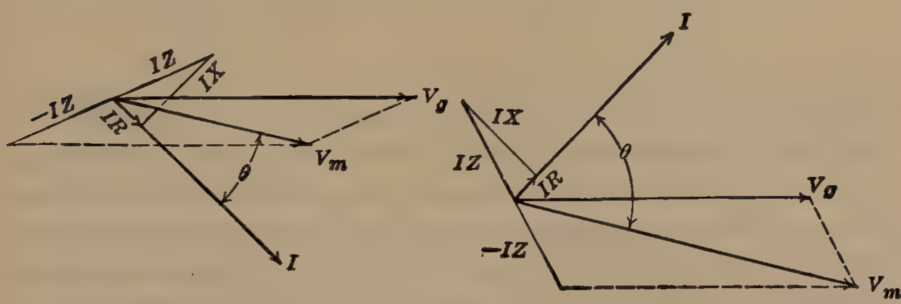


FIG. 325.—Synchronous motor taking power through resistance and reactance in series.

Assume, first, that the synchronous motor is underexcited and, therefore, taking a lagging current. Along the vector I (Fig. 326 (a)), the IR drop is laid off in phase with I; at right angles to I and leading, the IX drop is laid off. The vector sum of the IR and IX drops is equal to the IZ drop in the line. Obviously, the motor voltage must be equal to the generator voltage *minus* the IZ drop, vectorially considered. IZ is reversed and added to V_g, therefore, giving V_m, the motor voltage. It will be observed that numerically V_m is considerably less than V_g.



(a) Lagging current; motor voltage less than generator voltage.

(b) Leading current; motor voltage greater than generator voltage.

FIG. 326.—Effect of line impedance on synchronous-motor voltage.

If the motor now be overexcited, I will lead the voltage V_m. By subtracting IZ from V_g (Fig. 326 (b)), the motor voltage V_m becomes numerically *greater* than V_g.

This gives a method of controlling the voltage at the end of a transmission line. If the voltage at the receiving end of the line tends to change because of a change in the generator voltage or in the line drop, it may often be held substantially constant by

varying the excitation of a synchronous motor placed at the receiving end of the line. In practice, synchronous motors are often installed for purposes of regulation only. At the Los Angeles end of the 240-mile Big Creek Line, two 15,000-kv-a. synchronous condensers are installed, their sole function being to hold the voltage in Los Angeles at the proper value. If the load were removed and no such regulating devices existed, this voltage would rise to values considerably in excess of that at the generating station 240 miles away, due to the line charging current's flowing through the line reactance.

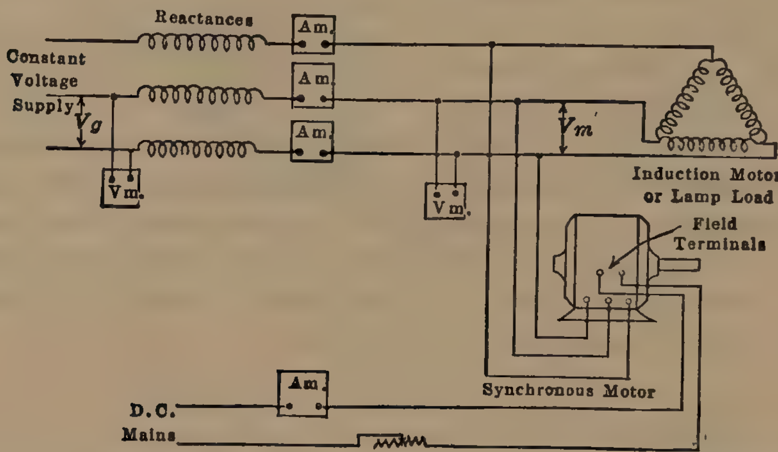


FIG. 327.—Synchronous motor for controlling voltage at end of transmission line.

Also, at the present time (1928), three 30,000-kv-a., 600-r.p.m., 13,800-volt synchronous condensers are being installed at the Plymouth Meeting substation of the Philadelphia Electric Company, to control the voltage of the 220-kv. line from the new Conowingo hydroelectric development on the Susquehanna River.

Even without the adjustment secured by altering the field current, a synchronous motor tends to maintain constant voltage at the end of a transmission line having reactance. If the voltage at the motor terminals drops, its back e.m.f. tends to exceed the terminal voltage, and the motor must then take a leading current in order to operate. This leading current, flowing through the line reactance, tends to maintain the motor voltage, as a leading current flowing through reactance tends to produce a rise of voltage from generator to load. On the other hand, a rise

of voltage at the motor terminals tends to cause the motor to operate underexcited. This increases the drop from generator to load and tends to cause the voltage at the load to decrease.

The effect of the synchronous motor on voltage control may be shown by a laboratory experiment, the connections for which are given in Fig. 327. A synchronous motor, running either light or partly loaded, is supplied from constant-potential mains through three series reactances, one in each main. A lamp load or an induction-motor load is connected in parallel with the synchronous motor. Vary the lamp load or the induction-motor load and maintain the synchronous motor terminal voltage V_m constant by varying its field current. It will be found that the

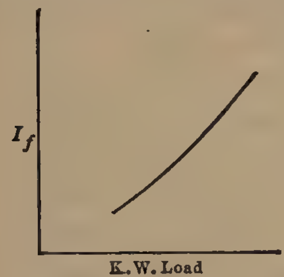


Fig. 328.—Relation of field current to load at motor, motor voltage constant.

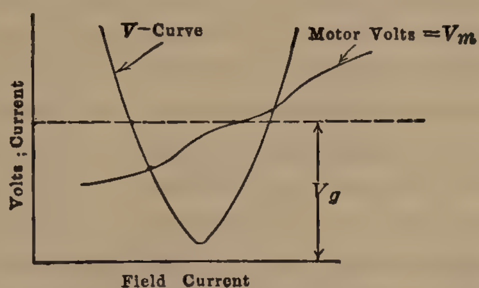


Fig. 329.—Effect of field current on motor voltage at constant load.

field current must be materially increased as the load is increased. Figure 328 shows the general trend of the curve giving the relation between the field current and the load.

It is also instructive to keep the lamp load or the induction-motor load constant and at the same time to obtain a V-curve and find the relation of V_m to the synchronous-motor field current. The results of such a test are shown in Fig. 329. V_m is considerably lower than V_g for low values of field current, but after unity power factor is reached, V_m exceeds V_g. (Further discussion of transmission line regulation is given in Chap. XIII, p. 433.)

166. Industrial Applications of Synchronous Motor.—Single-phase synchronous motors are rarely used in practice. Like the single-phase induction motor, the direction in which they rotate is determined by the direction in which they are started. Unlike the polyphase synchronous motor, they will not start

by induction motor action but must be brought up to speed by other means. Polyphase synchronous motors are commonly used.

The inherent disadvantages of the synchronous motor are that it requires a direct-current supply for its excitation, its starting torque is very small, and the motor is very sensitive to system disturbances and may fall out of step when these occur. On the other hand, the ease with which its power factor can be controlled is a distinct advantage, often outweighing all the disadvantages. The fact that its speed is constant is of little moment, since induction motors, especially in the larger sizes, have only 1 or 2 per cent. speed regulation.

The synchronous motor is used only in the larger sizes where the cost of attendance per kilovolt-ampere is low. It should not, moreover, be used where there are too sudden applications of the load, as it may drop out of step under such conditions. An important field of use is in connection with motor-generator sets where a large unit is required and where any sudden changes of load are partly absorbed by the inertia of the direct-current armature. A few such motors, situated at various points in a large system, may make it possible to operate the generating station and many of the transmission lines and substations at high power factor, in spite of low power factor in the consumers' loads.

Even with these advantages of the synchronous motor, electrical engineers often prefer to use induction motors for motor-generator sets, because of their simplicity and greater reliability.

Other common applications of synchronous motors are drives for rubber mills and for ammonia compressors, which are used in making artificial ice.

Electric Propulsion.—Synchronous motors are also coming into use for the electric propulsion of cargo and merchant ships. Such ships, when under way, operate at a constant speed, and the constant-speed characteristic of the synchronous motor is not a disadvantage, therefore. As such motors can be operated at unity power factor, the weight of motor, generator, and connecting leads is smaller than when induction motors are used. This matter of weight is important in marine work. The air-gap of synchronous motors is considerably greater than that of induction motors, and the mechanical difficulties which a short air-gap

involves, such as very accurate alignment, etc., are not present when synchronous motors are used. Owing to the salient-pole feature of the synchronous motor, stator coils may be replaced without removing the rotor. Also, the field windings on the salient poles are less subject to injury than the embedded conductors in the rotor of an induction motor. The dampers of synchronous motors used for electric propulsion are designed to give moderately high torque on starting, reversing, etc. Both the speed and the voltage of the generator may be varied, so that the motors have different starting characteristics from those existing at constant frequency and constant voltage.

167. Frequency Changers.—It is sometimes necessary to supply electric power from one electric system to another electric system of different frequency. A common method is to use synchronous-motor-alternator sets. The synchronous motor and the alternator must have a different number of poles, the number of poles in each being proportional to the frequency of the system to which the particular machine is connected. For example, if the frequency is being changed from 60 to 25 cycles, the number of poles of the synchronous motor must be to the number of poles of the alternator in the ratio of 60/25 or 12/5. The highest speed at which this ratio of frequencies can be obtained will require a set having a 24-pole synchronous motor and a 10-pole alternator. The set will operate at only 300 r.p.m.

Except in very large units, electrical machines operating at this very low speed would be costly. A 10-pole, 4-pole combination gives either a frequency ratio of 60/24 cycles or a frequency ratio of 62.5/25 cycles and operates at 750 r.p.m. Because of its greater speed, this combination is often used, even if it does not give an exact 60/25 cycle ratio.

It is often difficult to synchronize such a set, as it must be synchronized with both systems. If the alternator voltages are out of phase with their respective line voltages, the synchronous motor must be made to slip a pole at a time until the alternator voltages are in phase with their respective line voltages. The load is shifted either by advancing the phase of the system supplying the power, as by opening the turbine governors, or by retarding in some manner the phase of the voltage in the system receiving the power.

168. Synchronous Motors of Very Small Size.—Because of their absolutely constant-speed characteristics, synchronous motors are very useful for driving such devices as must be held in absolute synchronism with the supply frequency. Such uses involve the measurement of slip in the induction motor (see p. 314), the driving of oscillograph mirrors, stroboscopic devices, mechanical rectifiers, synchronous electric clocks, etc.

As the power required of such motors is extremely small and the matter of low power factor is of no moment, they are often made to operate without direct-current excitation. In Fig. 330 (*a*) and (*b*) are shown motors of this type. In (*a*), the four-pole armature consists of a cruciform-shaped piece of iron with the

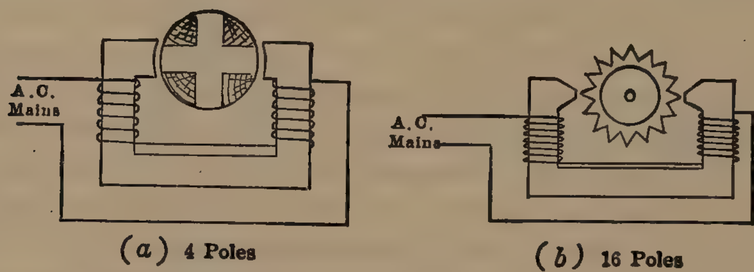


Fig. 330.—Miniature synchronous motors.

spaces filled with wood to make the armature cylindrical. The field is made up of U-shaped laminations and is excited from the alternating-current supply.

When the armature is brought up to speed, two diametrically opposite armature poles are attracted to the field poles as the flux is increasing. Because of the inertia of the armature, it continues to rotate when the flux is passing through zero. The next pair of poles is then attracted by the flux as the latter increases in the opposite direction. Such a motor will, therefore, run at constant speed, provided the frequency is constant.

A 16-pole motor operating on the same principle is shown in Fig. 330 (*b*).

These motors really operate on the principle of maximum permeance, although it will be recognized that they are salient-pole synchronous motors of the rotating-field type, having no direct-current field excitation. Their excitation is produced by armature reaction.

CHAPTER XII

RECTIFIERS: THE SYNCHRONOUS CONVERTER

169. Methods of Obtaining Direct Current from Alternating Current.—At the present time, over 90 per cent. of electrical energy is generated and transmitted as alternating current. A very large percentage of this energy is utilized as alternating current; for example, to operate alternating-current motors, electric furnaces, and many other types of electrical appliances, for illumination purposes, etc. There are many cases, however, where the electrical energy must be in the form of direct current, even although the available supply of energy is alternating current. For example, direct current must be used for charging storage batteries, for electrolytic work, for telephone exchanges, etc. The direct-current series motor is practically the only type of motor that can be used for street-railway work, and it is also used in railway electrification. In the congested city districts, where the consumers' loads are large and close together, direct-current power is preferable to alternating-current power, as capacitive effects in the underground cables are not present when direct current is used, and inductive effects in the system are also absent with direct current. Furthermore, in such loads, the importance of continuity of service requires that a large storage-battery reserve be available. This, again, is an additional reason for supplying direct-current service in such districts. Also direct-current motors are well adapted to elevators and to printing-press drive which form a considerable portion of the power load in a large city.

As the power supply in the above cases is almost always alternating current, this alternating current must in some manner be changed to direct current. There are several methods of accomplishing this, the most common being the following:

1. Mechanical rectifier—commutating type.
2. Mechanical rectifier—vibrating type.

3. Mercury-arc rectifier.
4. The tungar rectifier.
5. Electrolytic rectifier.
6. Induction- or synchronous-motor-generator sets.
7. Rotary or synchronous converter.

170. Types of Rectifiers and Converters.—(*a*) *Rectifying Commutator.*—The rectifying commutator is a commutator driven by a synchronous motor. The segments are so connected that when the alternating current reverses, the connections to the direct-current circuit are simultaneously reversed, as shown in Fig. 331. A unidirectional current is thus obtained. As the brushes cannot have zero width, it is difficult to commutate at the point of zero current, and the current and voltage are rarely

Fig. 331.—Commutating-type rectifier.

zero at the same time. Hence, such devices spark more or less and so are limited to small currents and voltages.

(*b*) *Vibrating Rectifier.*—The vibrating rectifier (Fig. 332) is based on the same principle as the rectifying commutator, except that the circuit connections are reversed by contacts which are opened and closed, synchronously, by alternating-current magnets and a polarized armature. This type of rectifier ordinarily is designed for use on 110-volt, 60-cycle circuits. The circuit voltage is reduced by means of a step-down transformer, the secondary of which has a middle tap. This secondary excites two series-connected, alternating-current magnets, which are so connected that they both have the same polarity on corresponding ends at every instant. The vibrator is a soft-iron bar magnet, pivoted below these alternating-current magnets, each of its two ends being directly beneath one of the alternating-current magnets. This bar magnet is excited by direct current taken from battery terminals and has, therefore, a fixed polarity. Assume that, at some particular instant, the right-hand end of

the transformer secondary is positive. By following through the circuits in Fig. 332, it is seen that both the lower ends of the alternating-current magnets are north poles. Also, the left-hand end of the bar magnet is a north pole, and its other end is a south pole. This left-hand end is, therefore, repelled downward, and the right-hand end is attracted upward. This closes the

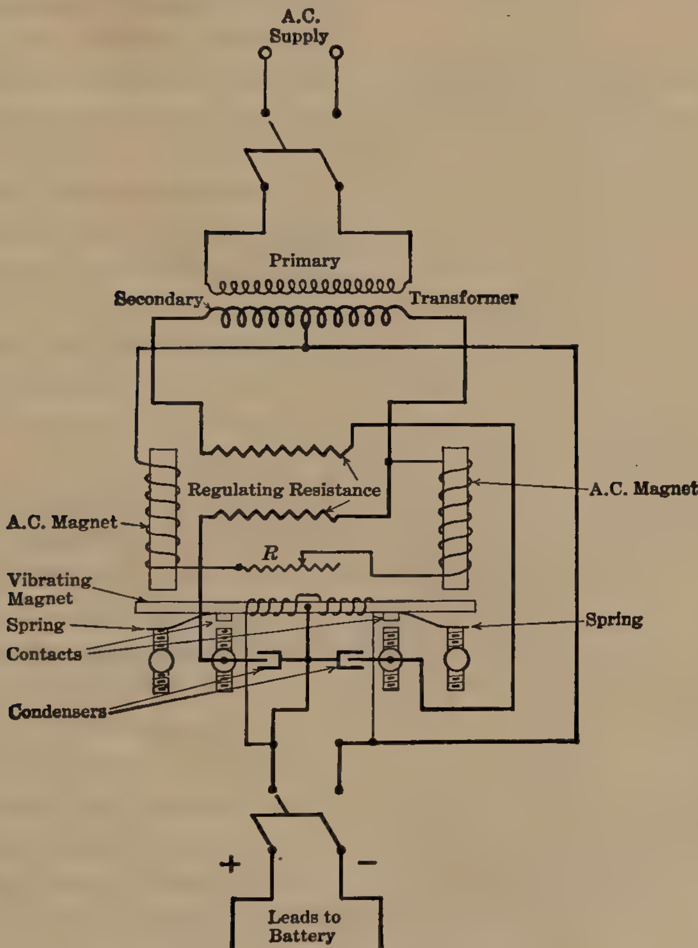


FIG. 332.—Vibrating rectifier.

left-hand contact, which allows current to flow into the left-hand battery terminal, assumed to be positive. During the next half-cycle, the left-hand end of the transformer is positive and the right-hand end of the magnet is repelled. This closes the right-hand contacts, and current still flows into the positive or left-hand side of the battery. Hence, the battery receives a uni-

directional current and may be charged from alternating-current supply. The contact should open when the current is zero. This adjustment is made by means of the resistance R, which shifts the phase of the current in the alternating-current magnets. Condensers are connected across the contacts in order to minimize sparking. It makes no difference how the battery is connected, as the direction of excitation of the vibrating magnet causes the current always to flow into the positive battery terminal.

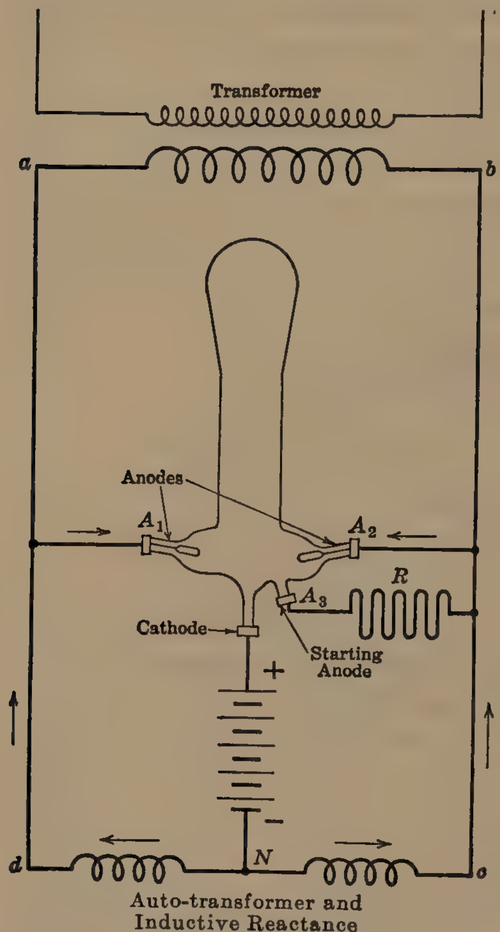


Fig. 333.—Mercury-arc rectifier for low voltages.

This type of rectifier is designed for 8 amp. at from 8 to 10 volts. Owing to difficulties, due primarily to wave form, it has not been entirely satisfactory in practice.

(c) *Mercury-arc Rectifier.* The mercury-arc rectifier has already been mentioned in connection with the constant-current transformer (see p. 259).

It operates on the principle of electron emission, described in Chap. XIV, combined with ionization of gas or vapor molecules by collision (see p. 464 and Vol. I, p. 244, Par. 178). In order to obtain the best results, the tube or tank must have a very high vacuum, 0.005 to 0.0001 mm. of mercury. The lower terminal (Fig. 333) is the cathode to which current goes from the tube. This cathode consists of a pool of mercury. The two terminals A_1, A_2 are the iron anodes from which the current enters the tube. A_3 is a starting anode, by means of which the mercury arc is established. When an arc occurs between an anode

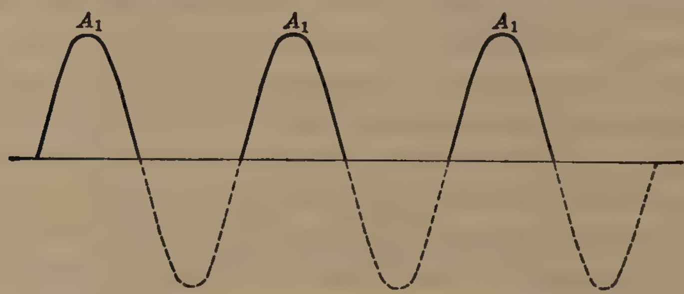
and a cathode, it concentrates at a single spot on the mercury pool, called the *cathode spot*, and its temperature is raised by 2000 to 3000° C. This causes a copious emission of electrons from this cathode spot. These are attracted to the anode which is positive at the time and are repelled by the anode terminal which is negative at this time. Because of their high velocities, these electrons by collision ionize the mercury vapor, the negative ions going to the positive anode, and the positive ions to the cathode spot, thus causing current flow from anode to cathode. Furthermore, the resulting bombardment of the cathode spot by these heavy positive ions striking at high velocity maintains it at the necessary high temperature. By the foregoing action, current can thus pass from the positive anode to the cathode.

The anodes are comparatively cool, 400 to 800° C., which is below the temperature at which they can emit electrons freely. Hence, at the time an anode terminal is negative, the electrons in its neighborhood are all drawn to the cathode, ionization by collision ceases, and no current can pass from cathode to anode. Likewise, current cannot pass from anode to anode. It is thus seen that the rectifying action occurs at the anodes, not at the cathode. Current, therefore, can *enter* the tube from either anode, A_1, A_2 (Fig. 333), depending upon which side of the transformer secondary ab is positive. The current can then pass through the cathode to the load. If only one anode were used, the negative half of the alternating-current wave would be eliminated in each cycle, and the resultant wave would appear as shown in solid line (Fig. 334 (*a*)). This condition of operation could not be maintained with the mercury arc, because the arc is extinguished as soon as the current becomes zero.

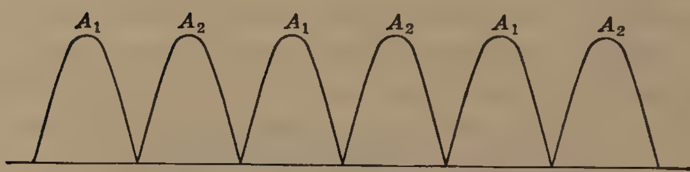
To obtain a continuous flow of current through the tube, two anodes A_1, A_2 are necessary, one anode being connected to each end of the transformer secondary. When one end of the transformer becomes negative, the other becomes positive, so that either one anode or the other is always positive. Current is always entering the tube, therefore, from either one anode or the other. Were there no inductance in circuit, the rectified wave under these conditions would appear as shown in Fig. 334 (*b*). The portions of the wave marked A_1 are due to anode A_1, and those marked A_2 to anode A_2. Each of these portions reaches

the zero value twice for each cycle of current supply. This would cause the arc to be extinguished. By introducing inductance in the circuit, however, the current is held over the zero point, and the resulting wave is similar to that shown in Fig. 334 (*c*), being more or less pulsating in character.

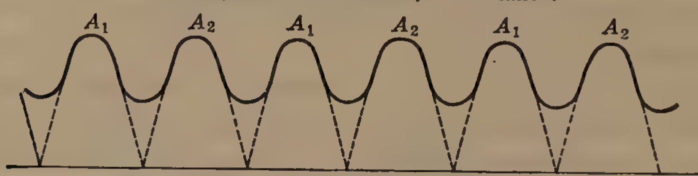
The direct current leaves the cathode, enters the positive terminal of the battery to be charged (or other translating device), and flows to the neutral of the auto-transformer.



(*a*) Negative half of wave cut off.



(*b*) Negative half rectified; no reactance.



(*c*) Negative half rectified; reactance producing a pulsating uni-directional current.

FIG. 334.—Rectified alternating-current waves.

The operation of the auto-transformer is as follows: Assume that, at some particular instant, terminal *b* of the transformer secondary (Fig. 333) is positive, and terminal *a* negative. Current obviously attempts to pass from *b* to *a* through some external circuit. One path is by way of the anode A_2, the tube, the cathode, and through the battery to the neutral N of the auto-transformer. As some of this current must return to terminal *a* of the transformer secondary, it attempts to pass through the winding Nd of the auto-transformer. A part of the current does

pass through this winding and in so doing creates a flux in the core of the auto-transformer which induces an e.m.f. in the winding Nc. The direction of this e.m.f. is such as to cause the remainder of the current to flow from N to c. This current flows through the local circuit NcA_2. This, it will be remembered, is the principle of the autotransformer (see p. 245, Par. 119).

The anode A_3 is for starting purposes only. When the tube is tilted, a conducting stream of mercury is established between A_3 and the cathode. The resulting current flow vaporizes some

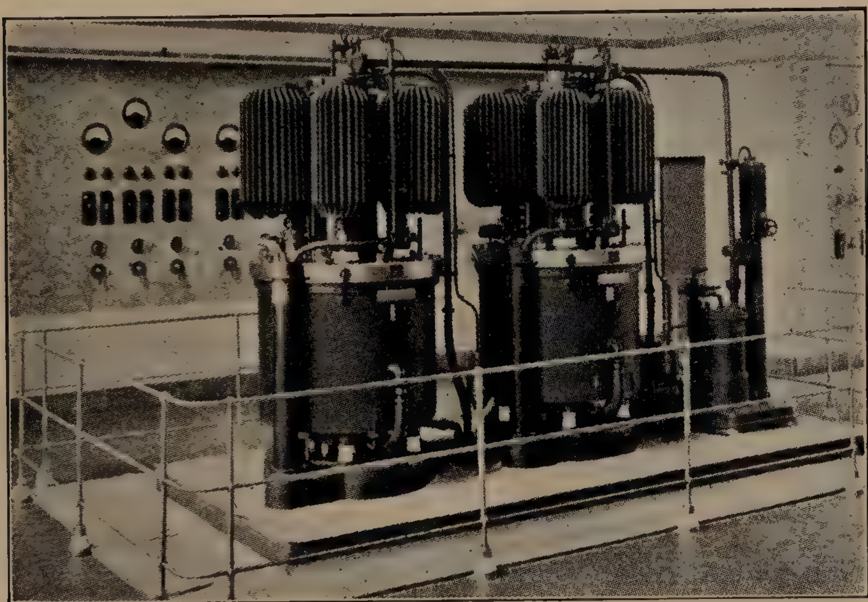


FIG. 335.—Brown-Boveri rectifiers with exhaust pump at the 1,000-kw., 600-v. substation of the Zurich Municipal Electrical Supply.

of this mercury and so establishes the arc. A ballast resistance R is necessary in order to limit the current at starting, since there is then a metallic path of low resistance between A_3 and the cathode.

Until recent years, this type of rectifier has not been developed in large capacities, the ratings being limited to approximately 50 kw., owing to the glass tubes' being fragile. Of late, however, the Brown Boveri Co.[1] of Baden, Switzerland, has been able, by employing iron tanks, to develop mercury-arc rectifiers in large

[1] MARTI, O. K., and H. WINOGRAD: "Mercury-arc Power Rectifiers," *Jour. A. I. E. E.*, p. 818, August, 1927.

power ratings. These rectifiers are built in capacities from 220 kw. at 220 volts direct current up to 2,700 kw. at 5,000 volts direct current. Two installed rectifiers of this type, having a total capacity of 1,000 kw., are shown in Fig. 335. These power rectifiers usually operate six-phase, so as to reduce the ripples in the direct-current voltage wave. The six anodes for six-phase operation show plainly above the tank (Fig. 335). The voltage drop in the mercury arc is about 25 volts and is practically independent of the current. Hence, the efficiency increases with the voltage rating. Also, this voltage characteristic gives high efficiency over wide ranges of load.

At 600 volts and over, these rectifiers have higher efficiencies than the synchronous converter, particularly at the lighter loads. At this time (1928), a total of 600,000 kw. of such rectifiers is installed in the United States. Although their use in this country is still in the experimental stage, their operation seems to have been eminently satisfactory where they have been tried.

(*d*) *Tungar.*[1]—The tungar is based on the following principle: An incandescent filament emits minute negative charges called *electrons.*[2] When the discharge of these electrons occurs in an electrostatic field, the electrons attain considerable velocity. If a gas is present, these electrons collide with the gas atoms and ionize them. That is, when an electron collides with an atom of gas, that atom is broken up into an electron and a positive ion. The region in which this action occurs then becomes *ionized.* Ionized gas is a conductor of electricity.

Figure 336 (*a*) shows a glass bulb containing an inert gas, usually argon, at reduced pressure, and also an ordinary coiled tungsten filament. Near the filament is a graphite anode. A transformer *ab* steps down the supply voltage, and the filament is connected across its secondary. The filament then becomes incandescent and tends to emit negative charges or electrons.

One terminal of the transformer secondary *c* and one end of the filament are connected to the transformer primary at *b*. The filament is then at practically the same potential as that of the power-supply line *b′b*. The voltage of the battery being

[1] For complete description, see by R. E. Russell: "The Tungar Rectifier," *Gen. Elec. Rev.*, 1917, p. 209.

[2] See Chap. XIV.

charged is somewhat less than the voltage between line $a'a$ and line $b'b$. The potential of the graphite anode, therefore, is different from the potential of point c, usually by approximately 5 or 6 volts. Consequently, during one half-cycle, the potential of the filament is negative with respect to that of the anode; and during the next half-cycle, its potential is positive with respect to that of the anode.

When the filament is negative, the negative charges or electrons are repelled by it, because like charges repel each other. These electrons attain a considerable velocity and break up the gas

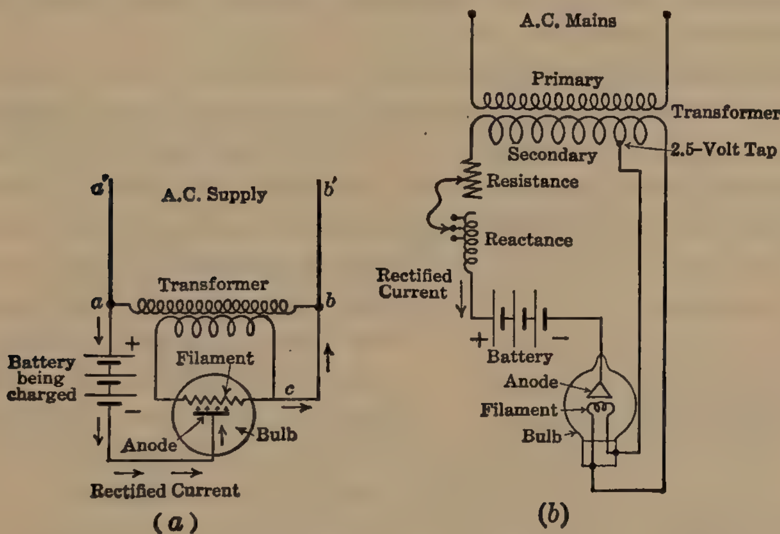


FIG. 336.—Tungar rectifier.

particles into ions. The region between the filament and the anode becomes conducting, and as a result current flows from a into the positive terminal of the battery, through the battery to the anode, to c and then to b.

When the filament is positive, the electrons or negative charges which it tends to emit due to its incandescence are attracted toward the filament, since positive and negative charges attract each other. Consequently, the electrons which produce the ionizing action are withdrawn from the region between the filament and the anode. As a result, the gas is no longer ionized, and it ceases to be a conductor. No current can flow, therefore, during this half-cycle. The current can flow only in one direction, therefore, from the graphite to the filament, and the device acts

as a rectifier. A very small and almost negligible part of the current is due to the electrons themselves, which act as carriers of negative electricity from filament to anode.

Figure 336 (*b*) shows the connections for one commercial type of low-voltage tungar, the switches and cutouts being omitted. Both the current to be rectified and the current, for heating the filament are supplied by the transformer secondary, the filament being connected between a 2.5-volt tap and one end of the secondary. Current regulation may be obtained by adjusting the resistance and the reactance. Where electrical connection between load and primary mains is permissible, an auto-transformer with taps may be used.

The devices shown in Fig. 336 eliminate the negative half of each wave, but this is not a serious disadvantage when ordinary batteries are being charged. A two-bulb rectifier supplying a continuous, pulsating current is also manufactured, however. The efficiency of the tungar rectifier is from 35 per cent. in the smaller sizes to 75 per cent. in the larger sizes. The capacities at present are not much in excess of 750 watts. (The kenotron and other thermionic valves are described in Chap. XIV.)

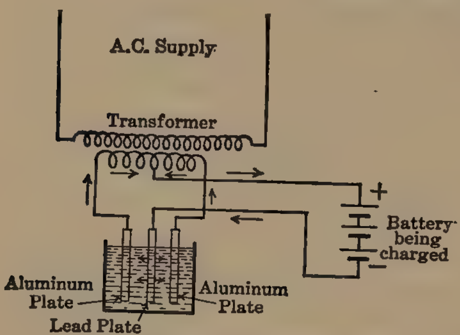


Fig. 337.—Electrolytic rectifier.

(*e*) *Electrolytic Rectifiers.*—Electrolytic rectifiers are based on the following principle: If a lead plate and an aluminum plate be immersed in a sodium- or ammonium-phosphate solution, current can pass from the solution to the aluminum. As soon as the current attempts to reverse and pass from the aluminum to the solution, a thin insulating film of aluminum oxide is instantly formed over the aluminum plate and acts as an insulator up to about 150 volts. This prevents the current's flowing from aluminum to solution, and such a device may be used, therefore, as a rectifier. Figure 337 shows such a simple rectifier, giving a continuous pulsating current like that shown in Fig. 334 (*b*).

Such rectifiers are of low efficiency, 60 per cent. and lower, and of small capacity. They are used primarily for charging low-

voltage batteries from alternating-current supply. Their advantage lies in their cheapness and simplicity.

(*f*) *Induction- or Synchronous-motor-generator Sets.*—With the exception of large capacity mercury-arc rectifiers, none of the foregoing devices is capable of converting alternating to direct current on the large scale required in modern power systems. To convert large amounts of power, induction-motor- or synchronous-motor-generator sets may be employed. The capacity of such units is limited only by the size in which it is possible to construct the direct-current generator. The disadvantage of a motor-generator set is that it requires two machines, with corresponding cost and floor space, and the overall efficiency is not extremely high, being the product of the efficiencies of the individual units of the set.

(*g*) *The Rotary or Synchronous Converter* is a single machine which converts alternating to direct current, or *vice versa*, and may be built to convert large amounts of power efficiently and economically. Because it has only one armature and one field, the synchronous converter usually costs less than an equivalent motor-generator set. Because the armature current is small, being the *difference* between the alternating and the direct currents, this type of machine has a high efficiency when operating under favorable conditions.

171. Principle of Synchronous Converter.—It has already been demonstrated that alternating current is generated in the armature coils of the ordinary direct-current generator. If taps be brought out properly from the armature winding to slip-rings, alternating current may be taken from this same winding, and the machine becomes an alternator. Such an alternator can obviously operate as a synchronous motor.

The synchronous converter is constructed like the ordinary direct-current generator, although the relative dimensions may be different. It has fixed poles, a rotating armature, a commutator, a shunt field, and usually a series field. In addition to the commutator, however, leads are taken from the armature to slip-rings, in the manner shown in Fig. 338 (also see Figs. 339 and 340). Figure 338 represents a two-pole, single-phase converter.

In the synchronous converter, as commonly used, alternating current is supplied to the slip-rings, and direct current is taken

from the commutator and brushes. If, however, the direct-current brushes be open circuited or removed, the machine becomes, under these conditions, a synchronous motor of the rotating-armature type. On the other hand, if direct current be supplied to the brushes and commutator, and the slip-ring brushes be disconnected, the machine becomes a shunt or compound motor.

If the machine be driven mechanically, and current be taken from the slip-rings only, it becomes an alternator. On the other

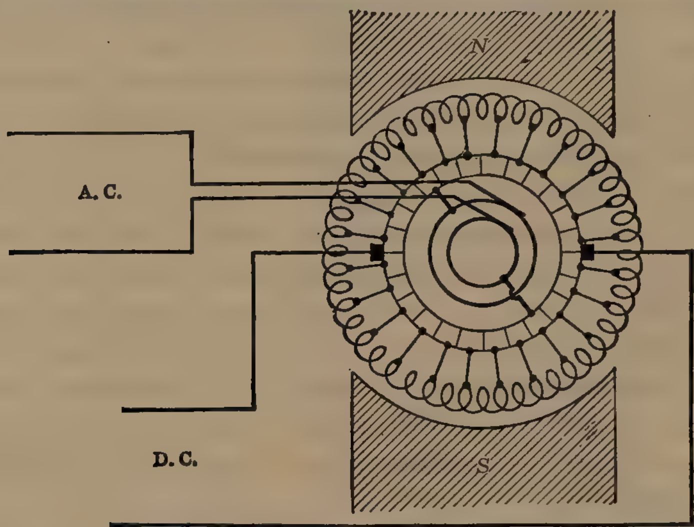


Fig. 338.—Two-pole, single-phase synchronous converter.

hand, if current be taken from the commutator only, it becomes a direct-current generator. Both alternating and direct current may be taken from it simultaneously, and it then becomes a *double-current* generator.

In the synchronous converter as ordinarily used, alternating current is supplied to the slip-rings so that the machine operates as a synchronous motor, so far as the alternating-current side is concerned. At the same time, direct current is taken from the commutator and brushes, and, therefore, this side of the machine has characteristics very similar to those of a shunt or compound generator. When operated in this manner, the machine is said to be a *direct* synchronous converter.

The machine, however, may take power from the direct-current supply, operating as a direct-current motor, and deliver alter-

nating current from the slip-rings. When operated in this manner, the machine is said to be an *inverted* synchronous converter. This is not the usual method of operation.

172. Polyphase Converters.—The output of a converter increases materially with the number of phases. For example, the rating of a six-phase converter is more than twice its rating when operated single-phase (see p. 401).

The connections of polyphase converters are comparatively simple. For example, the four-phase converter shown in Fig. 339 requires four slip-rings. The points at which the slip-rings

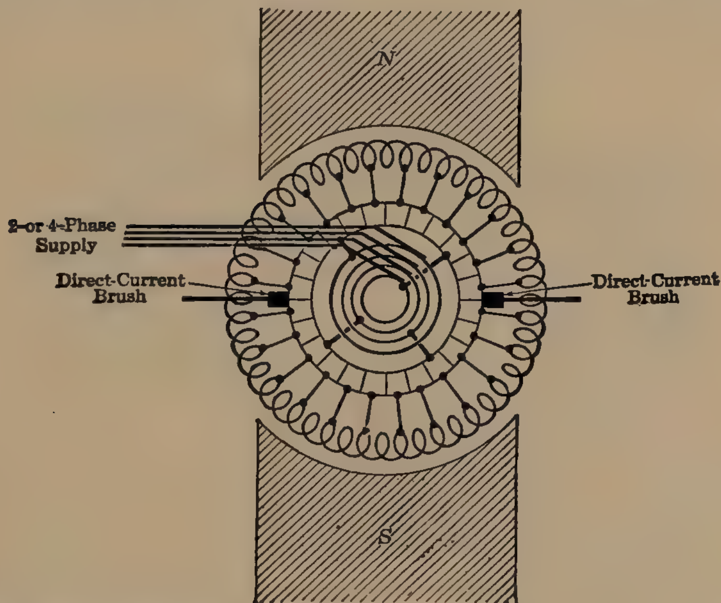


FIG. 339.—Two-pole, four-ring, four-phase synchronous converter.

are connected to the winding are 90 space-degrees apart in the two-pole type. If the machine has four poles, two taps from each ring to the winding are necessary. This is illustrated in Fig. 340, in which a three-phase, four-pole converter is shown. Two taps run from each ring to the winding; in this case, the taps are diametrically opposite. For example, if the tap from one ring connects to a portion of the winding which at some particular instant is under the center of a north pole, then there must be similar taps running from this same ring to every point of the winding which lies at that instant under the center of a north pole (see points *a*, *a*, Fig. 340).

A six-phase, six-pole synchronous converter will have six slip-rings and three taps from each slip-ring, making a total of 18 taps to the winding.

A simple rule for obtaining the number of taps to the winding is to remember that if the machine has n phases, there must be n slip-ring taps for every 360 electrical space-degrees, or for every *pair* of poles. (This does not hold for single-phase.) For example, in Fig. 340, there must be three taps for each pair of

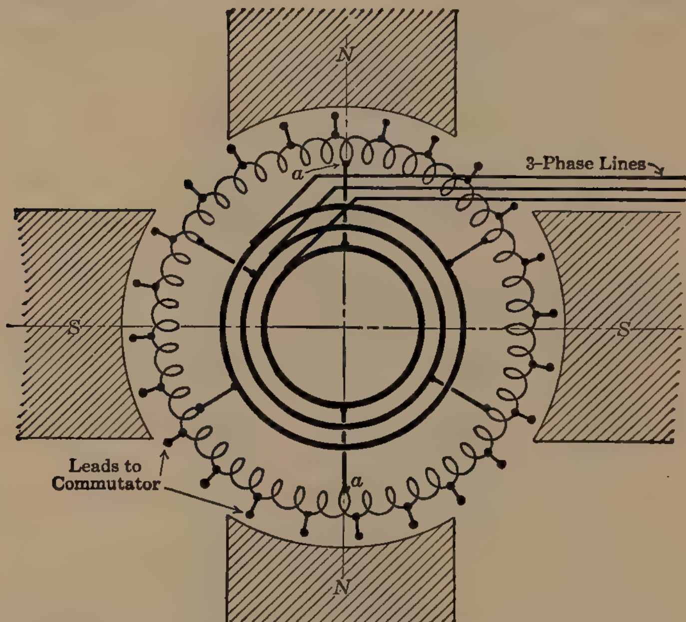


FIG. 340.—Three-phase, four-pole synchronous converter.

poles, or six taps in all. Figure 351 (p. 402) shows how the taps are brought out from the armature to the slip-rings in a 14-pole, six-phase converter.

It is to be noted that the slip-ring taps must be brought out at equidistant points along the winding, in order that the alternating voltages may be balanced. Hence, the direct-current windings that can be used for a synchronous converter are more or less restricted, for the number of coils must be divisible by the number of slip-ring taps.

173. Single-phase Voltage Ratios in Synchronous Converter. In a synchronous converter, both the alternating and the direct-current e.m.fs. are induced by the same system of conductors,

cutting the same field. There must be a fixed ratio, therefore, between the direct-current and the alternating-current *induced* e.m.fs.

In a single-phase converter, there are the same number of active conductors between the direct-current brushes as between the alternating-current slip-rings, as will be seen in Fig. 338. The same number of conductors, cutting the same field, gives both the direct-current e.m.f. and the single-phase e.m.f.

It will be remembered that the e.m.f. between the brushes of a direct-current generator is the sum of the e.m.f. waves generated in each of the individual conductors connected in series between the brushes. The resulting e.m.f. is the peak value of the result-

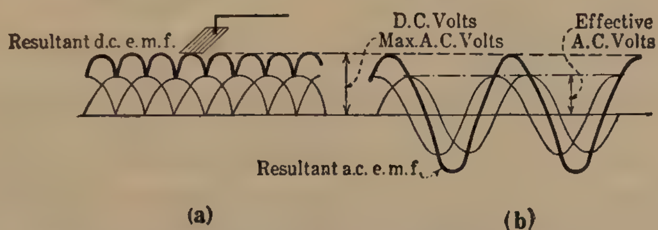


FIG. 341.—Relation between direct and alternating induced e.m.fs. in synchronous-converter armature.

ing wave, as is shown in Fig. 341 (*a*) (also, see Vol. I, p. 265, Pars. 190 and 191). For simplicity, Fig. 341 (*a*) shows only the wave resulting from two coils between slip-rings spaced 90° apart.

In a single-phase machine, there are just as many conductors between the slip-ring taps as between the brushes. The resultant alternating e.m.f. wave between slip-ring taps is, therefore, found by adding together the alternating waves, 90° apart, point by point, as shown in Fig. 341 (*b*). Comparing Figs. 341 (*a*) and 341 (*b*), it will be noted that the direct-current e.m.f. is equal to the *peak* value of the alternating e.m.f.

Therefore, in a single-phase converter, the direct-current induced e.m.f. is equal to the $\sqrt{2}$ *times the effective value of the single-phase alternating-current induced e.m.f.* This ratio may be modified by wave form as in the split-pole converter (p. 407).

174. Polyphase Voltage Ratios in Synchronous Converter.—It will be remembered that the total single-phase e.m.f. generated in an alternator armature is the vector sum of the individual

inductor e.m.fs., as shown in Fig. 342. In (*a*), the several conductors upon the surface of the armature are shown. In (*b*) are the vector e.m.fs. generated in the various conductors, together with their vector sum (also see p. 151, Par. 83). The total single-phase voltage is the diameter of a circle drawn to the proper scale, as shown in Fig. 342 (*b*). The three-phase

Fig. 342.—Relation of induced e.m.fs. to belt span, in a closed armature winding.

e.m.f. is the vector sum of the individual e.m.fs. included within a 120° arc (Fig. 342 (*b*)). The four-phase e.m.f. is the vector sum of the e.m.fs. included within a 90° arc, and the six-phase e.m.f. is the vector sum included within a 60° arc.

This gives a simple method for obtaining the various e.m.f. relations in a converter armature. Draw a circle (Fig. 343)

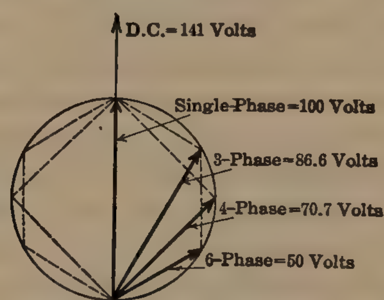


Fig. 343.—Relations existing among voltages in synchronous-converter armature.

whose diameter is 100 units. Let this represent a single-phase e.m.f. of 100 volts, effective. The direct-current e.m.f. will then be $\sqrt{2} \times 100 = 141.4$ volts, which is shown by extending the diameter. The three-phase e.m.f. is the length of a chord subtending an arc of 120°, or 86.6 volts. The four-phase e.m.f. is the length of a chord subtending 90°, or 70.7 volts. The six-

phase e.m.f. is the length of a chord subtending 60°, or 50 volts.

Below are tabulated these results.

ELECTROMOTIVE FORCES

Direct current	Single-phase	Three-phase	Four-phase	Six-phase
141.4	100	86.6	70.7	50
	Ratio of alternating-current e.m.fs. to direct-current e.m.f.			
	0.707	0.612	0.50	0.354

175. Current Ratios in Synchronous Converter.—The relations between the direct and alternating currents in a synchronous converter may be determined as follows:

Single-phase:

If the efficiency is assumed to be 100 per cent. and the power factor unity, neglecting voltage drops in the armature,

$$VI = V_1 I_1$$

$$\frac{I_1}{I} = \frac{V}{V_1} = \frac{141.4}{100} \qquad I_1 = 1.414I \tag{126}$$

where V and I are the direct-current voltage and current, respectively, and V_1 and I_1, are the single-phase voltage and current, respectively.

If the efficiency be η and the power factor $P.F.$,

$$VI = V_1 I_1 \times P.F. \times \eta.$$

The approximate single-phase current

$$I_1 = \frac{1.414I}{\eta \times P.F.} \tag{127}$$

In practice, the efficiency is from 92 to 96 per cent. and the power factor is rarely allowed to drop below 0.9.

Three-phase:

At 100 per cent. efficiency and unity power factor,

$$VI = \sqrt{3} V_3 I_3$$

where V_3 is the three-phase line voltage and I_3 the three-phase line current.

$$I_3 \doteq I\frac{V}{V_3\sqrt{3}}$$

neglecting voltage drops in the armature,

$$\frac{V}{V_3} = \frac{141.4}{86.6} = 1.63 \text{ (Par. 174)}$$

$$I_3 = 0.943I. \tag{128}$$

If the efficiency be η and the power factor $P.F.$, the approximate three-phase line current

$$I_3 = \frac{0.943\,I}{\eta \times P.F.}. \tag{129}$$

(With unity power factor and with the usual efficiency, $I_3 = I$, nearly.)

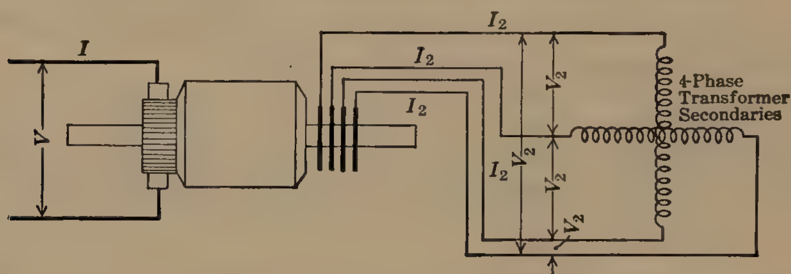


FIG. 344.—Currents and voltages in four-phase synchronous-converter armature.

Four-phase:

At 100 per cent. efficiency and unity power factor,

$$VI = 2\sqrt{2}V_2I_2$$

where V_2 is the voltage between adjacent lines and I_2 is the four-phase line current (Fig. 344).

$$I_2 = \frac{VI}{2\sqrt{2}\,V_2}, \quad \frac{V}{V_2} = 2$$

$$I_2 = I\frac{2}{2\sqrt{2}} = \frac{I}{\sqrt{2}} = 0.707I. \tag{130}$$

If the efficiency be η and the power factor $P.F.$, the approximate four-phase line current

$$I_2 = \frac{0.707I}{\eta \times P.F.}. \tag{131}$$

Six-phase:

The six-phase system may be considered as composed of two Y-systems, or two delta-systems, each having one-half the capacity of the six-phase system (see Par. 182, p. 409). Figure 345 shows a six-phase double-Y connection in which the six-phase voltages between adjacent lines and to neutral are V_6. A current I_6 flows in each line. As the six phases are all connected together at the neutral, this system may be split into two equal Y-systems (Fig. 345 (*b*)), each having V_6 volts to neutral. The output of each Y-system at unity power factor is $3V_6I_6$ watts.

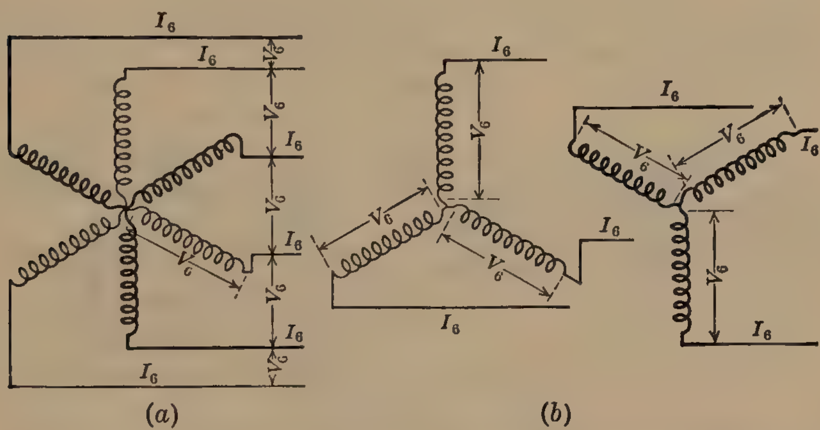


FIG. 345.—Currents and voltages in double-Y, six-phase system.

At 100 per cent. efficiency and unity power factor,

$$VI = 2(3V_6I_6) = 6V_6I_6$$

$$I_6 = \frac{V}{6\ V_6}I$$

but

$$\frac{V}{V_6} = \frac{141.4}{50} = 2.828 \text{ (Par. 174)}$$

$$I_6 = \frac{2.828I}{6} = 0.471I. \tag{132}$$

If the efficiency be η and the power factor *P.F.*,

$$I_6 = \frac{0.471I}{\eta \times P.F.}. \tag{133}$$

(With unity power factor and the usual efficiency, $I_6 = 0.5I$, nearly.)

Summarizing for unity power factor and 100 per cent. efficiency):

Number of slip-rings	Number of phases	Ratio $\frac{I_{AC}}{I_{DC}}$
2	1	1.414
3	3	0.943
4	4	0.707
6	6	0.471

Example.—A 500-kw. converter (Fig. 346) has an efficiency of 92 per cent. at full load and operates at a power factor of 0.94. The direct-current voltage is 550 volts. The alternating-current side is operated six-phase.

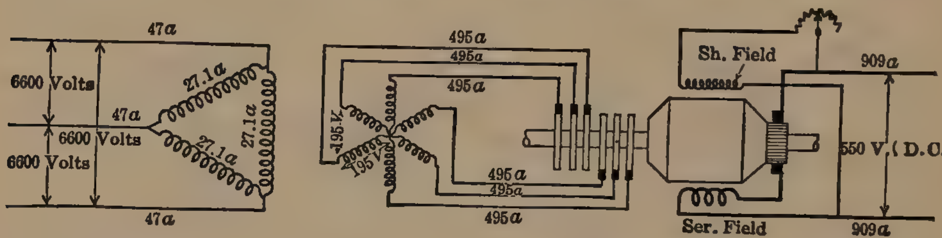


FIG. 346.—Currents and voltages in six-phase, 500 kw., synchronous converter and transformers.

Find the direct current and all the alternating-current line currents and voltages.

$V = 550$

$I = 500{,}000/550 = 909$ amp.

$V_6 = 550 \times 0.354 = 195$ volts between adjacent lines and to neutral.

From Eq. (133),

$$I_6 = \frac{0.471 \times 909}{0.92 \times 0.94} = 495 \text{ amp. per line.} \quad \textit{Ans.}$$

176. Conductor Currents in Armature of Converter.—It has already been pointed out that the synchronous converter has a high efficiency because the *net* current in each armature conductor is the *difference* between the alternating current and the direct current which would of themselves exist in that conductor. The reason for this is obvious. The alternating current entering through the slip-rings is a *motor* current, driving the machine as a synchronous motor, and is, therefore, in opposition to the induced e.m.f. The armature current which is delivered by the commutator to the brushes is a *generator* current and is, therefore,

in conjunction with the induced e.m.f. Both the alternating and the direct current utilize the same conductors, rotating in the same field. Under these conditions, the two currents must flow in opposite directions. The *net* current in each conductor must be the *difference*, therefore, between the motor current and the generator current.

The wave form for the resultant current in the various conductors is very irregular and differs for the different armature conductors. The value of the resultant current also differs in the different conductors.

Consider conductor a (Fig. 347) which lies midway between two slip-ring taps. First consider the direct current in this conductor as the conductor moves through successive positions 1, 2, 3, 4. If the load be assumed constant and the width of the brush be neglected, the direct current will be positive and will not vary as the conductor moves from (1) to (2) to (3). At (3), the brush position, the current reverses abruptly and then remains constant until the conductor reaches position (1). This is shown in Fig. 348 (a).

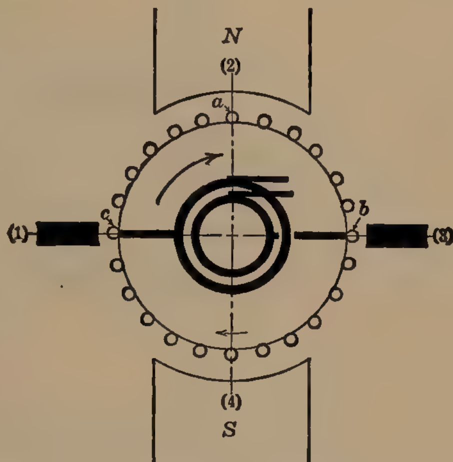


Fig. 347.—Relative positions of conductors and slip-ring taps.

The conductor a is midway between slip-ring taps, so that it is at the center of the alternating-current phase belt which is included between these slip-ring taps. The phase of the e.m.f. in a is the same as that of the resultant e.m.f. of the entire belt. This is evident from a study of Fig. 342 (p. 392), although a conductor at the exact center of the winding is not shown in that figure.

Assume that the current is in phase with the induced e.m.f. When a is in position (1) (Fig. 348), the alternating current in the entire phase belt is zero; when a reaches position (2), the current is a maximum; etc. This current is plotted in Fig. 348 (a), a sine wave being assumed. The alternating current is opposed to the direct current, since one is a motor current and the other

a generator current for the same induced e.m.f. The resultant current is found by adding the two currents, point by point, the result being shown in Fig. 348 (*b*). This resultant current is irregular in form, and its effective value is small compared to that of either of the component currents.

This resultant current, though periodic, is not a sine wave and, therefore, must be made up of a current wave of fundamental frequency and higher harmonics. As the current is assumed to be in phase with the induced e.m.f., the product of this current

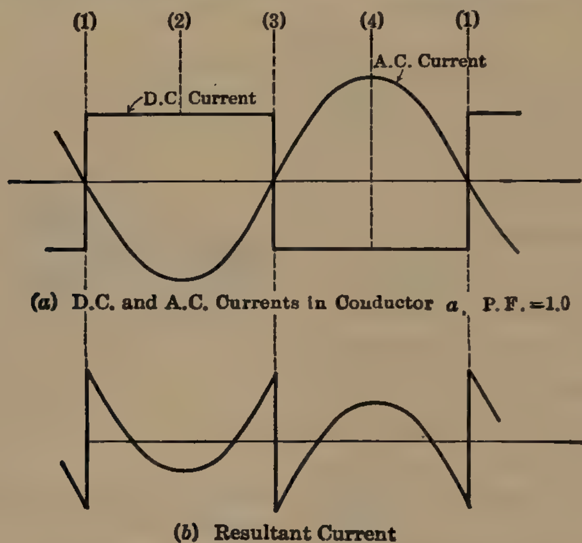


FIG. 348.—Current at unity power factor in a conductor midway between slip-ring taps.

of fundamental frequency and the induced e.m.f. gives the power necessary to supply the rotational losses, which include friction, windage, and core losses.

Next, consider conductor *b* (Fig. 347), at one of the slip-ring taps but in the same phase belt as *a*. As this conductor passes through the successive positions (1), (2), (3), and (4), the *direct* current is the same for each position of *b* as it was for the corresponding position of *a*. This direct current is plotted in Fig. 349 (*a*). The *alternating* current in *b* must be the same as in *a*, for the two are in the same phase belt and so are in series. When conductor *b* is in position (1), *a* is in position (4), and, therefore, the current in both *a* and *b* is a positive maximum, from Fig. 348. When *b* reaches (2) the current is zero, etc. This current is

plotted in Fig. 349 (*a*). The resultant current is shown in Fig. 349 (*b*).

It will be noted that the resultant current in conductor *b* is *distinctly greater* in magnitude than the current in conductor *a* (Fig. 348 (*b*)). The heating in the conductors nearer the slip-ring taps will, therefore, be greater than it is in the conductors midway between taps. On the other hand, it can be similarly shown that the heating in conductor *c*, in the same phase belt as

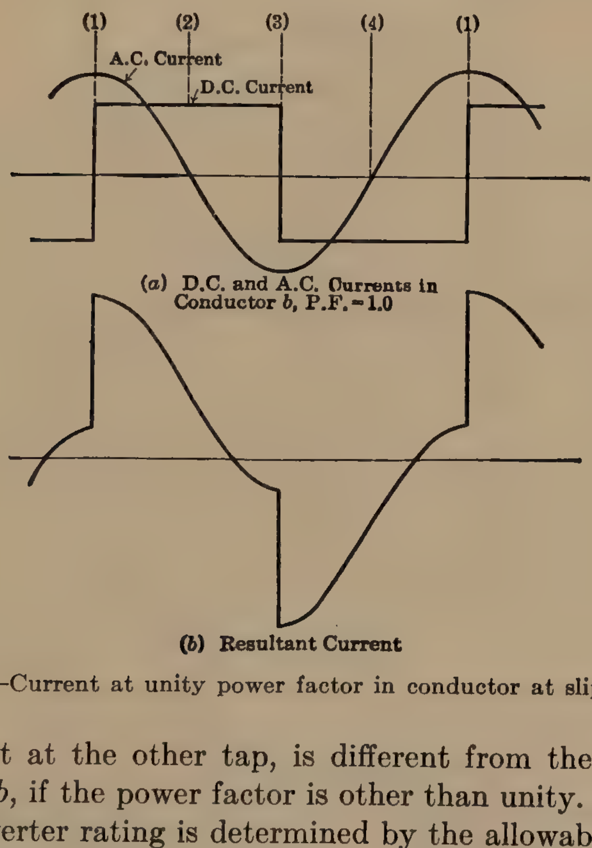


Fig. 349.—Current at unity power factor in conductor at slip-ring tap.

a and *b* but at the other tap, is different from the heating in either *a* or *b*, if the power factor is other than unity.

The converter rating is determined by the allowable temperature of the hottest part of its armature. Although the conductors midway between slip-ring taps are operating at temperatures lower than the allowable safe values, the converter rating must be adjusted to conform to the safe temperature limits of the conductors whose temperature is highest.

The greater the number of phases the greater will be the number of slip-ring taps. This will produce a lesser temperature range due to difference in position of the various armature con-

ductors, because the resultant of the direct and the alternating current for conductors located near the slip-ring taps, which conductors operate at the highest temperature, is decreased in magnitude. The average heating for all the conductors will be reduced, which will permit an increase in rating for the converter. The rating of a given converter increases rapidly with increase

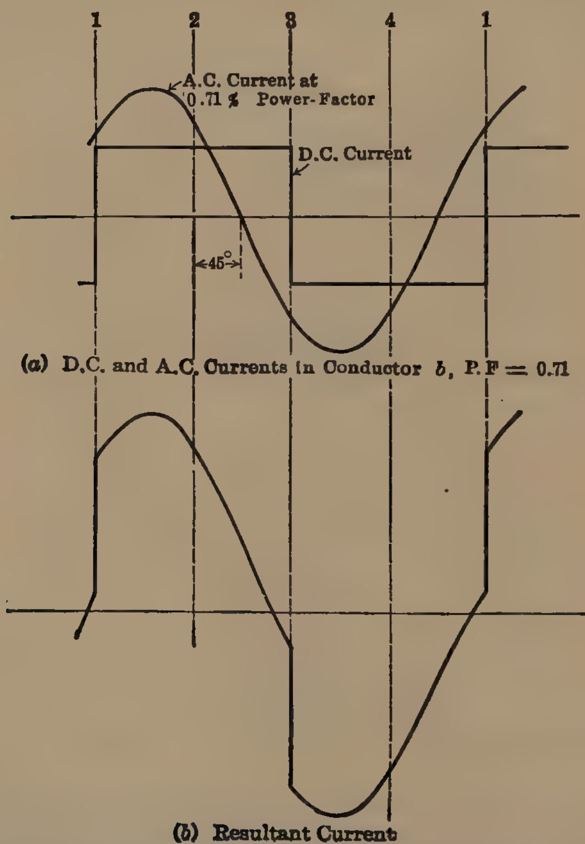


FIG. 350.—Effect of low power factor on current in conductor at slip-ring tap

in the number of phases, as shown in Par. 177 which gives the rating of a converter for different numbers of phases, the output as a direct-current generator being taken as unity.

177. Effect of Number of Phases and of Power Factor on Output of Synchronous Converter.—The considerable gain in rating obtained by operating a converter six-phase is the reason that six-phase converters are so commonly used. The advantage

obtained by operating twelve-phase is usually offset by the added wiring complications.

Number of phases	P. F. = 1.0	P. F. = 0.9
1	0.85	0.74
D. C.	1.00	1.00
3	1.33	1.09
4	1.65	1.28
6	1.93	1.45
12	2.18	1.58

178. Effect of Power Factor on Converter Rating.—The rating and efficiency of a converter decrease much more rapidly with decrease in power factor than is the case with other types of alternating-current machinery. This results from the rapid increase in the resultant current in the converter armature with phase displacement between the alternating and the direct-current waves. Assume that, in Fig. 349, the alternating current lags the induced e.m.f. by 45°. This corresponds to a power factor of 0.71. For the same power and e.m.f., the alternating-current wave must be increased to 1/0.71, or 1.41, times the value shown in Fig. 349. This current wave is shown in Fig. 350 (*a*). It is to be noted that the resultant wave shown in Fig. 350 (*b*) has been increased considerably in magnitude over the value shown in Fig. 349 (*b*). Hence, for the same heating in the two cases, it would be necessary to lower by a considerable amount the output of the converter operating at a power factor of 0.71. The Table in Par. 177 shows the large reduction in rating caused by lowering the power factor from unity to 0.9.

At values of power factor other than those near unity, the synchronous converter loses most of its advantages over the motor-generator set. A converter should be operated at a power factor which is, therefore, very nearly unity.

179. Armature Reaction in Converter.—At unity power factor, the resultant current in a converter armature is comparatively small, as shown in Fig. 348 (*b*). The armature reaction is correspondingly small, therefore, and there is practically no distortion of the field. As a result, the machine commutates

very much better than when operating as a direct-current generator carrying the same load. When the power factor decreases, the resultant armature current increases, as shown in Fig. 350 (*b*). As the rotational losses do not change to any great extent with change of power factor, the power necessary to overcome these losses changes only a small amount with change of power factor. Hence, the *energy component* of the fundamental of the resultant current changes only a small amount with change of power factor, since the power necessary to rotate the armature is equal

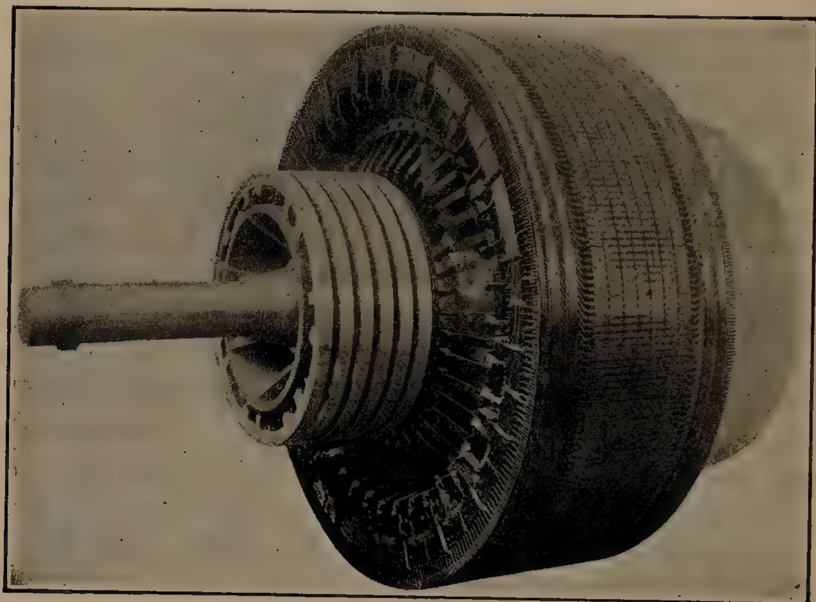


FIG. 351.—Armature of 4,000-kw., 625-volt, General Electric synchronous converter.

to this energy component multiplied by the back e.m.f. At power factors less than unity, therefore, practically the only current which is added to the energy current existing at unity power factor is a quadrature current, lagging or leading the induced e.m.f. by 90 time-degrees. Only the energy component or the component of current in phase with the induced e.m.f. produces *cross-magnetization* (see p. 164). When the converter is operating *direct*, any current in quadrature with the induced e.m.f. merely strengthens or weakens the field, depending upon whether the current lags or leads. Consequently, there is magnetizing action upon the fields when the current lags and demagnetizing action when the current leads (see Chap. XI,

Pars. 156 and 157, pp. 349 and 352). As a result, the added quadrature current merely strengthens or weakens the field but does not distort it. Hence, there is little or no sparking in a converter armature due to field distortion.

It will be remembered (see Vol. I, p. 327) that in a direct-current machine, an e.m.f. of self-induction exists in the armature coils which are undergoing commutation. It is desirable, therefore, that a counter e.m.f., opposite and equal to this e.m.f. of self-induction, be induced in these coils. Otherwise, sparking will exist even if there be no field distortion. In a direct-current generator, this counter e.m.f. is obtained either by moving the brushes ahead of the neutral plane or by the use of commutating poles. This counter e.m.f. assists the current in the coils undergoing commutation to reverse, and better commutation results. This same e.m.f. of self-induction exists in the converter coils which are undergoing commutation. Commutating poles are used in converters, therefore, particularly in those of large capacity, in order to improve commutation. The commutating poles need not be so strong as those which are required for a direct-current machine of the same rating, as there is little or no cross-magnetization to be neutralized.

The resultant current in the armature conductors of a converter, under ordinary conditions of operation, is considerably less than either the alternating or the direct current. A much larger commutator, therefore, in proportion to the armature, is required than would be necessary for a direct-current generator having an armature of the same size. Converter armatures have abnormally large commutators, as shown in Fig. 351.

180. Voltage Control.—The ratio of the direct-current e.m.f. to the alternating-current e.m.f. in a converter armature is fixed, regardless of field excitation. The ratio of *brush* voltage to *slip-ring* voltage, however, may be changed a limited amount by varying the field excitation. The brush voltage and the diametrical slip-ring voltage, increased by $\sqrt{2}$, differ from each other by the *impedance drop* through the converter armature. If this impedance drop changes either in phase or in magnitude, the ratio of brush voltage to slip-ring voltage changes. The impedance drop may be varied in phase and in magnitude by changing the excitation. Weakening the field below the value

which gives unity power factor makes the current lag, increases its value, and lowers the induced e.m.f. (see p. 354, Fig. 312). Strengthening the field above the value which gives unity power factor makes the current lead, increases its value, and raises the induced e.m.f. (see p. 351, Fig. 310). The effect of changing the field excitation is, therefore, to change the power factor, which, in turn, changes the magnitude and phase of the impedance drop in the armature, as has already been explained in connection with the synchronous motor (see pp. 349 and 352). The ratio of brush voltage to slip-ring voltage can, therefore, be changed in this manner. This ratio can be varied by only 2 or 3 per cent. above and below normal, and the voltage ratio and the power factor cannot be adjusted independently.

Series Reactance.—It was shown in Par. 165 (p. 370) that the voltage at the terminals of a synchronous motor can be raised by overexcitation and lowered by underexcitation, provided there is sufficient reactance in the circuit between the motor and the source of constant voltage. As the converter is operating on its alternating-current side as a synchronous motor, it has excitation characteristics similar to those of the synchronous motor. That is, *overexcitation* causes it to take a *leading* current, and *underexcitation* causes it to take a *lagging* current. With series reactance in the alternating-current line, therefore, the alternating voltage may be raised and lowered by changing the excitation (see Par. 165, p. 370). This may be accomplished by hand regulation of the shunt-field rheostat or automatically by means of a regulator or by compounding the machine.

Instead of using special series reactances, the transformers, which are usually necessary with a converter, may be designed to have sufficient leakage reactance for this purpose.

The disadvantage of this method of voltage control is that a change of voltage is accompanied by a change of power factor. Lowering the power factor by any considerable amount is not desirable, because of the decreased efficiency and output which result. The voltage and power factor cannot be changed independently. This method is usually limited, therefore, to less than 10 per cent. variation above and below the normal voltage.

Induction Regulator.—The induction regulator has already been described in connection with the induction motor (see p.

314). This type of regulator may be connected between the transformers and the converter, and the alternating voltage impressed on the converter terminals may be raised and lowered thereby. This changes the direct-current voltage by a corresponding amount. Under these conditions, the voltage may be raised independently of power factor, but the extra equipment is an objection to the use of the induction regulator. Also, it is difficult to build induction regulators capable of withstanding mechanically the shocks to which they are subjected during sudden heavy overloads and short circuits on the converter. This is particularly true of converters which supply electric railways with power.

Series Booster.—A low-voltage alternator is often connected to the shaft of the converter. This alternator has the same number

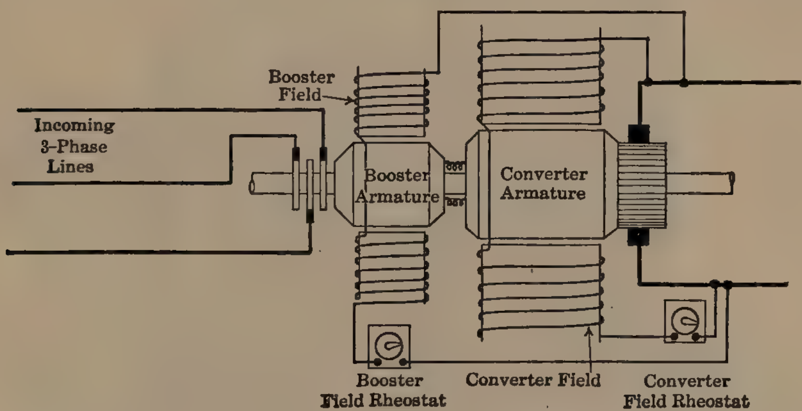


FIG. 352.—Synchronous converter with series booster.

of poles as the converter. The armature of the alternator is connected in series with the alternating-current lines supplying the converter, as shown in Fig. 352. By raising the field of the alternator or booster, the alternating voltage of the converter is raised. The converter voltage may be lowered, not only by decreasing the booster field but by reversing it as well.

When the booster voltage is assisting the converter voltage, the booster acts as an alternator and takes mechanical power from the converter armature. This increases the *energy* component of the resultant armature current in the converter and, hence, changes the cross-magnetizing effect of the armature. When the booster voltage bucks the converter voltage, the booster receives

electrical energy and delivers mechanical energy to the converter shaft. That is, it operates as a synchronous motor and tends to drive the converter mechanically. The energy current in the converter armature is decreased, therefore, and may even be reversed. This causes a variation of the cross-magnetization which, in turn, requires that the strength of the commutating

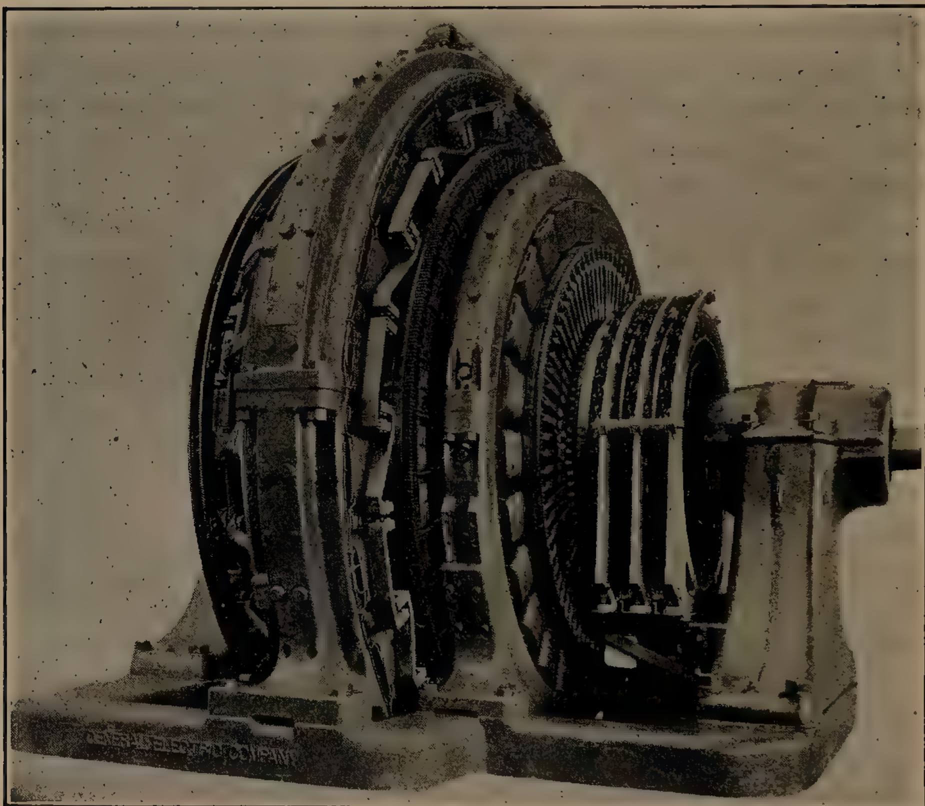


Fig. 353.—General Electric 3,500-kw., interpole-type, synchronous converter with synchronous booster.

poles be changed accordingly. This is accomplished by separate windings on the commutating poles, the current in these windings being controlled by the booster field rheostat.[1] The distinct advantage of this method of control is that the voltage may be varied independently of power factor. The objection to this type of voltage control is the additional machine. At this time, however, it is the most common method of controlling

[1] For further information, see "Standard Handbook," Sec. 9.

the voltage of large units. Figure 353 shows a converter having a booster generator.

Transformer Taps.—The converter voltage may be adjusted approximately to the desired value by taps on the transformers.

Owing to the arcing and burning of sliding contacts, the design of such apparatus for changing voltage under load is not simple. Tap changing mechanisms for changing transformer voltages under load have recently been developed (see p. 240), however, but so far they have not been applied to synchronous-converter operation. Such taps would have the disadvantage of not giving gradual changes in voltage. The use of taps for fixed adjustment of the converter voltage is, however, common.

Split-pole Converter.—The split-pole converter is based on the following principle:

The total direct-current e.m.f. generated depends on the *total flux* between brushes, irrespective of the manner in which this flux is distributed. The alternating e.m.f. depends on the form of the flux wave, as well as on the total flux. If the distribution of the flux be altered, therefore, without changing its total value, the alternating e.m.f. may be altered in value, but the direct-current e.m.f. will not be affected.

In the split-pole converter, the form of the alternating e.m.f. wave is varied by means of auxiliary poles adjacent to the main poles. The main poles are excited by the main-field winding, and the auxiliary poles by a separate winding. By changing the auxiliary excitation in conjunction with the excitation of the main winding, the wave form of the alternating e.m.f. may be changed, thus varying the ratio of the alternating-current to the direct-current e.m.f.

The brushes in a generator must be moved *forward*, in order that the machine may commutate in the fringe of a leading pole tip (see Vol. I, p. 327, Par. 221). To balance the e.m.f. of self-induction, the brushes of the split-pole converter must be moved forward, in order to commutate in the fringe of a leading pole tip (Fig. 354). This fringe must come from the main poles, for their flux is nearly constant in strength, whereas the flux of the auxiliary pole is varied over a wide range and may be reversed even. In a *direct* synchronous converter, therefore, the armature must rotate from *main* to *auxiliary* pole (Fig. 354), whereas,

in an *inverted* synchronous converter, the armature must rotate from *auxiliary* to *main* pole.

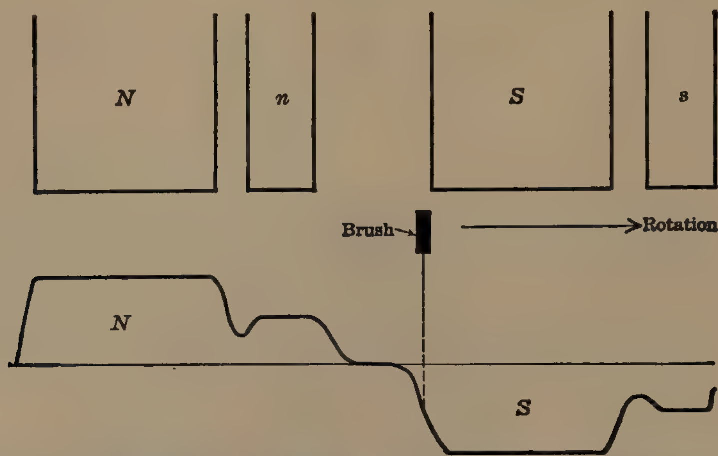


FIG. 354.—Relation between direction of rotation and position of auxiliary poles in split-pole direct synchronous converter.

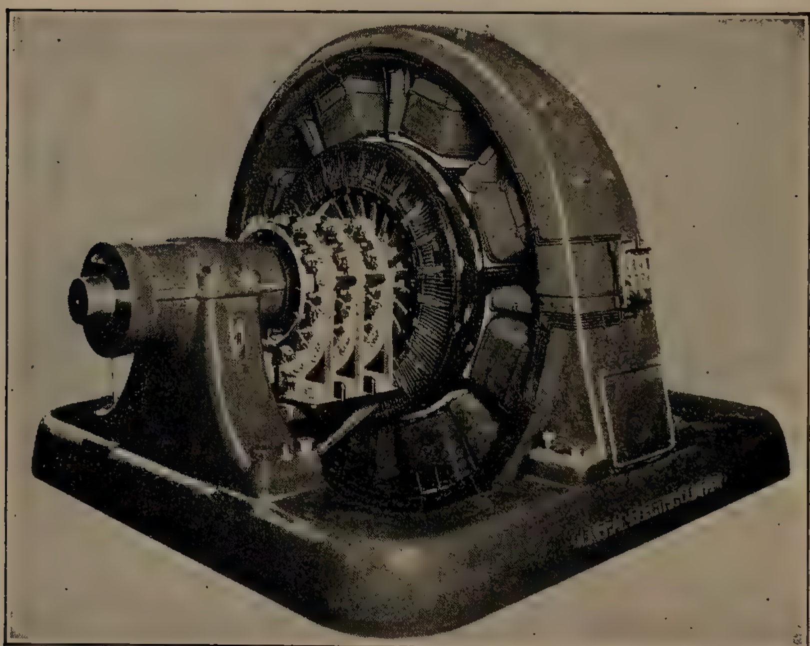


FIG. 355.—General Electric, eight-pole, 750-kw., 375-r.p.m., shunt-wound, split-pole synchronous converter.

Figure 355 shows a General Electric split-pole converter without commutating poles.

181. Experimental Determination of Voltage and Current Relations in Converter.—An instructive laboratory experiment is carried out with a converter connected in the manner shown in Fig. 356. The series reactances may be omitted if the transformers themselves have sufficient leakage reactance. Connect instruments to measure the three-phase input, a voltmeter to measure the transformer primary voltage, a voltmeter to measure the slip-ring voltage, ammeters to measure the currents between the transformer secondaries and the converter, direct-current instruments to measure the converter output, and a direct-current ammeter to measure the field current.

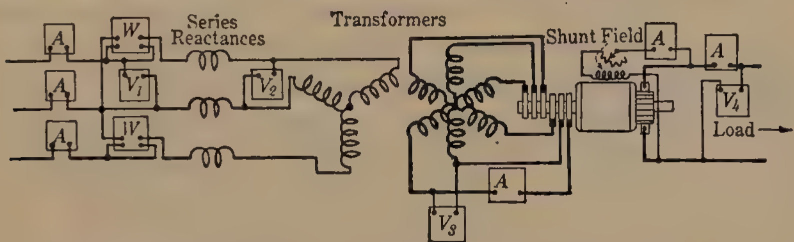


FIG. 356.—Connections for testing synchronous converter.

Keep the load on the converter constant at its rated value. Vary its field over the maximum range of operation, reading all instruments. With field current as abscissas, plot as ordinates:

1. Voltages V_1, V_2, V_3, and V_4.
2. Efficiency of the entire unit.
3. Power factor.

Also, check the currents by the equations of Par. 175 (p. 393). Note the effect of power factor on efficiency.

Other experiments may be performed using these same connections, such as keeping the field current constant at its normal no-load value ($P.F. = 1.0$) and noting the changes in efficiency and power factor as the load is increased. Plot efficiency and power factor as ordinates with output as abscissas.

182. Synchronous-converter Connections.—Transformers are usually necessary with synchronous converters. The direct-current voltage is always low, and the alternating voltage at the slip-rings must be still less. Transformers are necessary, moreover, for obtaining a six-phase from a three-phase system.

Usually, the transformer primaries may be connected either in Y or in delta. The most common six-phase connections for the

transformer secondaries are the "diametrical," the star, and the double-Y (see Fig. 345, p. 395). The difference between the diametrical and the star is that the secondaries are connected together at the neutral point in the star, whereas three separate secondaries are connected across diametrically opposite points in the diametrical connection. There is no difference between the double-Y and the star if the neutrals of the two Y-systems are connected together. Other than a slight effect on harmonics, and the fact that a neutral is available in the double-Y, there is little difference in the use of the three connections, except with the split-pole converter.

If the induced e.m.f. of the converter armature contains harmonics, there will be no circulatory current within the armature itself, for in the direct-current type of armature, such as is used for the converter, any e.m.f. induced under a given pole in one part of the armature is opposed by an opposite and equal e.m.f. induced under an opposite pole. If, however, the *line* voltage is practically sinusoidal and the *induced* e.m.f. of the converter contains harmonics, there will be unbalanced harmonic voltages. The current due to these unbalanced voltages will consist entirely of harmonics which contribute no energy but do heat the armature and transformers.

This effect is negligible in the ordinary converter, but in the split-pole type, the voltage control depends on the introduction of large harmonic voltages into the e.m.f. wave. When this type of converter is used, therefore, the transformer connections must be so chosen that as many as possible of the harmonic currents are eliminated. Most three-phase transformer connections eliminate the third-harmonic current and its multiples, with the following two exceptions: The primaries cannot be connected in delta if the secondaries are connected either diametrical, six-phase star, or double-Y, with the neutrals of the two Y-systems connected together, for the third-harmonic currents in the secondaries, due to unbalanced harmonic voltages, will cause third-harmonic currents to circulate in the primary delta, producing extra heating in the converter armature and in the transformers.

If the transformer primaries be connected in Y, with no neutral connection, no third-harmonic currents or multiples thereof can

flow into the Y, as these currents are all in phase with one another. In order that currents may flow to a common point, there must be phase difference, as the currents flowing toward the point must be equal to the currents leaving the point at any instant, or electricity will accumulate at the point. If no third-harmonic currents can flow in the transformer primaries, none can flow in their secondaries; hence, there will be no circulatory harmonic currents between the transformer secondaries and the converter armature if the primaries are connected in Y without a neutral connection to the main generator. If, however, the neutral of the transformer primaries be carried back to the main gener-

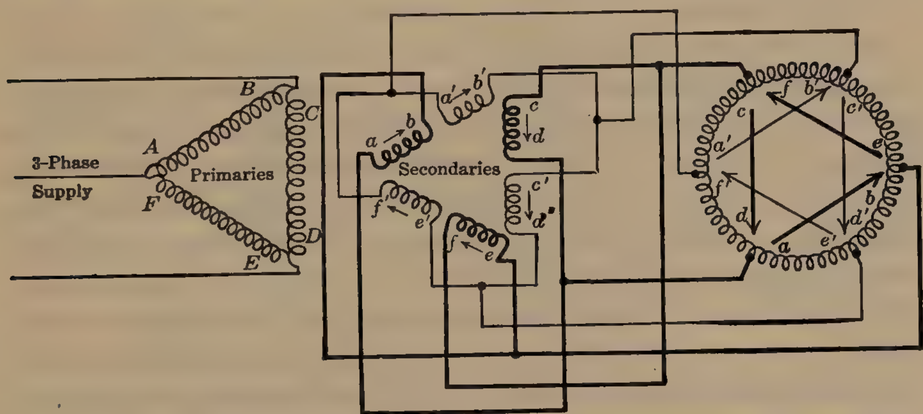


Fig. 357.—Double-delta connection of transformers to six-phase synchronous converter.

ator, the third-harmonic currents and multiples thereof can return to the generator through the neutral. The secondaries cannot be connected either diametrical, six-phase star, or double-Y with interconnected neutrals, therefore, if the primaries are connected in Y with a neutral return to the generator.

The harmonic currents other than the third and multiples thereof are not eliminated by three-phase connections, but they are reduced to small values by the use of series reactances or by using transformers having high leakage reactance.

Figure 346 (p. 396) shows the connections for a 500-kw. converter and transformers taking power from 6,600-volt, three-phase, 60-cycle supply and delivering 550 volts direct current. The transformer primaries are connected in delta, and the secondaries can be connected either diametrical, star, or double-Y. (If this were a split-pole type of converter, the primaries could not

be connected in delta, but they must be connected in Y without neutral return to main generator.) The advantage of the star and the interconnected double-Y connection is the fact that a neutral is accessible. The voltages and currents at each part of the system are shown. Unity power factor, 98 per cent. efficiency for the transformers, and 95 per cent. efficiency for the converter are assumed.

The double-delta connection of secondaries may also be used. Such a connection for a converter is shown in Fig. 357. The arrows point in the relative directions in which the voltages act. No neutral is available if this method of connecting the transformers is used.

183. Inverted Synchronous Converter.—When a converter operates from a direct-current source and delivers alternating current, it is known as an *inverted* synchronous converter. The direct-current side has characteristics very similar to those of a shunt or compound motor. The alternating-current side has characteristics very similar to those of an alternator. A converter when operating inverted has the same rating as when operating direct. When operating from the alternating-current supply, the speed of the converter must be in synchronism with the supply and, hence, constant. When operating alone from the direct-current supply, the speed is determined by the back e.m.f. and the flux, just as in any direct-current motor, and the speed may vary. In fact, at times there is a tendency for the inverted converter to race, so that inverted converters should have speed-limiting devices. An inductive load on the alternating-current side weakens the field through armature reaction, in the same manner that the field of an alternator is weakened under similar conditions. The weakening of the field increases the speed of the converter. This increased speed causes the current to lag still more ($\tan \theta = 2\pi fL/R$), because of the increased frequency. As the effect is cumulative and may cause the armature to reach dangerous speeds, the necessity for using a speed-limiting device is obvious.

A centrifugal device is often used to trip the circuit breaker when the speed exceeds the safe value. Another method, not often used, is to have an exciter on the converter shaft. As the speed increases, the exciter voltage increases, and the converter

field is strengthened. This tends to check the increase of speed of the converter.

Inverted converters will operate satisfactorily in parallel on the alternating-current side, any converter being made to take more load by weakening its field.

184. Starting Synchronous Converter from Alternating-current Side.—There are several methods of starting direct synchronous converters, some of which are similar to the methods used with the synchronous motor.

If polyphase currents are supplied to the armature, a rotating field is produced about the armature (Fig. 358). This is similar to the rotating field of the induction motor, except that it is produced by a rotating armature about itself. If the armature speed is below synchronism, this field cuts the pole faces and the damper windings (Fig. 362) and induces currents. A reaction results between the rotating field and these induced currents, producing rotation.

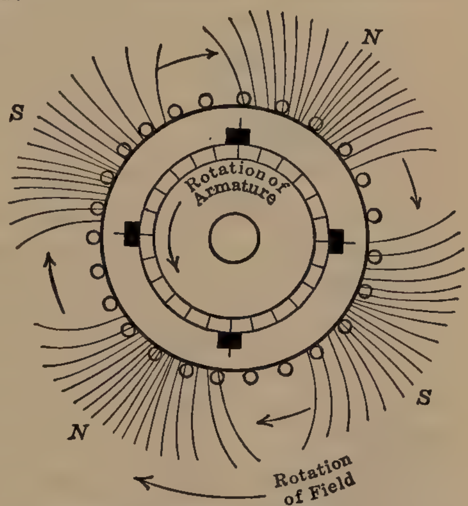


Fig. 358.—Relative directions of rotation of the armature and of the rotating field produced by the armature.

When starting the converter in this manner, several precautions are necessary. The armature is the primary, and the shunt field coils are the secondary of a transformer, the secondary having a very large number of turns. The rotating armature field, therefore, induces very high voltages in the field coils on starting and tends to puncture them. To reduce this voltage, the field is usually split into sections by a field-splitting or sectionalizing switch. Figure 359 shows the connections of a three-pole switch used to sectionalize the field circuit of a four-pole converter into four parts. *This sectionalizing switch should be open when starting from the alternating-current side.*

If there be a switch short circuiting the series field, this should be opened, as otherwise the currents generated in the series field by the transformer action of the armature will cause undue heat-

ing. If there be a series-field shunt or diverter, this should be opened for the same reason.

The rotating field produced by the armature *cuts* the armature conductors (Fig. 358) just as if the armature were rotating and cutting the flux of a stationary field, as in the direct-current generator. This field induces voltages in the armature coils. Some of these coils are short circuited by the brushes, so that sparking results under the brushes, even though there is no direct-current load. This sparking may not be severe, as the rotating field is comparatively weak in the interpolar spaces where the brushes are, because of the high reluctance of the air path at these points. If interpoles are used, however, the reluctance of the interpolar space is reduced very materially so that sparking becomes severe. Consequently, brush-raising devices are usually installed on interpole machines, to lift the brushes on starting and so eliminate this sparking. One brush in a positive brush holder and one brush in a negative brush holder are usually left on the commutator to supply the field excitation. In order to reduce the sparking caused by these two brushes' short circuiting armature coils in which e.m.fs. are induced by the rotating field, these brushes are often beveled so that the time of short circuiting is reduced to a minimum. Converters are started at reduced voltage obtained from taps on the transformer secondaries, although starting compensators are used, at times, in the units of smaller size.

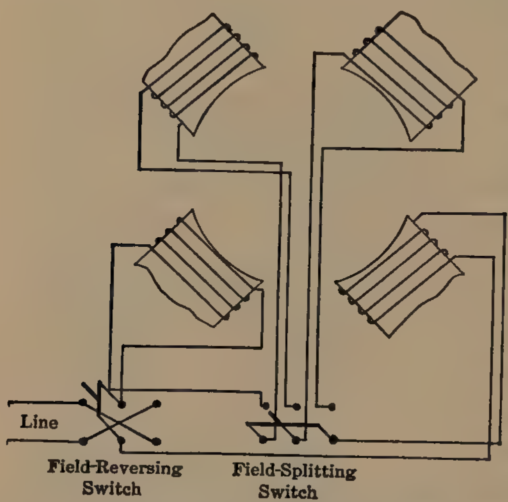


FIG. 359.—Connections of shunt field and shunt field-splitting switch.

As a rule, converters excite their own shunt fields. The armature rotates in a direction *opposite* to that of the rotating field which is set up about it (Fig. 358). As the armature, therefore, approaches synchronism, the rotation of this field becomes slower and slower with respect to the field structure, as the rotating field

rotates in one direction and the armature in the opposite direction. The field poles themselves, which are magnetized alternately north and south by this field, become more and more slowly magnetized as the armature approaches synchronism. Finally, due to hysteresis action (see Chap. XI, p. 363, Par. 161), the poles themselves become permanently magnetized through armature reaction, and the armature pulls into synchronism in a manner similar to that of the salient-pole synchronous motor when started in this manner.

When the shunt-field switch is closed, the field produced by the shunt winding may oppose the field built up in the field poles by armature reaction. Consequently, there is a tendency for the armature to slip a pole. Should the armature slip a pole, the direct-current voltage at the brushes reverses. This reverses the shunt-field current, which again causes the converter to slip a pole. This action, unless checked, may continue indefinitely. It may be stopped by reversing the shunt-field current by means of the field-reversing switch (Fig. 359).

It often happens that the direct-current field is not strong enough to cause the armature to slip a pole, because the field voltage may be low, due to the alternating voltages, being reduced through the starting taps. The tendency exists, however, and due to the resulting distortion of the pole flux, the brushes are no longer in the commutating zone. The brush voltage is thereby reduced, which again lowers the tendency to slip a pole. The converter will continue to run under these conditions, but it will take a large current at low power factor, will spark at the brushes, and its operation will be unsatisfactory. By reversing the field current, however, normal operating conditions can be obtained.

185. Methods of Obtaining Correct Polarity.—It is important that the converter always come up with the same direct-current polarity, as it may be operating in parallel with other apparatus. As has just been pointed out, the converter may build up with either polarity. If this polarity happens to be wrong, there are several methods of correcting it.

Below are given some of these methods. The starting compensator or transformer taps are assumed to be in the starting positions.

(*a*) Open the shunt-field circuit and then open the line switch long enough for the converter to slip one pole. This can be determined very readily with a stroboscope. Close the field switch and then throw the alternating-current switch quickly into the running position. With a little practice this operation can be readily performed.

(*b*) Reverse the shunt field by means of the field-reversing switch. This causes the machine to slip a pole and so reverses the direct-current voltage, making it correct. If left this way, the machine will continue slipping, one pole at a time, as has just been pointed out. The shunt-field switch must be thrown back immediately, therefore, to its original position.

(*c*) When the converter is first connected across the alternating-current line, the rotating field produced by the armature cuts the armature conductors and generates alternating currents in these conductors, as has already been pointed out. The brushes are stationary and the field rotating, so there is no commutating action. There is an alternating e.m.f. of line frequency, therefore, across the brushes at the instant of starting. The armature rotates in a direction *opposite* to that of its rotating field, because of the reaction with the pole-face currents. This is illustrated by Fig. 358. The rotating field about the armature is shown as rotating clockwise. A conductor, such as the pole faces, when placed in this field, would tend to rotate clockwise. That is, if the armature were held stationary, the field structure would tend to rotate in the direction of the rotating flux produced by the armature, or in a clockwise direction. The torque produced by this rotating flux is, therefore, in such a direction that it tends to cause the field structure to rotate in a clockwise direction. The field structure, however, is fixed in position, and the armature is free to rotate. The *reaction* between the two remains unchanged. Consequently, the armature will rotate in a *counter-clockwise* direction. The relative motion between armature and field structure is the same as if the armature were stationary and the field were free to rotate.

As the speed of the armature increases, the field produced by it must rotate slower and slower in *space*, although it does not change its speed relative to the armature. The brushes tend to become more and more nearly stationary with respect to this

rotating field, so that their commutating action becomes greater and greater. The frequency of the e.m.f. across the brushes becomes less and less and, when the armature finally pulls into synchronism, becomes zero, and a direct-current voltage exists across the brushes.

If a direct-current voltmeter be connected across the brushes, its pointer will tend to oscillate at line frequency when the alternating current is first switched on. As the armature speeds up, this frequency becomes less and less, and the pointer is soon able to follow the slow oscillations. When the frequency of oscillation becomes very low and the pointer is just going through zero in the positive direction, the field switch should be closed. This insures the converter's coming in with the correct polarity. A zero-center type of voltmeter is desirable when this method is employed.

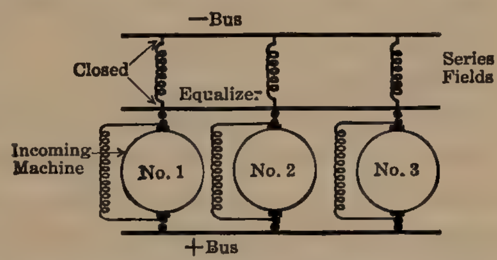


FIG. 360.—Method of obtaining correct polarity by closing equalizer and series-field switches.

(*d*) If the converter operates in parallel with others, and equalizers are used, a weak field of the correct polarity may be produced in the field of the incoming converter by closing a line and an equalizer switch, as indicated in Fig. 360. This tends to make the armature reaction build up fields of the correct polarity and so insures the converter's coming in properly.

186. Starting Synchronous Converter by Means of an Auxiliary Motor.—As was pointed out in Chap. XI, one method of starting a synchronous motor is to bring it up to speed with an auxiliary motor and then synchronize (see p. 363, Par. 161). This same method may be used with the converter. The methods of synchronizing are identical with those used with the alternator (see p. 205, Par. 101). This method of starting is practically obsolete.

187. Starting Synchronous Converter from the Direct-current Side.—If sufficient direct-current power is available, the converter may be started from the direct-current side, starting as a shunt motor. When started in this manner, the series field should be short circuited, as it will oppose the shunt field when

the machine operates as a motor and will, therefore, reduce the starting torque. The transformer secondaries are short circuits on the direct-current armature at starting, as the frequency is zero and their resistance is very low. This is particularly true if the brushes happen to be resting on commutator segments which are connected directly to the slip-ring taps. The transformers should, therefore, be disconnected. The proper speed is obtained by adjusting the shunt field. As there is practically no voltage control in the simple converter when operating in this manner, it is not always possible to adjust the alternating voltage to a value equal to that of the line. To prevent any disturbance which may result from synchronizing at a voltage other than bus-bar voltage, some of the starting resistance is often left in the armature circuit until after the machine has been synchronized.

188. Parallel Operation of Synchronous Converters.—Synchronous converters may be operated in parallel on the direct-current side, just as shunt and compound generators are similarly operated. If one series-field winding be used on each machine, only one equalizer is necessary. If the machine is a three-wire converter and is compounded, there will be two series fields, as shown in Fig. 361. In this case, two equalizer switches are necessary (see Vol. I, p. 440, Fig. 369). The loads are shifted by changing the voltages of the converters, either by field control or by any of the other methods already described.

Better operation is obtained if each converter has its own transformer bank, rather than by having a single bank supplying all the converters. This introduces more or less reactance between converters and stabilizes their operation. It may even be necessary to install series reactances in the transformer leads.

The alternating side of a converter may be accidentally opened by a circuit breaker or otherwise, while the direct-current side may still be connected to a source of power, such as other converters or a storage battery across the bus-bars. The converter will then tend to operate as a shunt motor, usually with a weakened field, due to the differential action of the compound winding. Under these conditions, the converter may tend to race (see p. 412). Converters, therefore, are usually equipped with reverse-energy relays on the direct-current side, or else the direct-current

breakers are interlocked with the alternating-current ones, so that the direct-current side will be opened simultaneously with the alternating-current side.

The *resultant* current in the converter armature conductors produces the torque which overcomes the stray-power losses of

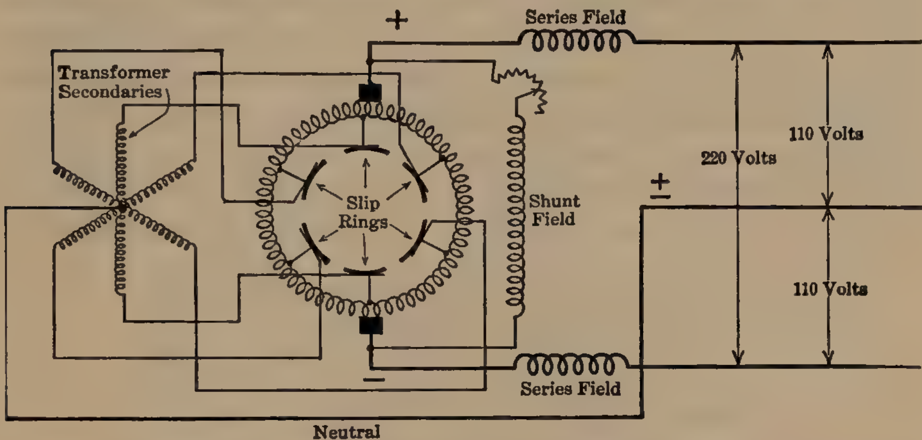


Fig. 361.—220-volt, three-wire direct-current system obtained from 220-volt synchronous converter employing six-phase star connection.

the converter. This resultant current is the *difference* of two nearly equal currents, as has already been demonstrated. A small percentage change in either the motor current alone or the generator current alone produces a large percentage change in

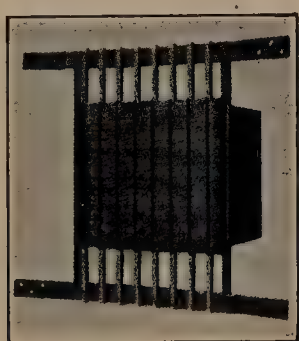


Fig. 362.—Main pole with damper winding.

this torque current. The converter is very sensitive, therefore, to line disturbances, such as fluctuations of voltage or of frequency. Accordingly, it has a much greater tendency to "hunt" than even the synchronous motor. For this reason, converters

always have amortisseur or damper windings or grids built around and into the poles, as shown in Fig. 362. The action of these windings is the same as in the synchronous motor described on p. 362 (Par. 160), except that the windings are now stationary in space. The armature which produces the rotating field rotates at synchronous speed in one direction, and the rotating field itself rotates at synchronous speed in the opposite direction with respect to the armature. Under normal operation, therefore, the field is stationary in space with respect to the amortisseur windings.

189. Three-wire Converter.—It is pointed out in Vol. I (Chap. XIV) that the neutral of a three-wire-system may be

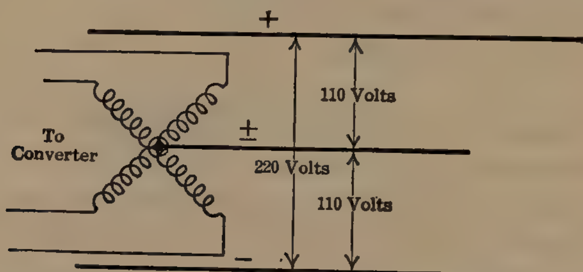


Fig. 363.—Direct-current neutral obtained from neutral of four-phase, diametrically connected transformers.

obtained by the use of two or more slip-rings connected to the direct-current armature (see Vol. I, p. 457). A reactance coil is connected across these slip-rings, and alternating current flows through this reactance. The direct-current neutral is connected to the middle point of this reactance, and the direct current of the neutral divides and passes back into the armature through the reactance. The reactance has a low resistance and has practically no effect on the direct current.

It is to be noted that a synchronous converter with the proper transformer connections provides a neutral point for just such a direct-current neutral. For example, if a six-phase double-Y, or a six-phase star (Fig. 361), a four-phase star (Fig. 363) or a three-phase Y-connection of transformer secondaries be used, an excellent neutral is provided.

In the first two of these connections, the direct current flows in opposite directions through the two halves of each transformer secondary, so there is no direct-current magnetizing action on

the core. In the Y-connection, however, this is not the case, and the magnetizing action of the direct current, acting in conjunction with that of the alternating current, produces a pronounced unsymmetrical cyclic magnetization of the iron. This is undesirable, as it results in an increased magnetizing current whose positive and negative values will be unequal and dissimilar, and the transformer losses are increased. By splitting each transformer secondary into two sections, a,a', b,b', and c,c' (Fig. 364) and connecting as indicated, it will be observed that the direct current flows in opposite directions in the two halves

FIG. 364.—Zigzag connection of three-phase secondaries for eliminating direct-current magnetizing of transformer cores.

of each secondary winding and, consequently, has no appreciable magnetizing effect.

Figure 361 shows the complete connection for a six-phase, 220-volt, three-wire converter, having two series fields and with the direct-current neutral connected to the neutral of the six-phase, star-connected transformer secondaries. Figure 363 shows a direct-current system supplied by a four-phase converter, the neutral being obtained through two star-connected transformers.

CHAPTER XIII

TRANSMISSION OF POWER BY ALTERNATING CURRENT

190. Transmission Systems.—To transmit power economically over considerable distances, it is necessary that the voltage be high. High voltages are readily obtainable with alternating current. As high as 20,000 volts may be generated directly. For voltages in excess of this it is desirable to use transformers, as it is difficult to insulate the generators for these higher voltages. The transmission voltage is usually too high for commercial uses, but for purposes of distribution it may be stepped down to the desired value by the use of transformers.

Direct-current voltages for commercial power can be raised and lowered only by machines having rotating commutators. The efficiency of such apparatus is not high, and operating difficulties are encountered in connection with the commutators, even at comparatively low voltages. Hence, alternating current is nearly always used for transmission purposes. (The one exception is the Thury[1] system in Europe.) Where considerable power is involved, polyphase systems are used because of the many advantages of polyphase over single-phase systems. For example, polyphase motors are considerably cheaper and lighter than single-phase motors of equal rating and, as a rule, have better operating characteristics. The output of generators when operating polyphase is much greater than when operating single-phase (see p. 104).

Of the polyphase systems, the three-phase system is generally used for transmission, although the employment of two-phase for distribution purposes is not uncommon. The three-phase system has the advantage that it requires the least number of conductors of all the polyphase systems; the voltage unbalancing even with unbalanced loads is not usually serious; and for a given voltage between *conductors*, with a given power transmitted a

[1] See Vol. I, p. 358, and also "Standard Handbook," Fifth Ed., Sec. XI.

given distance with a given line loss, the three-phase system requires only 75 per cent. as much copper as either the single-phase or the two-phase system.

The single-phase system is used in railroad electrification, where single-phase power is supplied at the trolley. The most notable examples of this are the New York, New Haven & Hartford Railroad and the Norfolk and Western Railway.

When the voltage is so high as to make transformers necessary, the power is usually generated at 6,600 or 13,200 volts. This voltage is not so high as to make difficult the proper insulation

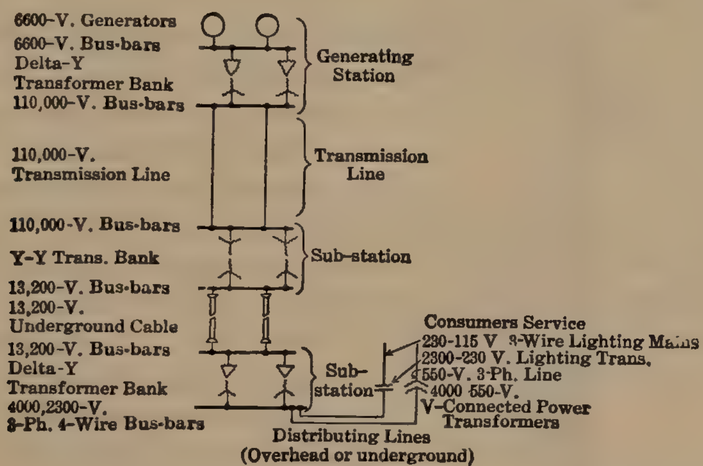


FIG. 365.—Typical connections of a power system.

of the generators, and at the same time the armature conductors, the bus-bars, and the leads running from the generator to the bus-bars do not become too large.

The transmission voltage is largely determined by economic considerations. Although a high voltage reduces the conductor cross-section, the saving in copper may be offset by the increased cost of insulating the line, by the increased size of transmission-line structures, and by the increased size of generating and sub-stations, due to the large clearances required by the high-voltage leads and bus-bars. A rough basis for determining the transmission voltage is to use 1,000 volts per mile of line.

Because of the danger involved, it is not usually permissible to carry high-voltage transmission lines through thickly populated districts in order to reach the distributing substations.

The voltage is usually stepped down to about 13,200 or 26,400 volts at substations located at the outskirts of the city and thence carried into the city underground, or occasionally overhead, at 13,200 volts.[1]

Figure 365 shows a typical system. No attempt is made to show switches, circuit breakers, etc. Power is generated at 6,600 volts and is delivered directly to the 6,600-volt bus-bars. It is then stepped up to 110,000 volts, the transmission voltage, by delta-Y transformer banks whose secondaries are connected to the 110,000-volt bus-bars. The power then passes out over the duplicate transmission lines to a substation located in the outskirts of the district where the power is to be utilized. It is then stepped down to 13,200 volts by Y-Y-transformer banks and delivered to the 13,200-volt bus-bars at this substation. The power then leaves these 13,200-volt bus-bars for the various distributing substations in the district. One distributing substation is shown. Here the voltage is stepped down to a three-phase, four-wire system. This system has 4,000 volts between conductors, or 2,310 volts to neutral, for distribution to the consumers.

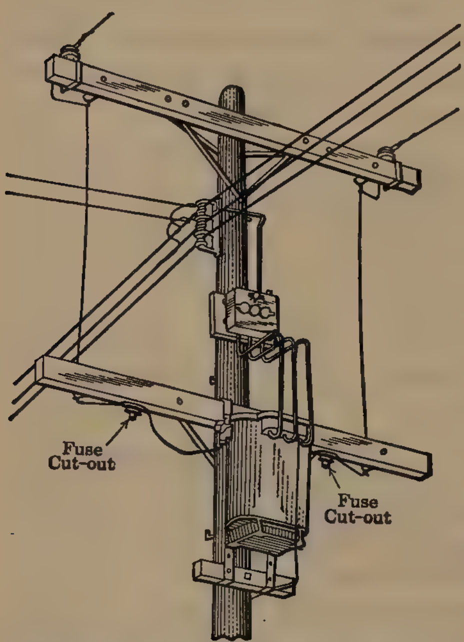


Fig. 366.—Typical 2,300–230/115-volt lighting transformer and secondary three-wire mains.

Usually, the lighting and the power loads are connected to separate feeders, in order to avoid the annoying flickering of the

[1] In 1927, the Commonwealth Edison Company of Chicago installed 6 miles of special 132,000-volt, hollow conductor cable, and at the same time the New York Edison Company installed 12 miles of such cable. In both installations, the operation of these cables appears to be very satisfactory. See P. Torchio, "132,000-volt, Single-conductor, Lead-covered Cable," *Jour. A. I. E. E.*, p. 118, February, 1928.

lamps when motors are thrown on or off the line. The lighting loads are usually supplied by 10/1 transformers located on the poles, from whose secondaries 230–115-volt, three-wire systems are obtained (Fig. 366). The two wires coming from the top crossarm to the crossarm next beneath and going through the fuse cutouts to the transformer are the 2,300-volt lines. In a four-wire, three-phase, 4,000-volt system, the primary of this transformer would be connected between one line conductor and neutral. The 230–115-volt secondary wires leave the front side of the transformer and feed three vertically arranged conductors of the three-wire secondary mains, which supply the local lighting loads. The *power* consumers are usually connected to the secondaries of V-connected or delta-connected transformers or are connected to the secondaries of three-phase transformers located at the consumer's premises. In order that the secondary mains may not be too large, 440 and 550 volts are generally used for the power loads (also, see Vol. I, p. 443, Fig. 372).

In the substation, other power-transforming apparatus may be installed, such as constant-current transformers, motor-generator sets, synchronous converters, or mercury-arc rectifiers for obtaining direct current, etc.

191. Transmission-line Reactance, Single-phase.—In making line calculations for the transmission of direct-current power, the resistance alone needs to be considered. In making similar calculations for alternating-current lines, it is necessary to take into consideration not only the line resistance but the line reactance as well. In cables and in overhead lines operating at high voltage, it is also necessary to consider the capacitance between conductors.

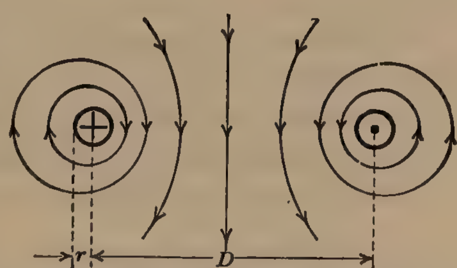


Fig. 367.—Magnetic field between the two conductors of single-phase line.

Figure 367 shows the cross-section of a two-conductor, single-phase line. As the current at any instant flows in opposite directions in the two conductors, the circular paths of the magnetic lines set up about one conductor must always go in a direction opposite to that for the other conductor. That is, when one

magnetic field is acting in a clockwise direction, the other must be acting in a counterclockwise direction. This causes the two fields to act in conjunction in the area between the two conductors, as shown in Fig. 367. Thus, two parallel wires form a rectangular loop of one turn, through which flux is set up by the current in the wires. This flux links the loop, and the circuit has inductance, therefore. It might appear that this inductance would be negligible, because the loop has but one turn and the flux path is entirely in air. It must be remembered, however, that the cross-sectional area of the flux path is *large*, usually being from 1 to 20 ft. wide and several miles long. Although the flux density is small, the total flux linking the loop is usually considerable.

It can be shown that the inductance of such a loop is[1]

$$L = 2l\left(0.080 + 0.741 \log_{10} \frac{D}{r}\right) \text{ mil-henrys.} \qquad (134)$$

where D is the distance between conductor centers, and r is the radius of each conductor, both expressed in the same units. l is the length of the line in miles. The reactance of the loop is

$$X = 2\pi fL \qquad (135)$$

where f is the frequency in cycles per second.

It is usually more convenient to consider the inductance of a single conductor only. The inductance per single conductor is obviously one-half the value given in Eq. (134), which applies to the two conductors of the circuit.

The reactance per mile then becomes

$$X = 2\pi f\left(80 + 741 \log_{10} \frac{D}{r}\right)10^{-6} \text{ ohms per mile.} \qquad (136)$$

Appendix I (p. 544) gives values of the reactance at 60 cycles per second for solid and stranded conductors, at various spacings. The reactance for stranded conductors is slightly less than the corresponding values given for solid conductors. The reactance at other frequencies may be found by direct proportion.[2]

[1] The derivation of these various transmission-line equations is found in R. R. LAWRENCE, "Principles of Alternating Currents" (see front of this volume, p. II).

[2] For more complete tables, see "Standard Handbook," Sec. XI, Fifth Ed.

Example.—A single-phase transmission line is 40 miles long and consists of two 0000 solid conductors spaced 4 ft. on centers.

Find: (*a*) the inductance of the entire line and the reactance per conductor at 25 cycles per second; (*b*) at 60 cycles per second. (*c*) If a 200-amp., 60-cycle current flows over this line, find the total reactance drop.

The diameter of 0000 conductor is 460 mils; the radius, $r = 0.230$ in.

$$\frac{D}{r} = \frac{48}{0.230} = 209$$

$$\log_{10} 209 = 2.32 \text{ (p. 540).}$$

The inductance per mile

$L' = 2(0.080 + 0.741 \times 2.32) = 3.60$ mil-henrys (from Eq. 134).

(*a*) The total inductance

$L = 3.60 \times 40 = 144$ mil-henrys or 72 mil-henrys per conductor. *Ans.*

The reactance per conductor at 25 cycles

$$X_1 = 2\pi 25 \times 72 \times 10^{-3} = 11.3 \text{ ohms.} \quad \textit{Ans.}$$

(*b*) The reactance per conductor at 60 cycles

$$X_2 = 2\pi 60 \times 72 \times 10^{-3} = 27.1 \text{ ohms.} \quad \textit{Ans.}$$

(*c*) The total reactance drop

$$V = 27.1 \times 200 \times 2 = 10{,}840 \text{ volts.} \quad \textit{Ans.}$$

192. Transmission-line Reactance, Three-phase.—In transmission-line problems, it is convenient to consider the reactance

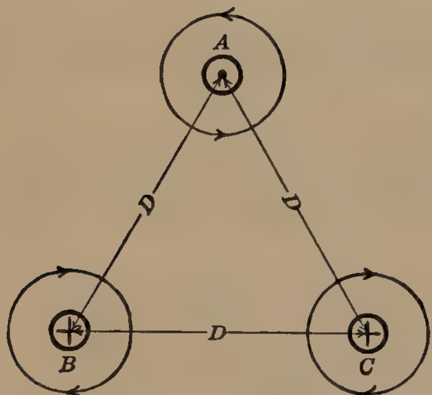


FIG. 368.—Three symmetrically spaced conductors of three-phase line.

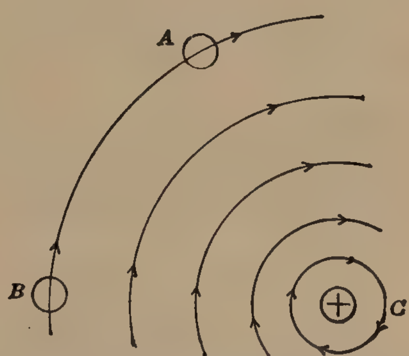


FIG. 369.—Magnetic field produced by conductor C does not link loop AB.

of the individual conductor, rather than the reactance of the looped line or of the entire circuit. The convenience becomes more apparent when three-phase lines are considered. In Fig. 368 are shown the three conductors of a three-phase line, symmetrically spaced. That is, each conductor is at an apex of the same equilateral triangle. The current at the instant shown is flowing

outward in conductor A and inward in conductors B and C. The field produced by each conductor is indicated. These fields are continually changing, due to the cyclic variation of the currents in the three phases, and this causes a rotating field in the region between the conductors. This rotating field is similar to the rotating field of the polyphase induction motor, and, as it cuts all three conductors, it induces e.m.fs. in them.

In treating this problem, however, it is simpler to consider the reactance of each conductor separately. If the spacing is symmetrical, the flux produced by each conductor does not induce any e.m.f. in the circuit composed of the other two conductors. For example, Fig. 369 shows the circular field produced by conductor C acting alone. As none of its lines links the circuit AB, conductor C does not induce any e.m.f. in loop AB. Likewise, conductor A induces no e.m.f. in loop BC, and conductor B induces no e.m.f. in loop CA, provided the conductors are symmetrically spaced.

In the three-phase case, therefore, the reactance per conductor is found by Eq. (136), (p. 426) or by consulting the tables (p. 544). The distance between the centers of conductors is used for D.

The value of D to be used with unsymmetrical spacing is given in Par. 195 (p. 432).

Example.—A three-phase line consists of three 0000 solid conductors placed at the corners of an equilateral triangle, 4 ft. on a side. Find the reactance drop per conductor per mile when a 25-cycle alternating current of 120 amp. flows in the conductors.

$$X = 2\pi 25\left(80 + 741 \log_{10} \frac{48}{0.23}\right)10^{-6} \text{ ohms.}$$
$$= 157(80 + 741 \times 2.32)10^{-6}$$
$$= 157 \times 1{,}800 \times 10^{-6} = 0.282 \text{ ohm.}$$

The voltage drop

$$V = 120 \times 0.282 = 33.8 \text{ volts.} \quad \textit{Ans.}$$

Instead of calculating the reactance X, it may first be found in Appendix I (p. 544) for 60 cycles per second, its value being 0.677 ohm. The 25-cycle reactance is $^{25}/_{60}$ of this value and is equal to 0.282 ohm.

193. Transmission-line Capacitance, Single-phase.—If a *direct-current* voltage be applied to a transmission line under no-load conditions, no current flows after the first few moments,

except the almost negligible leakage current. If an *alternating* voltage be applied to a transmission line, considerable current may flow, even if there be no appreciable leakage and no connected load. This current is the *charging current* of the line and leads the voltage by almost 90°. The line acts as a condenser, the conductors being the plates, and the air the dielectric. Each conductor becomes charged, first positively and then negatively, which results in an alternating current.

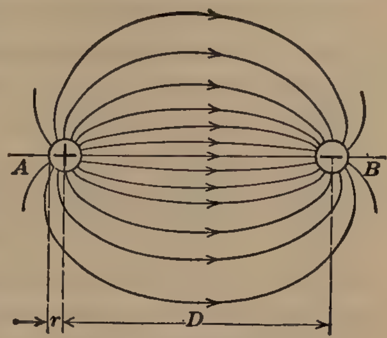
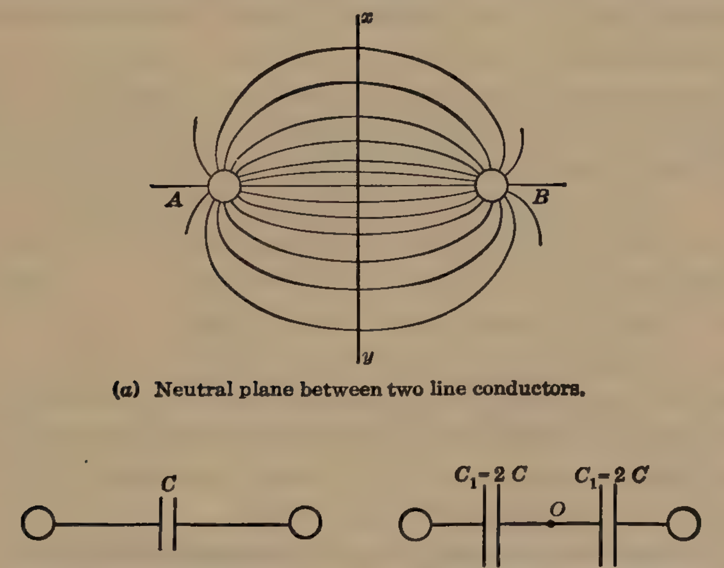


FIG. 370.—Electrostatic flux between line conductors.

This is illustrated by Fig. 370, which shows conductors A and B of a single-phase line. At the instant shown, conductor A is positive, and conductor B is negative. The electrostatic flux



(a) Neutral plane between two line conductors.

(b) Line capacitance replaced by a single condenser.

(c) Line capacitance replaced by two series-connected condensers.

FIG. 371.—Substitution of equivalent condensers for transmission-line capacitance.

existing in the field between A and B is shown. The capacitance *between conductors* of such a line can be shown to be approximately

$$C = \frac{0.0194}{\log_{10} \frac{D}{r}} \ \mu\text{f. per mile}^1 \qquad (137)$$

[1] See footnote 1, p. 426.

where D is the distance between conductor centers, and r is the radius of each conductor, both expressed in the same units.

The simplest method of treating transmission-line problems is to work with voltages to *neutral* and with capacitances to *neutral*.

In Fig. 371 (a), an imaginary plane surface xy is shown midway between conductors A and B and perpendicular to the plane of the conductors. The electrostatic field between this surface and each conductor is the same. As the plane bisects every electrostatic flux line, the potential difference between conductor A and any point in the plane is equal to the potential difference between conductor B and this same point. That is, the potential of every point on the plane xy is midway between the potential of conductor A and that of conductor B. Hence, every point in this surface is at the same potential, and xy is an equipotential surface. The plane xy may be replaced by a thin conducting plate of infinite breadth without disturbing the electrostatic field. Each conductor has the same capacitance to this plate. This capacitance must be *twice* the capacitance between the conductors themselves. That is, the capacitance C between conductors (Fig. 371 (b)) may be replaced by two equal capacitances C_1, C_1, connected in series (Fig. 371 (c)) where $C_1 = 2C$. The joint capacitance of the two capacitances C_1, C_1 in series is obviously just equal to that of the single capacitance C. The point O is the neutral of the system, its potential being the same as that of the plate xy.

If the capacitance to neutral is used when calculating the charging current, the voltage to *neutral* must also be used. With half the voltage and twice the capacitance, the charging current per conductor is the same as if the total voltage and the capacitance between conductors had been used.

The capacitance to neutral may be found by multiplying Eq. (137) by 2.

$$C_1 = \frac{0.0388}{\log_{10} \dfrac{D}{r}} \ \mu\text{f. per mile to neutral.} \tag{138}$$

Obviously, the line charging current is

$$I_c = 2\pi f C_1 E 10^{-6} \text{ amp. per mile of line.}$$

where f is the frequency in cycles per second, E is the voltage to *neutral*, and C_1 is the capacitance to neutral in microfarads per mile of line.

Appendix J (p. 545) gives amperes per mile of line, per 100,000 volts to neutral, at 60 cycles per second, for various sizes of conductor and various spacings.

Example.—A 40-mile, 60-cycle, single-phase line consists of two 000 conductors spaced 5 ft. apart. What is the charging current if the voltage between wires is 33,000 volts?

The diameter of 000 wire is 410 mils.

The radius

$$r = 0.205 \text{ in.}$$

$$\frac{D}{r} = \frac{60}{0.205} = 293$$

$$\log_{10} 293 = 2.47$$

$$C_1 = 40 \times \frac{0.0388}{2.47} = 0.628\ \mu\text{f.}$$

The charging current

$$I_c = 2\pi 60 \times 0.628 \times \frac{33{,}000}{2} 10^{-6} = 3.91 \text{ amp.} \quad \textit{Ans.}$$

194. Transmission-line Capacitance, Three-phase.—Figure 372 shows the three conductors A, B, C of a three-phase line,

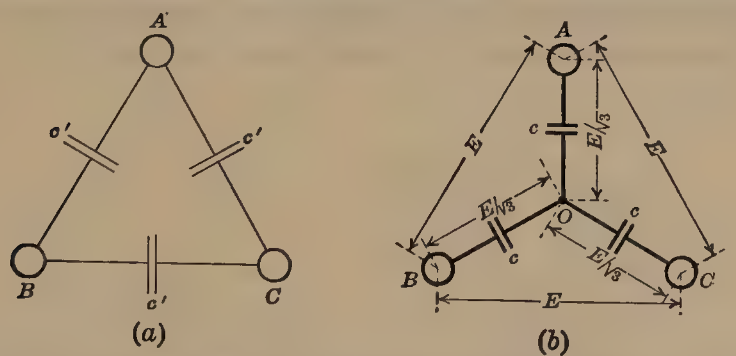


FIG. 372.—Delta-capacitance of a three-phase system replaced by an equivalent Y-capacitance.

these conductors being symmetrically spaced. There is capacitance between each pair of conductors, which can be represented by three equal capacitances c', c', c' (Fig. 372 (*a*)) connected in delta. In determining the capacitive relations in this type of system, it simplifies the problem to substitute an equivalent Y-system for the delta-system. It is obvious that any delta-load may be replaced by an equivalent Y-load. This

is the same as considering that each conductor has capacitance c to a fictitious neutral O (Fig. 372 (*b*)). In the actual line, the neutral may be the ground. The voltage across each of these condensers c is $E/\sqrt{3}$ where E is the line voltage.

Equation (138) (p. 430) may then be applied to finding the capacitance to neutral c, D being taken as the distance between conductor centers. The voltage to neutral $E/\sqrt{3}$ is used for determining the charging current per conductor.

Example.—Assume that a third wire be added to the system of Par. 193 to form a symmetrical spacing and that the system is operated three-phase, 33,000 volts between conductors. Find the charging current per conductor.

$$r = 0.205 \text{ in.}$$

$$\frac{D}{r} = \frac{60}{0.205} = 293$$

$$\log_{10} 293 = 2.47$$

$$c = 40 \times \frac{0.0388}{2.47} = 0.628\ \mu\text{f.}$$

Volts to neutral $= 33{,}000/\sqrt{3} = 19{,}070$ volts.
The charging current per conductor

$$I_c = 2\pi 60 \times 0.628 \times 19{,}070 = 4.52 \text{ amp.} \quad \textit{Ans.}$$

This may be checked by Appendix J (p. 545).

195. Three-phase System; Conductors Spaced Unsymmetrically.—If the conductors in a three-phase system are *not* symmetrically spaced, being located at the corners of a triangle

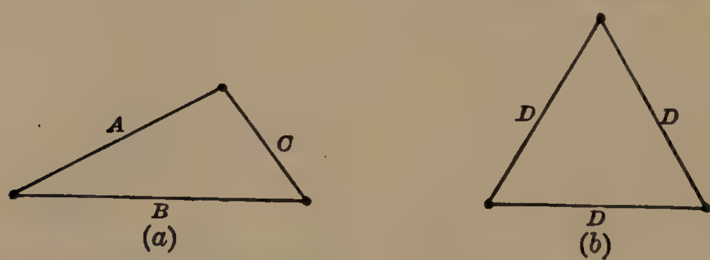


FIG. 373.—Unsymmetrical spacing and equivalent symmetrical spacing.

whose sides may be of any length, as A, B, and C (Fig. 373 (*a*)), the side D of the equivalent equilateral triangle (Fig. 373 (*b*)) may be found as follows:

$$D = \sqrt[3]{ABC}. \tag{139}$$

This value of D should be used as the distance between the conductor centers of the equivalent system in transmission-line calculations.

196. Single-phase Line Calculations.—It determining the voltage drop in an alternating-current line, both the resistance and the reactance must be taken into consideration. The voltage to supply the resistance drop is in phase with the current, and the voltage to supply the reactance drop is in quadrature with the current and leading.

In making transmission-line calculations, it is convenient in all cases to work to neutral. Figure 374 shows a single-phase line

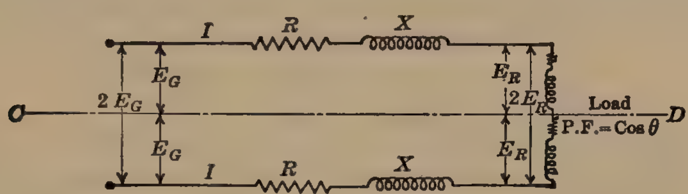


Fig. 374.—Single-phase line having resistance and reactance.

which has a resistance per wire of R ohms and a reactance per wire of X ohms. The load takes a current I amp. at a power-factor cos θ, and the *total* voltage at the load or receiver is $2E_R$. The voltage to *neutral* at the receiver is, therefore, E_R. The *total* voltage at the sending or generating end is $2E_G$.

If this system be split along the line CD, two systems result, one of which is shown in Fig. 375. Each of these two systems transmits one-half the total power, and the sending-end and

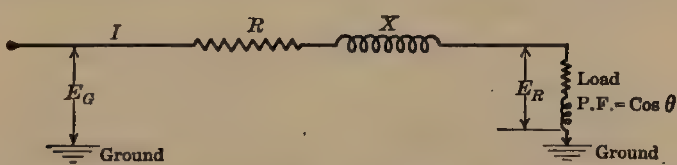


Fig. 375.—Single-phase line and voltages to neutral.

receiving-end voltage of each system is half the voltage between conductors. The voltage at each end is now the voltage to neutral. The ground is assumed to be the return conductor. The return conductor need be merely hypothetical, however, for under balanced conditions (Fig. 374), no current flows back through the ground, as each half of the system acts as a return for the other half. The voltage drop through the ground is, therefore, zero. That is (Fig. 375), for purposes of calculation, the ground may be considered as having zero resistance and zero reactance.

Let it be required, in Fig. 375, to determine the generator voltage E_G when the load voltage E_R, the current I, and power-factor $\cos\theta$ are given. The vector diagram is shown in Fig. 376 (*a*). The component of voltage to supply the IR drop is laid off in phase with the current I; the component to supply the IX drop is laid off 90° ahead of the current I. The resultant of these two components is the component to supply the IZ drop or to supply the actual voltage drop per conductor. The voltage at the generator E_G is the vector sum of E_R and IZ. In Fig. 376 (*b*), the IR and IX components are added to E_R vectorially.

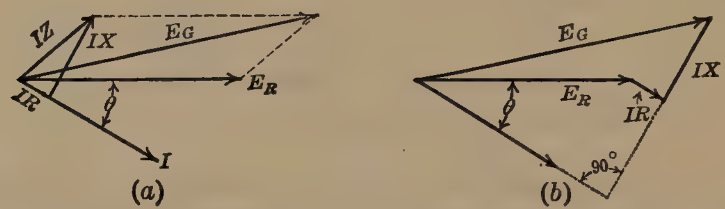


FIG. 376.—Vector diagrams for single-phase transmission line.

torially. It will be seen that this figure is similar to Fig. 166, (Chap. VII, p. 173), and its geometrical solution is identical.

$$E_G = \sqrt{(E_R \cos\theta + IR)^2 + (E_R \sin\theta + IX)^2}. \qquad (140)$$

If the current leads E_R,

$$E_G = \sqrt{(E_R \cos\theta + IR^2) + (E_R \sin\theta - IX)^2}. \qquad (141)$$

These voltage relationships may also be determined by means of complex notation. E_R is taken along the axis of reals. With lagging current,

$$\dot{E}_G = E_R + I\,(\cos\theta - j\sin\theta)(R + jX). \qquad (142)$$

With leading current,

$$\dot{E}_G = E_R + I\,(\cos\theta + j\sin\theta)(R + jX). \qquad (143)$$

Example.—It is desired to deliver 4,000 kw., single-phase, at a distance of 25 miles, the load voltage being 33,000 volts, 60 cycles, and the power factor of the load being 0.85, lagging current. The conductors are spaced 4 ft. apart. The line loss shall not exceed 10 per cent. of the power delivered. Determine: (*a*) the size of conductor; (*b*) the resistance drop per conductor; (*c*) the reactance drop per conductor; (*d*) the voltage at the sending end; (*e*) the line regulation. Neglect capacitive effects.

(*a*) The line loss $= 4{,}000 \times 0.10 = 400$ kw. $= 400{,}000$ watts.

The loss per conductor $= 400{,}000/2 = 200{,}000$ watts.

The current $I = \dfrac{4{,}000{,}000}{33{,}000 \times 0.85} = 142.5$ amp.

$$I^2R' = (142.5)^2R' = 200{,}000 \text{ watts.}$$

$$R' = \frac{200{,}000}{(142.5)^2} = 9.85 \text{ ohms.}$$

$$\text{Resistance (per mile)} = \frac{9.85}{25} = 0.394 \text{ ohm.}$$

From Appendix H (p. 542), the wire having the next lowest resistance per mile is 000 A.W.G., the resistance of which is 0.333 ohm per mile. *Ans.*

(*b*) Total resistance per conductor

$$R = 25 \times 0.333 = 8.34 \text{ ohms.}$$
$$IR = 142.5 \times 8.34 = 1{,}188 \text{ volts.} \qquad \textit{Ans.}$$

(*c*) From Appendix I (p. 544), for 000 conductor and 48-in. spacing, the reactance per conductor is 0.692 ohm per mile.

Total reactance per conductor, $X = 25 \times 0.692 = 17.3$ ohms. *Ans.*

The reactance drop

$$IX = 142.5 \times 17.3 = 2{,}470 \text{ volts.} \qquad \textit{Ans.}$$

(*d*) Applying Eq. (140), using volts to neutral ($E_R = 16{,}500$ volts),

$$\cos\theta = 0.85 \qquad \theta = 31.8° \qquad \sin\theta = 0.527$$

$$E_G = \sqrt{(16{,}500 \times 0.85 + 1{,}188)^2 + (16{,}500 \times 0.527 + 2{,}470)^2}$$
$$= \sqrt{(15{,}220)^2 + (11{,}170)^2} = \sqrt{357 \times 10^6} = 18{,}900 \text{ volts.}$$

Ung Eq. (142),

$$\dot{E}_G = 16{,}500 + 142.5(0.85 - j0.527)(8.34 + j17.3)$$
$$= 16{,}500 + 1{,}001 + j2{,}095 - j626 + 1{,}299$$
$$= 18{,}800 + j1{,}470$$
$$|E_G| = \sqrt{(18{,}800)^2 + (1{,}470)^2} = 18{,}900 \text{ volts } (\textit{check})$$

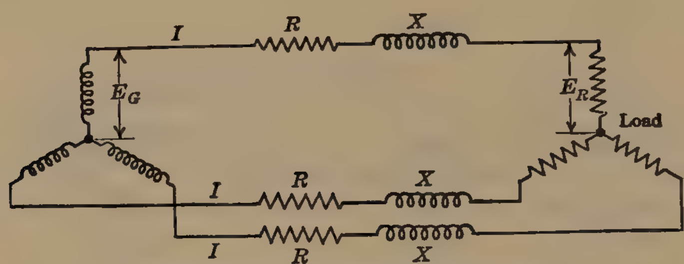
The voltage at the generator $= 2 \times 18{,}900 = 37{,}800$ volts. *Ans.*

(*e*) The line *regulation* is defined as the *rise in voltage when full load is thrown off the line, divided by the load voltage.*

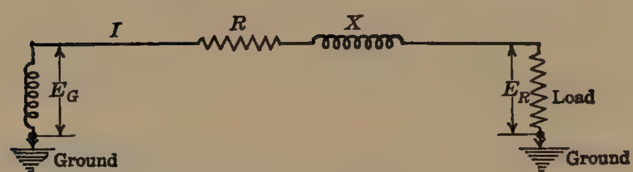
$$\text{Regulation} = \frac{37{,}800 - 33{,}000}{33{,}000} \text{ or } 14.4 \text{ per cent.} \qquad \textit{Ans.}$$

197. Three-phase Line Calculations.—The advantage of working transmission-line problems to neutral is much more obvious in three-phase lines than in single-phase lines. Figure 377 (*a*) shows a three-phase system, each conductor of which has a resistance of R ohms and a reactance of X ohms. The voltage to neutral at the load is E_R, and the voltage to neutral at the sending end is E_G. In order to determine the line characteristics, one

phase is removed (Fig. 377 (*b*)), and its characteristics determined. Under the condition of balanced load, which is assumed, the relations in all three phases are similar, so that the results obtained with one phase may be applied to the other two. As each pair of wires is the common return of the third wire, no current returns through the ground under the balanced conditions assumed. As the voltage drop between the load neutral and the generator neutral is zero, the ground may be considered as a return conductor of zero resistance and of zero reactance, as was



(*a*) **Three-phase transmission line having resistance and reactance.**



(*b*) **One phase of 3-phase line.**

Fig. 377.—Three-phase line having resistance and reactance.

done in the single-phase case. The load need not necessarily be Y-connected, as indicated in Fig. 377 (*a*). The same method is used even if the load be delta-connected and there be no neutral. The delta-load is replaced by an equivalent Y-load, and the computations are made for one phase only.

Example.—Solve the problem of Par. 196, assuming three-phase transmission, other conditions remaining the same. Power to be delivered, 4,000 kw.; load voltage, 33,000 between conductors; distance, 25 miles; frequency, 60 cycles; load power factor, 0.85, lagging current; spacing of conductors, 48 in.; allowable line loss, 10 per cent. of power delivered. Find (*a*), (*b*), (*c*), (*d*), and (*e*) (Par. 196). (*f*) Find the sending-end voltage when the power factor is 0.70 leading current.

(*a*) The power per phase $= 4{,}000/3 = 1{,}330$ kw.

The voltage to neutral $E_R = 33{,}000/\sqrt{3} = 19{,}070$ volts.

$$\text{Current per conductor } I = \frac{1{,}330{,}000}{19{,}070 \times 0.85} = 82.3 \text{ amp.}$$

Allowable loss per conductor $= 1{,}330 \times 0.10 = 133$ kw. $= 133{,}000$ watts.

Resistance per conductor $R' = 133{,}000/(82.3)^2 = 19.64$ ohms.

Resistance per mile $= 19.64/25 = 0.786$ ohm.

From Appendix H (p. 542), the wire having the next lowest resistance per mile is No. 1 A. W. G., the resistance of which is 0.665 ohm per mile. *Ans.*

(*b*) Total resistance per conductor

$$R = 25 \times 0.665 = 16.6 \text{ ohms}$$

$$IR = 82.3 \times 16.6 = 1{,}365 \text{ volts.} \quad \textit{Ans.}$$

(*c*) From Appendix I (p. 544), for No. 1 wire and 48-in. spacing, the reactance is 0.734 ohm per mile.

Total reactance per conductor

$$X = 25 \times 0.734 = 18.35 \text{ ohms.}$$

The reactance drop

$$IX = 82.3 \times 18.35 = 1{,}510 \text{ volts.} \quad \textit{Ans.}$$

(*d*) From Eq. (140), using volts to neutral ($E_R = 19{,}070$ volts)

$$\cos\theta = 0.85 \quad \theta = 31.8° \quad \sin\theta = 0.527$$

$$E_G = \sqrt{(19{,}070 \times 0.85 + 1{,}365)^2 + (19{,}070 \times 0.527 + 1{,}510)^2}$$
$$= \sqrt{(17{,}580)^2 + (11{,}560)^2} = \sqrt{443 \times 10^6} = 21{,}000 \text{ volts.}$$

Using Eq. (142),

$$\dot{E}_G = 19{,}070 + 82.3(0.85 - j0.527)(16.6 + j18.35) = 21{,}000 + j564$$

$$|E_G| = \sqrt{(21{,}000)^2 + (564)^2} = 21{,}000 \text{ volts.}$$

The voltage between conductors at the sending end

$$E'_G = \sqrt{3} \times 21{,}000 = 36{,}400 \text{ volts.} \quad \textit{Ans.}$$

$$\text{(e) Regulation} = \frac{21{,}000 - 19{,}070}{19{,}070} = \frac{1{,}930}{19{,}070} \text{ or } 10.1 \text{ per cent.} \quad \textit{Ans.}$$

$$\text{(f)} \qquad I = \frac{1{,}333{,}000}{19{,}070 \times 0.70} = 99.6 \text{ amp.}$$

Using Eq. (143),

$$\dot{E}_G = 19{,}070 + 99.6(0.70 + j0.715)(16.6 + j18.35) = 18{,}920 + j2{,}460 \text{ volts.}$$

$$|E_G| = \sqrt{(18{,}920)^2 + (2{,}460)^2} = 19{,}060 \text{ volts.} \quad \textit{Ans.}$$

With a leading current at this power factor, the sending- and receiving-end voltages are equal. With more line reactance or with a lower power factor, it is possible for the receiving-end voltage to be greater even than the sending-end voltage (see p. 371).

198. Lines Having Considerable Capacitance.—Heretofore, the line capacitance has been considered negligible in its effect on the regulation. In long lines of high voltage, the charging

current, due to the line capacitance, may have a very considerable effect on the regulation. Its tendency is to cause the voltage to rise from the sending end to the receiving end. The capacitance of the usual line is distributed uniformly along the line. The calculations are very considerably simplified, however, if the total capacitance C to neutral be divided, one-half being concentrated at the sending end, and one-half at the receiving end, in parallel with the load (Fig. 378). This assumption introduces little or no error in the results, even for the longest existing 60-cycle lines. The condenser at the sending end has no effect on the regulation, but its charging current $I_c/2$ must be added vectorially to the line current I in order to obtain the total current supplied by the generator. The current $I_c/2$ taken by the condenser at the load must be added vectorially to the load

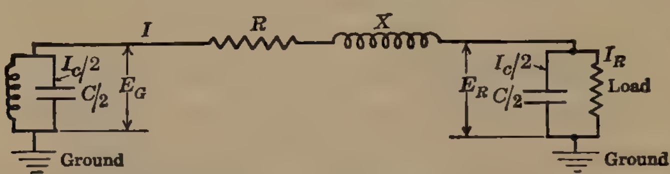


FIG. 378.—Transmission line having resistance, reactance and capacitance.

current I_R in order to obtain the total line current I. The problem is then treated by the methods already outlined.

Example.—It is required to deliver 30,000 kw., three-phase, at 0.80 power factor at a distance of 100 miles, with a line loss not exceeding 10 per cent. of the power delivered. The voltage at the load is 120,000 volts, 60 cycles, and the lines are arranged at the apexes of an equilateral triangle, 12 ft. on a side. Determine: (*a*) the voltage between conductors at the sending end; (*b*) the line regulation; (*c*) the total power supplied by the generating station; (*d*) the efficiency of transmission.

The power per phase

$$P = \frac{30,000}{3} = 10,000 \text{ kw.}$$

The volts to neutral at the load

$$E_R = \frac{120,000}{\sqrt{3}} = 69,300 \text{ volts.}$$

The current per conductor at the load

$$I_R = \frac{10,000,000}{69,300 \times 0.80} = 180.5 \text{ amp.}$$

The power loss per conductor $= 10,000 \times 0.10 = 1,000$ kw. $= 1,000,000$ watts.

The conductor resistance $R' = 1{,}000{,}000/(180.5)^2 = 30.7$ ohms.

$$\text{Resistance per mile} = \frac{30.7}{100} = 0.307 \text{ ohm.}$$

From Appendix H (p. 542), the wire having the next lowest resistance per mile is 0000, the resistance per mile of which is 0.264 ohm.

The conductor resistance $R = 100 \times 0.264 = 26.40$ ohms.

From Appendix I (p. 544), the reactance per conductor per mile for 0000 wire and 144-in. spacing is 0.810 ohm.

Total reactance $= 100 \times 0.810 = 81.0$ ohms.

The charging current for 0000 wire with 144-in. spacing and 100,000 volts to neutral is, from Appendix J (p. 545), 0.523 amp. per mile.

The total charging current for the above line is

$$I_c = 0.523 \times \frac{69{,}300}{100{,}000} \times 100 = 36.2 \text{ amp.}$$

As only one-half the line capacitance is assumed at the receiving end, the charging current flowing over the line, $I_c/2 = 36.2/2 = 18.1$ amp.

In order to find the total line current, however, this 18.1 amp. must be added vectorially to the 180.5 amp. of load current. The load current, therefore (Fig. 379), is resolved into an energy component $I_R \cos \theta = i_1 = 180.5 \times 0.8 = 144.4$ amp., and a quadrature component $I_R \sin \theta = i_2 = 180.5 \times 0.6 = 108.3$ amp.

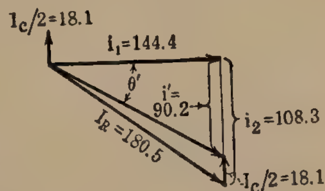


FIG. 379.—Effect of line charging current on total line current.

As the quadrature component, 108.3 amp., lags the load voltage by 90 °, and the charging current, 18.1 amp., leads the load voltage by 90°, the resulting quadrature component is

$$i' = 108.3 - 18.1 = 90.2 \text{ amp.}$$

The total line current

$$I' = \sqrt{(144.4)^2 + (90.2)^2} = 170 \text{ amp.}$$

Let θ' be the angle between this current and the voltage.

$$\operatorname{Cos} \theta' = \frac{144.4}{170} = 0.849 \quad \sin \theta' = \frac{90.2}{170} = 0.530.$$

(*a*) The voltage to neutral at the sending end

$$\begin{aligned} E_G &= \sqrt{(69{,}300 \times 0.849 + 170 \times 26.4)^2 + (69{,}300 \times 0.530 + 170 \times 81.0)^2} \\ &= \sqrt{(58{,}800 + 4{,}490)^2 + (36{,}700 + 13{,}770)^2} \\ &= \sqrt{(4{,}000 + 2{,}550) \times 10^6} = 81{,}000. \quad \textit{Ans.} \end{aligned}$$

The voltage between conductors

$$E = \sqrt{3} \times 81{,}000 = 140{,}300 \text{ volts.} \quad \textit{Ans.}$$

$$\text{(b) Line regulation} = \frac{81{,}000 - 69{,}300}{69{,}300} = \frac{11{,}700}{69{,}300} \text{ or 16.9 per cent.} \quad \textit{Ans.}$$

(*c*) The line loss

$$P_c = 3 \times (170)^2 \times 26.4 = 2{,}290{,}000 \text{ watts.}$$

The total generator power

$$P_G = 30{,}000 + 2{,}290 = 32{,}290 \text{ kw.} \quad \textit{Ans.}$$

(*d*) The efficiency

$$\eta = \frac{30{,}000}{32{,}290} \text{ or } 92.9 \text{ per cent.} \quad \textit{Ans.}$$

199. Solution by Complex Quantities of Lines Having Considerable Capacitance.—Transmission lines may be solved by Eqs. (142) and (143) (p. 434), if the line capacitance is negligible. If the line capacitance is not negligible, it is necessary merely to add one-half the total line charging current $\dfrac{+jI_c}{2}$ to the total

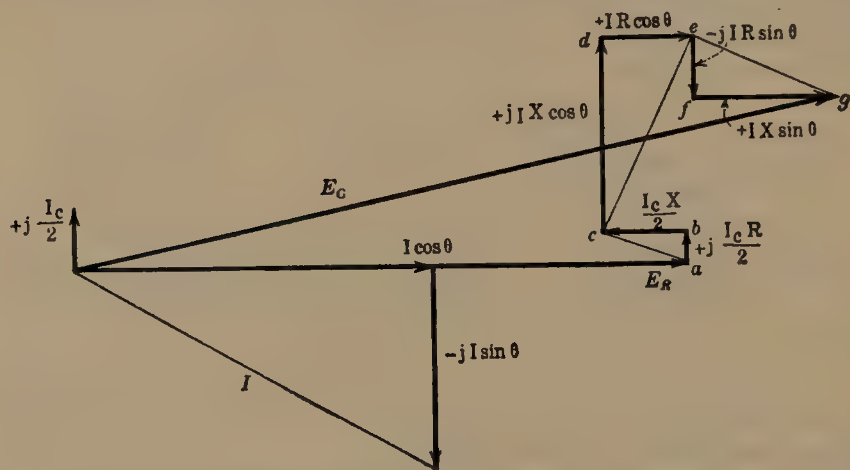


FIG. 380.—Complete vector diagram for transmission line.

current (see p. 438). Since this charging current is constant and nearly independent of the load, it is preferable, for purposes of analysis, to treat it as an independent current. The equation for the generator voltage then becomes

$$\dot{E}_g = E_R + I(\cos\theta \pm j\sin\theta)(R + jX) + j\frac{I_c}{2}(R + jX). \quad (144)$$

The minus sign is used for lagging current, and the plus sign for leading current.

This equation, when expanded, becomes

$$\dot{E}_g = E_R + IR\cos\theta + jIX\cos\theta - jIR\sin\theta + IX\sin\theta + \frac{jI_cR}{2} - \frac{I_cX}{2} \quad (145)$$

the negative sign being taken.

The position of each of these vectors is shown in Fig. 380. Since $jI_c/2$ is assumed constant, triangle *abc* is constant; each side of triangle *cde* is proportional to the energy current of the load, $I \cos \theta$, and, hence, to the kilowatts taken by the load; each side of triangle *efg* is proportional to the quadrature current of the load, $I \sin \theta$, and, hence, to the quadrature kilowatts. For example, if the load power factor becomes unity, $I \sin \theta$ becomes zero, and triangle *efg* disappears. The foregoing relations make the diagram (Fig. 380) very useful in analyzing the effects of varying load, power factor, etc. on such lines.[1] Applying Eq. (144) to the example (Par. 198),

$$\dot{E}_G = 69{,}300 + 180.5(0.80 - j0.60)(26.4 + j81.0) + j18.1(26.4 + j81.0) = 80{,}400 + j9{,}320$$

$$|E_G| = \sqrt{(80{,}400)^2 + (9{,}320)^2} = 81{,}000 \text{ volts.} \quad \textit{Ans.}$$

The power at the generator is readily found.

The energy current is $180.5 \times 0.80 = 144.4$ amp.

The load quadrature current is $-j180.5 \times 0.60 = -j108.3$ amp.

The total quadrature current is $-j108.3 + j18.1 = -j90.2$ amp. Then, by Par. 43 (p. 66),

$$P_G = 3[80{,}400 \times 144.4 - 9{,}320 \times 90.2] = 3[11{,}610 - 840] \times 10^3 \text{ watts}$$
$$= 3 \times 10{,}770 = 32{,}310 \text{ kw.} \quad \textit{Ans.}$$

200. Corona.—Figure 381 shows a tapered conductor whose diameter at the large end is about ½ inch. This conductor tapers gradually to a point. It is suspended vertically in air, with its tip about 18 in. from a conducting sheet or plate, which is grounded. The secondary terminals of a high-voltage transformer are connected, the one to the tapered conductor and the other to the plate.

A low voltage is first applied to the transformer, and the voltage is then gradually increased. When the secondary voltage is in the neighborhood of from 3,000 to 4,000 volts, a bluish discharge occurs from the pointed tip of the conductor. This may be plainly seen if the room be darkened. As the voltage is increased, the bluish discharge forms farther up on the conductor and surrounds it in a ring. When the voltage reaches the neighborhood of 100,000 volts, this bluish discharge may have formed on the rod up to a point where the diameter of the rod is about ⅜ in.

[1] See "Standard Handbook," Fifth Ed., Sec. 11.

Meanwhile, the discharge from near the pointed end, and the accompanying hissing sound, will have become quite vigorous.

This bluish discharge is called *corona.* It occurs when the electrostatic stress in the air exceeds about 75,000 volts maximum

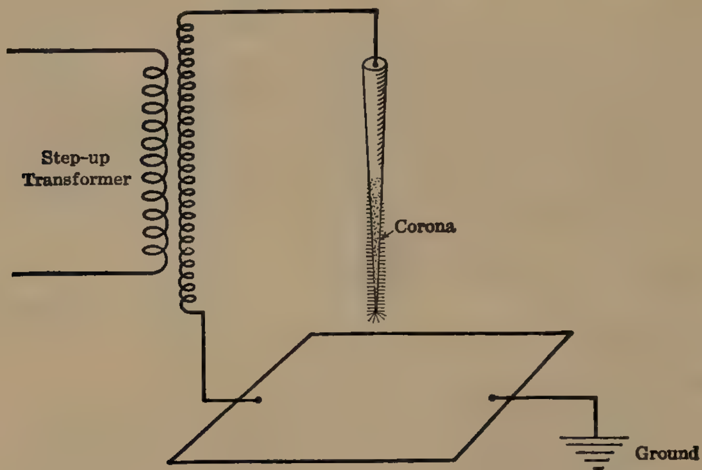


FIG. 381.—Formation of corona on tapered conductor.

per inch, or 53,000 volts effective per inch. At this voltage gradient, the number of electrostatic lines per unit area becomes too great for the air to withstand (see Vol. I, Chap. IX, pp. 244–246). This is the reason why corona first appears at the sharp point. The electrostatic flux lines are more concentrated at points. This is illustrated in Fig. 382, which shows a conducting body suspended in air, the potential of the body being considerably above ground potential. The electrostatic lines leaving this body are indicated. They are much more dense at those parts of the surface having a smaller radius of curvature.

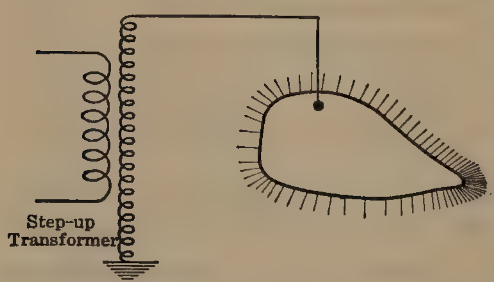


FIG. 382.—Effect of radius of curvature on distribution of electrostatic lines.

When air is so highly ionized that corona forms, its dielectric strength is practically nil, and the air may be considered as broken down or disrupted electrically. Under these conditions,

the air becomes a partial conductor and is practically valueless as an insulator.

Corona is always accompanied by the production of ozone, the odor of which is readily detected. In the presence of moisture, nitrous acid forms when corona occurs. The acid and ozone may attack metals and other substances, such as insulating materials. When corona occurs, the resulting ozone is very active chemically.

Corona is accompanied by a dissipation of energy. If a transmission line be operated at a sufficiently high voltage, corona loss occurs. Where a line is long, the loss becomes serious and must be considered when the line is designed. The loss may be reduced by increasing the diameter of the conductors and thus increasing their radius of curvature. This fact favors aluminum for transmission-line conductors, other factors being equal. At the present time, aluminum conductors having a steel cable for the center are in common use (aluminum cable—steel reinforced). This type of conductor has large diameter, and the steel core gives it high tensile strength (see Appendix, *H.*, p. 543). Figure 383 shows the conductors of a high-voltage line illuminated by the corona discharge.[1]

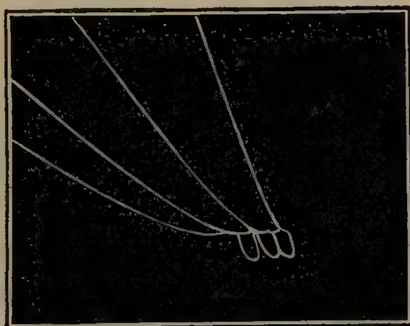


Fig. 383.—Illumination of transmission line by corona.

LIGHTNING ARRESTERS

201. Multigap Arresters.—Abnormal voltage rises occur in power systems due to lightning discharge, switching, short circuits, and other disturbances. These voltage rises may damage the system and any connected apparatus, by puncturing insulation, by producing insulator flash-overs, arcing grounds, etc. It is, therefore, highly desirable to relieve the line of such disturbances whenever possible. This is done by means of lightning arresters, whose function is to relieve any abnormal

[1] For a more complete discussion, see F. W. Peek, "The Law of Corona and the Dielectric Strength of Air," *Trans. A. I. E. E.*, Vol. XXX (1911) p. 1889.

voltage rise by passing current to ground and, therefore, preventing damage to the system and connected apparatus.

Lightning arresters are connected between the line to be protected and the ground. They must have five properties. They should be practically an open circuit when the line is at normal voltage. They should provide an easy path to ground for the discharge; the time lag of discharge should be practically zero. They should be able to absorb the energy of the discharge. They should be able to suppress the dynamic arc which follows the transient discharge and which the power of the system tends to maintain.

One simple type of arrester for low voltage is shown in Fig. 384. A number of cylinders made of non-arcing metal are connected

FIG. 384.—General Electric multigap lightning arrester.

between the line and ground. There is a small air-gap between adjacent cylinders. A carbon rod of high resistance is shunted from the conductor across approximately three-fourths of these cylinders, a medium resistance across approximately one-half the cylinders, and a comparatively low resistance across a little over one-quarter of the cylinders. The cylinder spacing is such that the full line pressure which exists across the last five cylinders cannot jump the series gaps. Any considerable increase of line voltage, however, causes a discharge through the resistance, across these five gaps, and thence to ground. If the discharge becomes sufficiently heavy, the voltage drop through the high resistance becomes excessive and increases the voltage across the next four gaps, which then break down and assist in the discharge. In a very heavy discharge, the voltage drops across all the resistances become large, and the discharge passes to

ground through the entire series of gaps. When the line returns to normal voltage, the cooling effect of the large number of cylinders, combined with the rectifying property of their metallic vapors, tends to prevent the dynamic arc's being sustained. Such arresters are suited only for low voltages (up to 5,000 volts) and can absorb only small amounts of energy.

202. Horn Gaps.—For high voltages, the horn gap (Fig. 385) is used occasionally. The gap consists of two horns, each mounted on an insulator, and the gap itself is located between

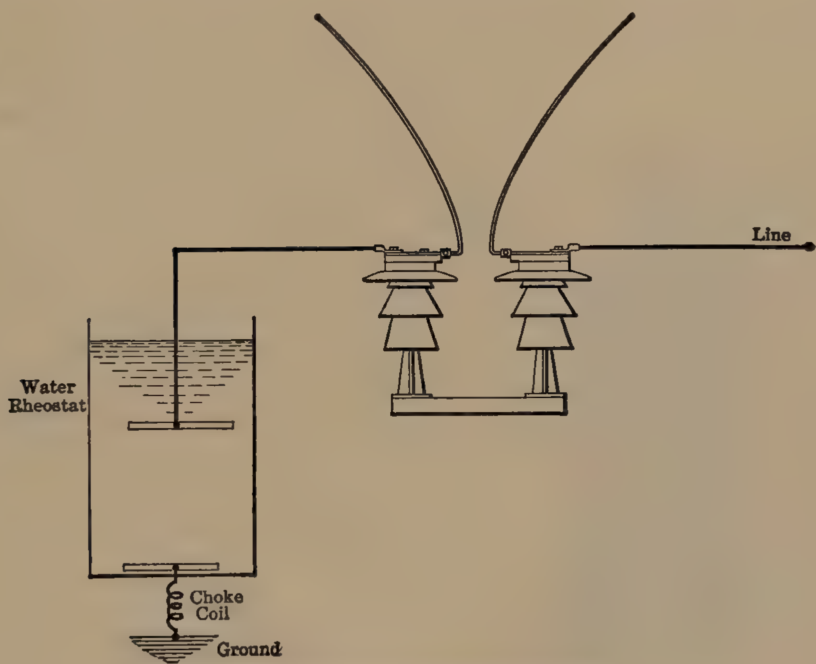


FIG. 385.—Horn-gap lightning arrester.

the lower parts of the horns. One horn is connected directly to the line to be protected, and the other is connected through a resistance, usually water, and a choke coil to ground. The gap is so set that ordinary operating voltages cannot jump it. When the voltage rises so that it is from 150 to 200 per cent. of its normal value, it jumps the gap, and the disturbance passes to ground. The resistance and choke coil limit the current and so prevent the line's being grounded by the arc. The function of the horns is to break the arc. An arc tends to rise because of its heat and, also, because of the well-known law that a current

tends to form a loop as large as possible, in order to make the permeance of the magnetic circuit a maximum (see Vol. I, p. 14, Par. 17).

Horn gaps are not altogether satisfactory, because they often arc over unnecessarily; the protection which they afford is insufficient because of the comparatively low discharge rate of the resistance and choke coil; they do not always suppress the dynamic arc which follows the transient discharge. This results either in a permanent arcing ground or in the destruction of the gap.

203. Aluminum-cell Arrester.—The aluminum-cell arrester superseded the horn-gap type, because of its very high discharge characteristics with slight increase in voltage above a critical value. It is based on the following principle: If aluminum be immersed in certain electrolytes and a direct-current voltage be impressed on the aluminum and the electrolyte, no appreciable current flows, except for an instant. This is due to the fact that the current builds up a very thin film of aluminum oxide on the plate, which acts as an insulator. This film builds up with alternating current as well as with direct current. The oxide, however, constitutes an insulator and also a dielectric of almost infinitesimal thickness. Considerable capacitance, therefore, exists between the plate and the electrolyte. This would result in a considerable charging current through the arrester, with alternating current, if it were connected directly across the line.

Fig. 386.—Cross-section of aluminum-cell arrester.

This film is an excellent insulator up to approximately 340 volts effective. When the voltage exceeds this critical value, the film breaks down and allows a large current to pass. When the voltage again drops below this critical value, the film re-forms and stops the current flow. Hence, such a device is an electrical safety valve. Its characteristics are, therefore, ideal for a light-

ning arrester. Several cells are always connected in series, the number of cells depending on the voltage. The oxide films prevent any discharge as long as the line voltage is normal. If the line voltage becomes abnormally high, the aluminum films are broken down, and the discharge passes readily to ground. When conditions again become normal, the films re-form, and no power arc can follow the discharge.

In practice, the aluminum is in the form of cones, the proper number being clamped together in a stack (Fig. 386). Each cone is about half filled with electrolyte. The entire stack is immersed in oil, as the oil acts as an excellent insulator and, also, absorbs the energy of the discharge.

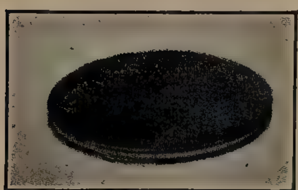
Were the stack connected directly across the line, the charging current to the stack would cause considerable heating in the cell and would, therefore, reduce its capacity for absorbing the energy of the discharge. Consequently, there is a small horn gap in parallel with a sphere gap in series with each arrester. The gaps are very short in comparison with the arcing distance of the circuit, so that they have little effect on discharges occurring during abnormal voltage rise. The use of spheres for the gaps increases the speed of the gaps in discharging high-frequency impulses (see Fig. 388).

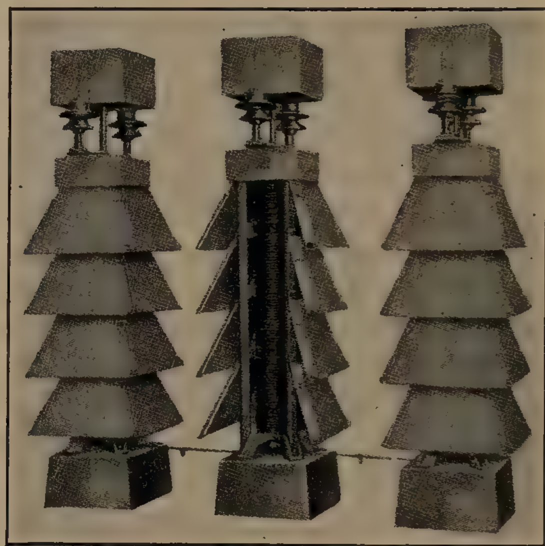
In time, the film dissolves, and it is necessary to re-form it about every 24 hr. This is done by closing an auxiliary gap, which allows the arrester to charge through a carbon-rod resistance and so re-forms the film. The purpose of the carbon rod is to limit the charging current and to damp out high-frequency disturbances that might otherwise occur when the arrester is being charged.

In arresters whose rating exceeds 12 kv., there are three stacks for a grounded three-phase system, each stack being connected between its respective conductor and ground. In a non-grounded system, the three line stacks are connected in a common Y, and a fourth stack, called the *ground stack*, is connected between the neutral of the Y and ground (see Fig. 396, p. 456).

204. Oxide-film Arrester.—The disadvantages of the aluminum-cell arrester are: The film dissolves, and the arrester must be charged at least once a day; it must be protected against freezing; if located in too warm surroundings, the film dissolves

readily; the oil carbonizes and short circuits the cones. The *oxide-film* arrester, developed by the General Electric Company, does not have these disadvantages. The individual unit (Fig. 387 (*a*)) consists of a porcelain annulus, about 7½ in. (19 cm.) in diameter and ⅝ in. (1.59 cm.) thick, over which two metal discs are crimped. The inner surfaces of the discs are coated with oxide films, which puncture at about 300 volts. The space between the discs is filled with lead peroxide. A number of these units, depending on the line voltage and other factors, such as lightning conditions, are stacked in series and connected between conductors and ground in series with a hemispherical gap at the line end (Fig. 387 (*b*)). When the potential exceeds 300 volts per unit, the oxide film punctures, permitting the passage of the charge to ground through the lead peroxide. The heat developed at the point of discharge causes the lead peroxide to change to litharge and red lead, which are fairly good insulators. After the voltage has returned to normal, these lead compounds seal the puncture. In time, the original film is replaced by litharge and red lead, which may raise the discharge voltage. In service, however, the units need not be replaced for several years. Figure 387 (*b*) shows an oxide-film arrester ready to be connected to the 25,000-volt lines.



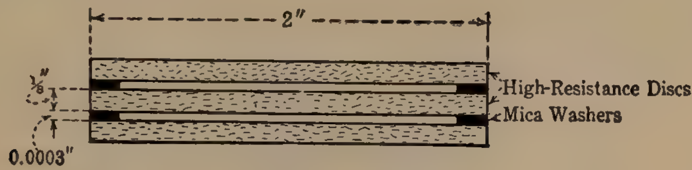


(*a*) (*b*)

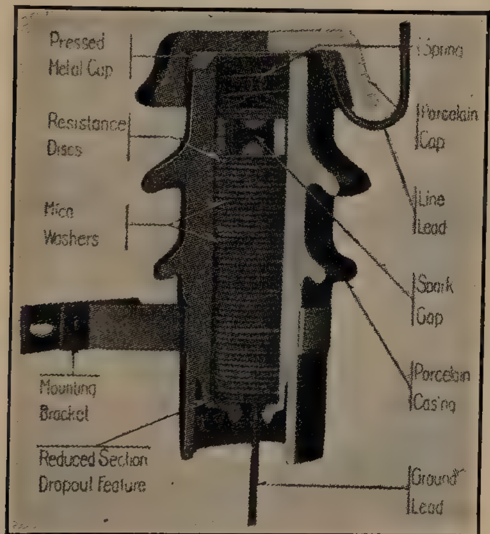
FIG. 387.—General electric oxide film lightning arrester.

205. Autovalve Lightning Arrester.—The autovalve lightning arrester, developed by Dr. Joseph Slepian of the Westinghouse

Electric & Manufacturing Company, depends on the following principles: The breakdown voltage of uniform air-gaps decreases with decrease in the distance between electrodes, until the separation of the electrodes is approximately 0.001 in. At this voltage, the gap breaks down at approximately 350 volts. Further decrease in the distance causes an *increase* in the breakdown voltage. That is, no uniform air-gap breaks down at less than 350 volts.



(*a*). Discs and mica spacers.



(*b*). Assembled unit.

FIG. 388.—Autovalve lightning arrester.

If such a gap exists between metal discs, which necessarily have high conductance, an arc, which develops localized high temperature, forms at breakdown. This causes a rapid emission of electrons (Chap. XIV), and the arc persists after the transient discharge is over, since it requires but 20 volts per gap to maintain the arc discharge. If, however, the discs are made of a material having high resistivity (but not an insulating material), the discharge takes the form of a glow over the entire gap. Ordi-

narily, no local high temperatures develop, and, hence, a high rate of thermionic emission does not result. Nevertheless, the discharge rate is very high under these conditions. When the voltage returns to normal, the voltage per gap becomes less than the e.m.f. necessary to maintain the glow discharge (350 volts), and the arrester again is virtually an open circuit.

The discs are circular, of 2-in. (5.1-cm.) diameter, and approximately ⅛ in. (0.3 cm.) thick. They have the appearance of carbon. The gaps, which are approximately 0.0003 in. (0.00076

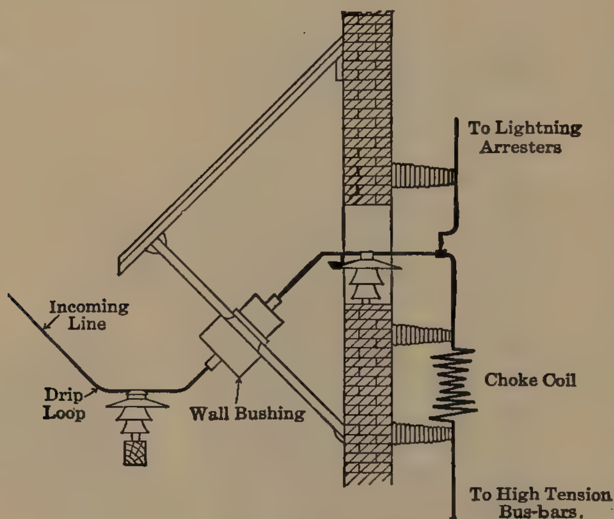


FIG. 389.—High-voltage entrance and connections to lightning arresters and bus-bars.

cm.), are formed by separating these discs with thin mica washers (Fig. 388 (*a*)). Because of the higher dielectric constant of the mica and its effect on the dielectric field, the charge always begins at the inner edge of the mica washer. This precipitates the discharge over the entire gap. Figure 388 (*b*) shows a 7,500-volt, autovalve arrester complete.

Lightning arresters should be connected to the incoming line where it enters the station, or even outside the station. Choke coils, consisting of a few turns of bare wire, are connected between the arresters and the station bus-bars (Fig. 389). When a surge reaches the station, it has a choice of two paths, the inductive path through the choke coil into the station and the condensive path through the arrester to ground. Obviously, a

surge, being of high frequency, will take the path to ground through the arrester, whose condensive reactance is low at high frequencies.

TRANSMISSION LINE CONSTRUCTION

206. Pin-type Insulators.—The success of any transmission line depends to a large extent on the insulators. Little or no difficulty is encountered in insulating low-voltage lines. Pin-type insulators are always used for such lines, because they are cheap, are easy to install, and act as rigid supports for the conductors. Pin-type insulators are made of glass, of porcelain, and of patented compounds.

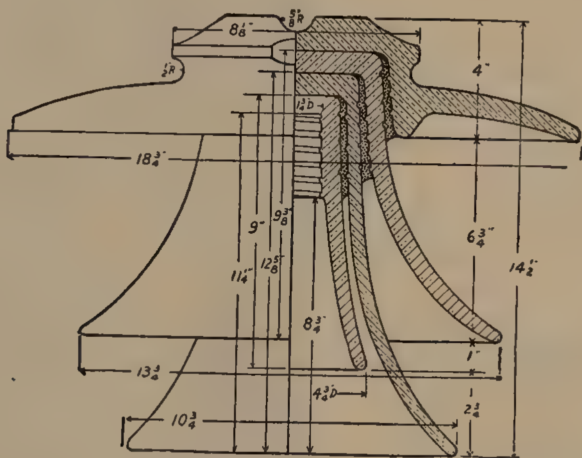


FIG. 390.—Typical 77,000-volt, pin-type insulator.

Glass is suitable for lines of light construction, such as telephone lines, and for power lines of moderate voltage. Its advantages up to 10,000 or 15,000 volts are its cheapness and the fact that cracks and flaws are readily detected. On the other hand, it is hygroscopic and breaks readily. At the present time, however, *Pyrex*-glass insulators have been developed to a point where they are being used on high-voltage lines.

Porcelain has excellent mechanical and electrical characteristics but is more expensive than glass. Internal flaws are not readily detected, and cracks in the porcelain cause rapid deterioration of the insulator. Porcelain is the principal material used for insulators on high-voltage power lines.

Patented compounds have good mechanical characteristics and are readily moulded to any desired form. They cannot, however, withstand the severe mechanical stresses combined with the electrical stresses and weathering encountered in power lines.

In the larger sizes of pin-type insulator, the insulator is made up in sections cemented together (Fig. 390). Pin-type insulators can be safely used for voltages up to about 66,000 volts, but for these high voltages they are large, expensive, and produce excessive torsion in the crossarms.

207. Suspension-type Insulators.—It seemed at one time as if the insulator would limit transmission voltages, as the pin type had practically reached its limit in size, weight, and cost. The introduction of the suspension-type insulator, however, has raised the limit of transmission voltages to more than three times the value possible with the pin-type insulator. With the suspension type of insulator, the conductor is suspended instead of being rigidly supported. A string of suspension insulators is made up of several units in series, the number of units depending on the voltage. A single unit can safely operate at from 16,000 to 25,000 volts, depending on local conditions. Under normal conditions, the insulator string acts as a flexible support for the

Fig. 391.—Section of link-type suspension insulator.

Fig. 392.—Arc-over, at 60 cycles, of six-unit string of link-type insulators.

conductor and offers little or no resistance to horizontal forces. Hence, the stresses in adjacent spans should be nearly balanced, or the string will be pulled out of the vertical line. When a span breaks, the string is thrown temporarily into the adjacent unbroken span as a strain or dead-end insulator. Suspension insulators are also used as strain insulators at dead ends, railroad crossings, etc. Figure 391 shows a section of a link-type suspension insulator in which the suspension loops link each other. Figure 392 shows a string of such insulators arcing over under high voltage (also, see Figs. 393, 394, and 395).

208. Transmission Structures.—There are three general types of transmission structures employed in this country—wooden poles, steel poles, and steel towers. Concrete poles are used occasionally.

Wooden poles are used on the lighter lines, especially where the voltage is low. Wooden poles have the advantage of being cheap, particularly when used near wooded sections. They are also light, easily fitted, and erected. On the other hand, their life is comparatively short so that they require frequent renewals. They are not sufficiently strong for heavy lines operating at high voltage. Owing to the limited height of wooden poles, the spans must be short.

Steel poles are ordinarily made of four main members supported and braced by lattice work (Fig. 393) and are usually set in concrete. This type of pole is strong and, if painted occasionally, has a long life. It does not require a wide right of way. It is particularly useful in mill yards and along railroad tracks, where the space is limited. Except for moderate heights, however, towers are cheaper than steel poles, especially in this country, where labor costs are high.

Steel towers are a development of the windmill tower so common in this country. They are ordinarily composed of four main members braced by light cross-members. They are stronger and more rigid than either the wooden or the steel pole. As they are made of a comparatively few standard members, riveted or bolted together, the labor costs are comparatively low. Owing to the spread of the four main members, they are able to resist the high torsional stresses such as would result from the breaking of the conductors on one side. Towers may be set in concrete

bases. This is necessary if the ground is marshy. A less expensive method is to rivet plates or feet on the bottom and bury the lower supports directly in the ground. The towers are usually

FIG. 393.—Steel pole carrying high voltage to East St. Louis & Suburban Railway Co. (*Archbold-Brady Co.*)

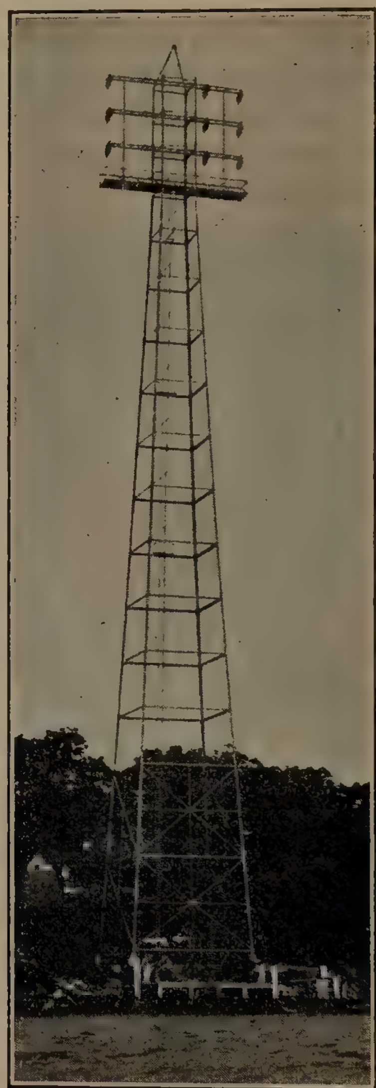


FIG. 394.—Archbold-Brady tower, 254 ft. high, at Thames River crossing, Montville, Conn.

shipped "knocked down" and are assembled on the spot by the erecting crew. Figure 394 shows a transmission tower of unusual height which supports the power lines of the Eastern Connecticut Power Company at a river crossing.

A cheaper form of transmission-line structure is the flexible tower. This form of tower is based on the principle that if the stresses in two adjacent spans are equal, the structure acts merely as a prop which supports the line but which need not resist longitudinal forces. Flexible towers (Fig. 395) are merely A-frames designed to withstand the maximum transverse stress which may occur but are not intended to withstand stress in the direction of the line. When these towers are used, an anchor tower about every mile is necessary, in order to take care of any unbalanced longitudinal forces which occur when conductors break. When suspension insulators are used, a steel ground wire is necessary at the top of the structure to give longitudinal support to the tower. The advantage of flexible tower construction lies in the fact that the towers are usually assembled complete in the shop and are easily erected.

Fig. 395.—132,000-volt single-circuit, A-frame, flexible tower of the Northern Ohio Traction & Light Company. (*Archbold-Brady Co.*)

SUBSTATIONS

209. Transformer Substations.—The function of the substation is to receive the electrical energy, usually at a voltage too high for commercial purposes, and to deliver this energy at other voltages and sometimes at other frequencies such as may be required for the district served.

The substation may be a transformer station only, receiving energy at a voltage of 26,400 volts, for example, and transforming it to 2,300 volts for general distribution. Figure 396 shows the wiring diagram of such a station. Two distribution lines leave

the station at 2,300 volts, one for lighting and one for power. Power loads and lighting loads should be kept separate, if possible, in order to avoid the flickering of lamps when the motor loads are thrown on and off the line. Usually, 2,300 to 230–115-volt transformers are used to step down the voltage for

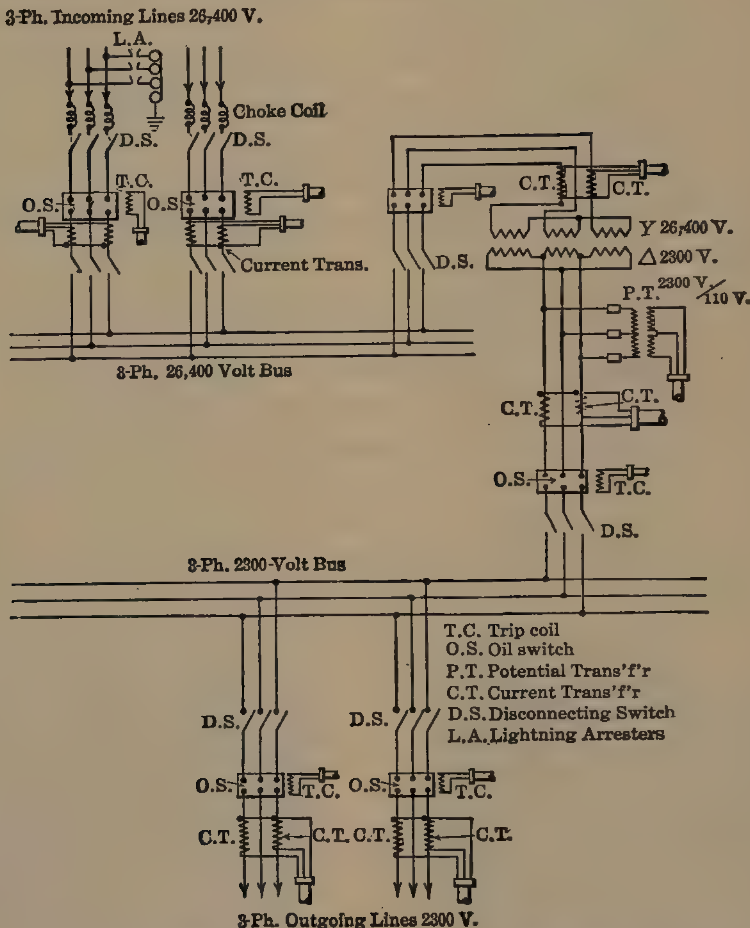


Fig. 396.—Typical connections for a transformer substation.

lighting purposes, a three-wire system being employed for the secondary (see p. 424, Fig. 366). Owing to the possibility of the low-voltage wires' coming in contact with high-voltage wires and so exposing the consumer to danger, one wire of the secondary of lighting circuits, usually the neutral, should be grounded at each consumer's premises. As motor loads are usually three-phase, two V-connected transformers, three single-phase transformers,

or a single three-phase transformer may be used for stepping down the voltage. In order to save secondary copper, motors are often operated at 440 or 550 volts. Some large consumers, employing a few large motors, may operate them at 2,300 volts and thus eliminate the step-down transformers.

210. Motor-generator and Synchronous-converter Substations.—It is often necessary to obtain direct current, either for power supply to a thickly populated district or for electric railways. As has been pointed out (p. 377, Par. 169), the mercury-arc rectifier, the synchronous-motor-generator set, the induction-motor-generator set, or the synchronous converter may be employed for changing the alternating-current supply into direct current. The rectifier has no rotating parts and in the higher voltages is very efficient as has already been pointed out. The advantage of the synchronous-motor-generator set is that its power factor may be controlled; its disadvantage is its tendency to fall out of step when line disturbances occur. The advantage of the induction-motor-generator set is that the induction motor tends to continue operating even when severe line disturbances occur; the induction motor does not require direct-current excitation; it is very rugged. Its principal disadvantages are that it takes lagging current and that at light loads its power factor is low.

The advantages and disadvantages of the synchronous converter as compared with motor-generator sets have already been discussed in Par. 170 (p. 387).

211. Oil Switches.—With an ordinary air-break switch, it is practically impossible to break a high-voltage circuit supplying any considerable amount of power. Special air-break switches are in use for interrupting high-voltage circuits, but the knife blades of these switches are from 4 to 6 ft. long, and the switch is provided with horn gaps. Such switches are suited only to outside mounting, where there is ample space for the resulting arc. The power rating of such switches is very limited. To interrupt high-voltage circuits, especially where the power is large, the switch contacts must be immersed in oil in order to quench the resulting arc. When the voltage is even moderately high, a separate compartment or a tank for each phase is necessary (Fig. 397 (*a*). In modern switches there are two sets of

contacts, one set for carrying the current which must have low contact resistance and the other for breaking the arc when the breaker is opened. The leaves of the laminated copper brush (Fig. 397 (*b*)) press on copper blocks when the switch is closed, giving low contact resistance to the current. The arc which occurs on opening the switch is broken at the four contacts which are plainly shown in Fig. 397 (*b*). These contacts open

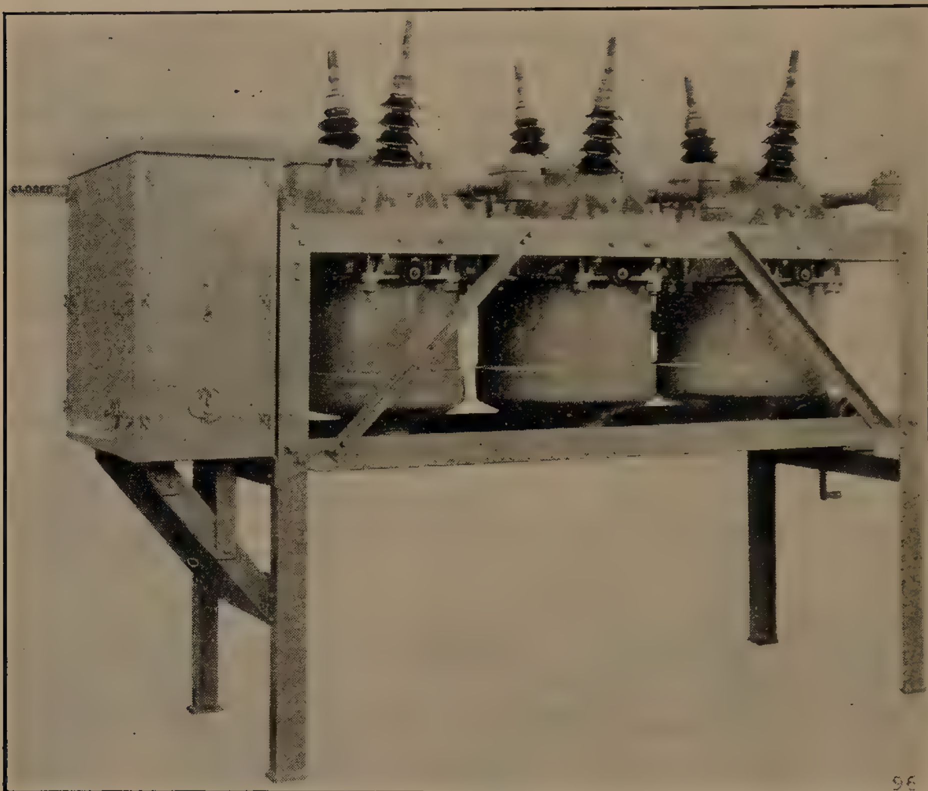


FIG. 397 (*a*).—Condit, three-pole, 25,000-volt, 800-amp., d.-c.-solenoid-operated, outdoor circuit breaker having interrupting capacity of 500,000 kv.-a.

after the current contacts open. By using four such contacts only one fourth the total energy is concentrated at each contact. The effect of the oil is to cool and quench the arc while the circuit is being opened. The heat of the arc tends to carbonize the oil so that it is occasionally necessary to renew it. During short circuits, the switch may be called upon to absorb a large amount of energy in a very short time. The resulting pressure within the switch compartments may be very high, so that it is neces-

sary to construct the tanks of heavily riveted or welded steel such as is shown in Fig. 397 (*a*). Even so, explosions of switch cells are not uncommon.

Due to the fact that carbonized oil may form a conducting path between switch contacts, there is always a possibility of injury to persons working on the supposedly dead side of the switch. It is always desirable, therefore, to have an air-break

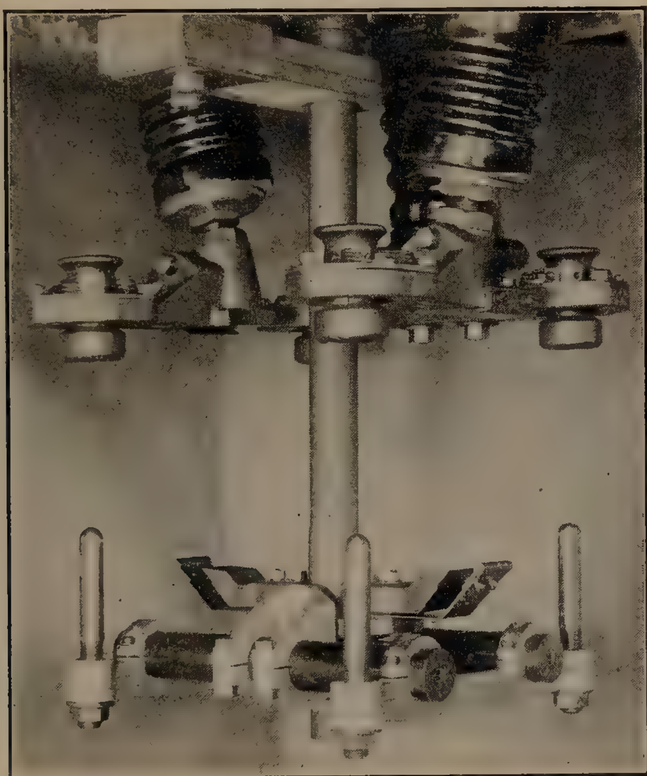


Fig. 397 (*b*).—Current-carrying, arcing details and insulation of Condit circuit breaker—contacts in open position.

disconnecting switch in each phase. The disconnecting switch may form a part of the switch, or it may be installed on a separate outside mounting (see wiring diagram, Fig. 396). The disconnecting switch is not called upon to interrupt the circuit under operating conditions but is opened only after the oil switch has interrupted the circuit.

Practically all oil switches operating at high voltages, or connected in circuits of considerable power, are operated by remote control. Both solenoids and motors, energized from a

low-voltage circuit and controlled from the switchboard by low voltage, are used to operate the oil switch. The switch of Fig. 397 is solenoid operated.

212. Arrangement of Apparatus in Substations.—The purpose of the substation building is to protect the equipment and the operator from the weather. The incoming high-voltage lines are brought in either through the roof, by means of roof bushings, or through the side walls by means of wall bushings (Fig. 389, p. 450). The incoming wires are bent to form drip loops so that water will not run down the wires into the station.

Fig. 398.—Automatic railway substation, New York Central Railroad Co., 110th St. General view of station showing 2000-kw. motor-generator, high-speed circuit breaker and automatic control panels. (*Courtesy of the General Electric Co.*)

The high-voltage bus-bars are usually located near the roof of the station so as to be out of the way. It is also desirable to place other high-voltage equipment, such as lightning arresters, oil switches, etc., on some form of balcony or else inside an enclosure so that the possibility of personal contact is minimized.

213. Automatic Substations.—In order to eliminate the cost of having an attendant in the smaller substations, automatic substations have been developed. These are particularly

adapted to electric railway work. After the trolley voltage in the vicinity of the station has fallen below a predetermined value and remained there for a minute or so, a combination of relays and switches starts up one of the synchronous converters or motor-generator sets, synchronizes them with the alternating-current line, and then connects the direct-current side to the trolley line. If the load on the station exceeds the safe load of the machine in service at that time, another machine starts up automatically, and after it is connected across the line, the field

FIG. 399.—One of two duplicate outdoor substations Wilson Dam, Sheffield, Alabama. Capacity of both: 260,000 kv-a., 154,000 (Y)/110,000 (Gr.)–11,500 volts. (*Courtesy of the General Electric Co.*)

rheostat operates to make it take its share of the load. Likewise, the machines drop out of service automatically after the load has fallen below a predetermined value. By means of pilot wires, the load despatcher in the central station is kept informed, by automatic means, of every operation in the substation. He, likewise, is able to start and stop the machine himself, his control being effected through the medium of the pilot wires. Figure 398 shows the interior of one of these stations.

214. Outdoor Substations.—When the voltage is high, the clearances required by the high-tension leads and bus-bars

within a substation may require a large building and, hence, a considerable investment. The investment in equipment and in buildings situated along transmission lines and supplying small loads may be large compared with the kilowatt-hours consumed. Substations for small loads would not be economically possible were it necessary to place all the apparatus within a building. Transformers, switches, lightning arresters have been designed so that it is possible to operate them out of doors. The building needs only to house the switchboard and the operator, if one is necessary. The oil switches, lightning arresters, transformers, and bus-bars can all be placed out of doors (see Fig. 397). The apparatus must be practically airtight to keep out moisture. Outdoor substations on a large scale are highly developed at the present time. Figure 399 shows an outdoor substation of moderate size.

CHAPTER XIV

ELECTRON TUBES

215. Electrons.—Electron tubes, vacuum tubes, or *thermionic valves*, as they are called, depend for their operation on the fact that, according to the latest investigations, electricity is atomic. That is, electricity is composed of extremely small *negative* charges called *electrons*. The charge of each electron is 1.59×10^{-19} coulomb, and its mass is 9.0×10^{-28} g. or 1/1,845 of that of the hydrogen atom. An atom of matter consists of a small, positively charged *nucleus*, with which the electrons are associated, the positive charge of the nucleus and the number of electrons depending on the substance. The *nucleus* itself is made up of the elemental positive charge, called the *proton*, which has practically all the mass of the atom and enough electrons to give a negative charge half that of the positive charge of the proton. In a *neutral* atom, the number of electrons associated with the nucleus is equal to the number of electrons in the nucleus, and, hence, the resultant charge of the entire atom is zero. In non-conductors of electricity, the electrons are very closely associated with the nucleus, and it is difficult to remove an electron from the atom. In the metals, which are conductors, a small proportion of the electrons appear to be free in the sense that they are able to pass easily from one atom to the next. But even so, the density of these free electrons in a metal is extremely large, being approximately 16,000 coulombs per cubic centimeter.

These free electrons in a metal are supposedly in constant motion and are continually colliding with one another and with the atoms of the metal, which are also in motion. Their motion is similar to that of the atoms of a gas in a confined space. As with the atoms of a gas, the velocities of the individual electrons at any instant differ widely, but their velocity as a whole gives an average velocity which is determined by the temperature of the metal.

216. Emission.—The surface of the metal is a boundary surface for the free electrons and is impervious to most of them. A force of repulsion is exerted at this surface on the electrons in contact with it, which turns back all those whose velocities fall below a certain critical value, while allowing those having velocities greater than this critical value to pass through. If the space outside the metal is evacuated, it will gradually become filled with electrons. These electrons collide among themselves, however, causing some of them to return to the metal through the surface. This process is accelerated by the *space-charge* effect of the electrons. That is, the electrons in the space outside the metal, all being negatively charged, mutually repel one another and drive some of the electrons back into the metal. A condition of equilibrium is reached when the number of electrons leaving the metal is equal to the number returning to it in any given time.

217. Critical Velocity.—The critical velocity which an electron must have in order to escape from a metal is of the order of magnitude of 10^8 cm. per second. It varies somewhat for the different metals. It is more usually expressed in terms of the difference of potential through which an electron must fall in order to acquire this velocity, because this difference of potential is the quantity actually measured. The energy of an electron carrying a charge e and having a mass m when it has fallen freely through a difference of potential V and acquired a final velocity v is

$$Ve = \tfrac{1}{2}mv^2. \tag{146}$$

The difference of potential corresponding to a velocity of 10^8 cm. per second is about 4 volts.

The emission of electrons from a metal is analogous to evaporation from a liquid. The surface tension of the liquid corresponds to the apparent repulsive force at the surface of the metal. The latent heat of evaporation corresponds to the work done by the electrons against the repulsive force at the surface.

At room temperature, the number of electrons emitted from a metal over any ordinary interval of time is very small, because the average velocity of the electrons within the metals is low, and only occasionally does one attain sufficient velocity to escape from the surface.

Raising the temperature of the metal increases the average velocity of the electrons and increases the emission, because more electrons obtain sufficient velocity to escape. The emission increases rapidly with increase in temperature and reaches astonishingly large values at white heat. Emission of electrons may also be produced by the action of ultra-violet light (photoelectric effect), X-rays, the various rays from radioactive substances, and by the impinging on the surface of high-speed electrons (secondary emission).

218. Richardson's Law.—Richardson, in 1901, showed that the emission per unit area is an exponential function of the temperature and is, also, a property of the material.

The current in amperes per square centimeter emitted from a body at temperature T in Kelvin degrees is given by the equation

$$i = a\sqrt{T}\,\epsilon^{-\frac{b}{T}} \tag{147}$$

where ϵ is the natural logarithmic base.

This relation is known as *Richardson's law* of emission. Values of the constants a and b are given in the following table:

Substance	a	b
Tungsten	23.6×10^6	5.25×10^4
Thorium	196×10^6	3.90×10^4
Barium oxide, Strontium oxide	15.0×10^4	2.0×10^4

Of the two constants, b is the more important at high temperatures, because it enters exponentially. The substances listed in the table are arranged in the order of their emissions at a high temperature, tungsten having the smallest emission.

The maximum temperature at which filaments may be operated depends on their melting points and their physical dimensions. The temperature is usually chosen to give a life of approximately 1,000 hr. This corresponds to current practice with incandescent lamps (see p. 506). The operating temperature for tungsten filaments 0.01 in. in diameter is approximately 2200° C.; for thoriated tungsten filaments, the temperature is about 1600° C.; and for platinum filaments coated with barium and strontium

oxides, is about 1300° C. At these temperatures, the emission is approximately the same for all these filaments.

219. Thermionic Efficiency.—In practice, the metal from which emission takes place is in the form of a long filament of small diameter which is heated by the passage of an electric current. The energy necessary to maintain this filament at a given high temperature is determined mainly by the heat radiation from it. The energy radiated varies as the fourth power of the absolute temperature, which is the Stefan-Boltzmann law of thermal radiation. *Thermionic efficiency* is defined as the ratio of the current emitted to the power input, both for the same surface area. For tungsten at 2200° C., this efficiency is approximately 0.01 amp. per watt, and for oxide-coated platinum at 1300° C., approximately 0.06 amp. per watt.

When a small fraction of thorium is alloyed with tungsten, the emission of electrons is greatly increased. A surface layer of thorium is apparently formed on the tungsten filament. This lowers the safe operating temperature to about 1600° C. and brings the thermionic efficiency up to about 0.05 amp. per watt. This filament is referred to as *thoriated tungsten.* Its thermionic efficiency is highly dependent upon the vacuum. Slight traces of gas greatly affect the surface layer of thorium and reduce the emission to that of pure tungsten. The chemicals, phosphorus or magnesium, used in completing the evacuation of the tube are deposited on its wall and are available for absorbing traces of gas formed during operation. When these deposits are completely oxidized, further formation of gas attacks the thorium surface layer and ends the life of the filament.

220. Space Charge.—It was shown, in Par. 216, that when a hot filament which emits electrons is placed in an evacuated chamber, the space becomes filled with electrons. The space is then called an *electron gas* or *space charge.* The density of the charge is not uniform but obviously is greatest next to the hot filament. Equilibrium is attained when in a given time as many electrons fall back into as are emitted by the filament.

221. Two-electrode Tube.—If a cold electrode, called the *anode* or *plate* (Fig. 400), is inserted in the evacuated chamber containing a heated filament, some electrons will reach it, and it will assume the potential of the space it occupies, which will be

slightly negative with respect to the filament. Let it be assumed for the present that the filament is heated in such a way that there is no fall of potential along it. If, now, the anode or plate is connected through a sensitive galvanometer back to the filament, a small current will flow. In the usual or conventional sense, its direction will be from the filament through the galvanometer to the plate (Fig. 400). Actually, the motion of the electrons, which are negative charges, constitutes the current. This motion is in the opposite direction or from the filament to the plate inside the tube.

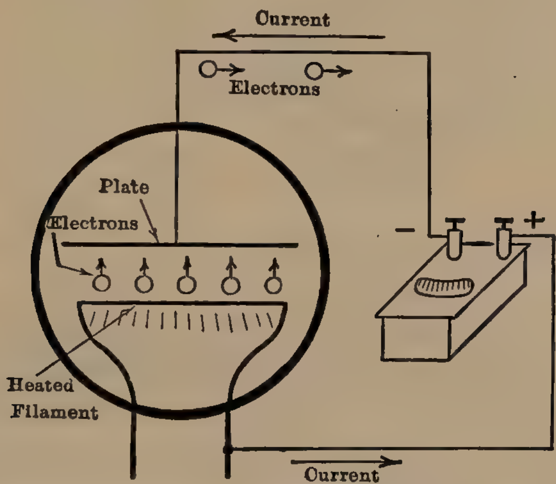


Fig. 400.—Emission of electrons from heated filament with no plate battery.

222. Child's Three-halves Power Law.—When a voltage is applied between plate and filament, making the plate positive with respect to the filament, the positive plate will attract the negative electrons, and a much larger current will flow than when the applied voltage is zero. This may be determined experimentally by connecting the tube in the manner shown in Fig. 401. The plate is made positive with respect to the filament by means of the battery B. The voltage E between the filament and plate may be varied to E', E'', etc., by the battery tap, as shown. A galvanometer or microammeter G is connected in circuit to measure the plate current. The manner in which the plate current varies with the temperature of the filament for different applied plate voltages is shown in Fig. 402. First apply E' volts between plate and filament. For low temperatures and the cor-

respondingly small emissions, the voltage E' is sufficiently large so that *all* the emitted electrons are attracted to the plate, and the current increases rapidly with the temperature along the curve

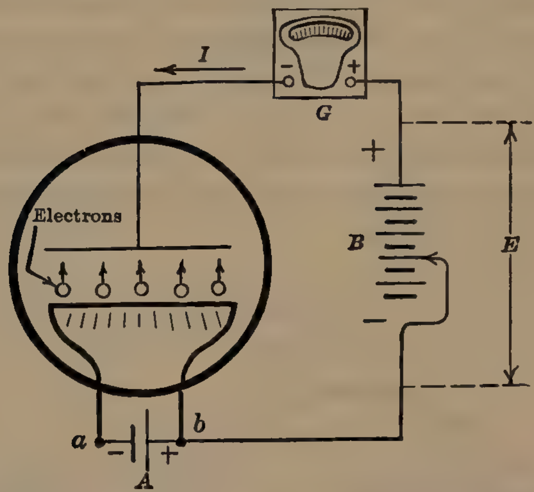


FIG. 401.—Emission of electrons from heated filament with plate battery.

oa. As the temperature and, hence, the emission increase, the density of the electron cloud between filament and plate also increases, and, hence, the reoulsive force on the electrons leaving

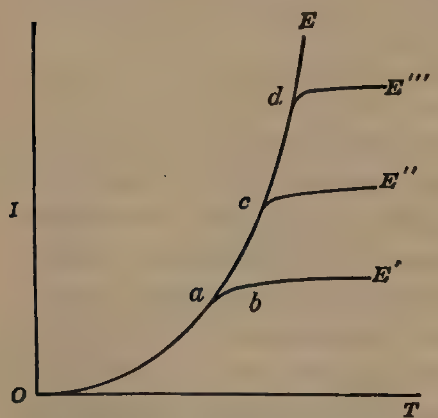


FIG. 402.—Plate current as function of temperature for different voltages.

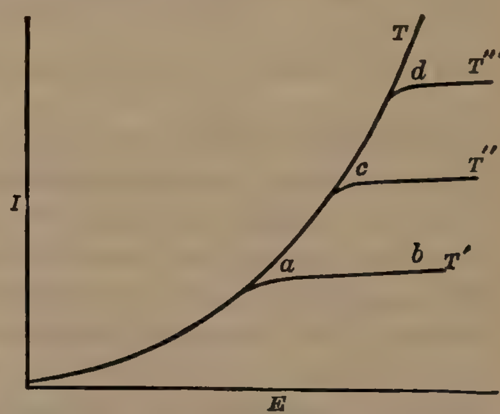


FIG. 403.—Plate current as function of voltage for different temperatures.

the filament, due to this negative space charge, increases. Ultimately, this repulsive force becomes greater than the attractive force due to the plate voltage E'. When this condition is

reached, the rate at which the electrons return to the filament is equal to the rate at which they are emitted by the filament, and the plate current becomes constant, as at b (Fig. 402). This is called *space-charge saturation.* If the plate voltage is raised from E' to E'' (Fig. 402), its attractive effect on the electrons increases. Obviously, it will require a greater space charge than before to drive the electrons back into the filament at the same rate as the filament is emitting them. Hence, *space-charge saturation* now occurs at c (Fig. 402), corresponding to a greater value of electron emission current.

The plate current may also be considered a function of plate voltage for different filament temperatures T', T'', etc., as shown in Fig. 403. At low plate voltages, the electron emission or current is limited by space charge and increases as the voltage to the three-halves power. This is called *Child's Three-halves Power Law.*

$$I = CE^{3/2} \tag{148}$$

The constant C is a function of the dimensions and relative spacing of filament and plate. At a given filament temperature, such as T', the voltage ultimately reaches such a value that its attractive force at the filament becomes larger than the mutual repulsive forces due to the space charge, and all of the electrons emitted are drawn to the plate. The current becomes constant and is, therefore, independent of the plate voltage. This condition for temperature T' is represented by the part ab of the curve. This is the true *filament saturation.* As the *filament temperature* is increased, the emission increases, and *filament saturation* occurs at higher values of plate current, as at c and d (Fig. 403).

From the foregoing, it is obvious that the plate current is a function of two quantities or parameters—plate voltage and filament temperature.

223. Edison Effect.—When the filament is heated by passing a current through it, there is a fall of potential along the filament because of the resistance drop. The various parts of the filament, therefore, will have different potentials with respect to the plate.

For example, in Fig. 401, the plate is at the potential of the plate battery B above the right end of the filament but is at a higher potential, equal to the sum of the voltages of the B and A

batteries, above the left end of the filament. If the B battery (Fig. 401) be removed, as in Fig. 400, and the plate connected to the positive end of the filament, its potential above the negative end of the filament will be the voltage of the A battery. Its potential above the various parts of the filament diminishes toward the positive end of the filament.

A current will flow in the plate circuit, larger than that discussed in Par. 221 but unequally distributed along the filament, being greatest at the negative end of the filament. If the plate be connected to the negative end of the filament, no current flows in the plate circuit, as the plate is negative to all other parts of the filament. Edison first noticed this effect in 1883, when he was developing the incandescent lamp. When he connected a plate P, sealed in the bulb near the filament F, through a sensitive galvanometer G to the negative terminal a of the filament F (Fig. 404), no current flowed through the circuit $PGaF$. If, however, he connected the plate to the positive terminal b of the filament F, a very appreciable current flowed through the circuit $PFbG$. At that time, nothing was known of electrons, and this current was referred to merely as the *Edison effect.* A study of the phenomenon was made by Fleming, in 1896, but its true significance was not understood until explained by J. J. Thomson and O. W. Richardson, in 1899 and 1901.

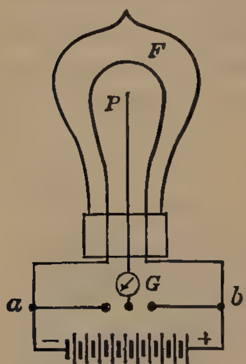


FIG. 404.—Edison effect.

224. Fleming Valve.—The most valuable property of the two-electrode tube is its characteristic of unilateral conduction. When the plate is positive with respect to the filament, it draws electrons from the filament, and a current flows from plate to filament which is roughly proportional to the three-halves power of their voltage difference, and the device has a finite although variable resistance. When the plate is negative with respect to the filament, the electrons are all driven back into the filament, and no current whatever flows. The resistance of the device becomes infinite. If an alternating voltage be applied to a two-electrode tube (Fig. 405 (a)), the resultant current is pulsating but unidirectional (Fig. 405 (b)). The negative loop is entirely

suppressed. The positive loop is somewhat distorted because of the variation of the resistance of the device with the current. The foregoing is called *half-wave* rectification. This rectifying action is identical in principle with that of the mercury-arc rectifier and the tungar rectifier described in Chap. XII. Fleming, in 1905, was the first to recognize this rectifying property of a two-electrode tube. He obtained a patent on its use as a detector of high-frequency oscillations, which become one of the fundamental patents in electron-tube development.

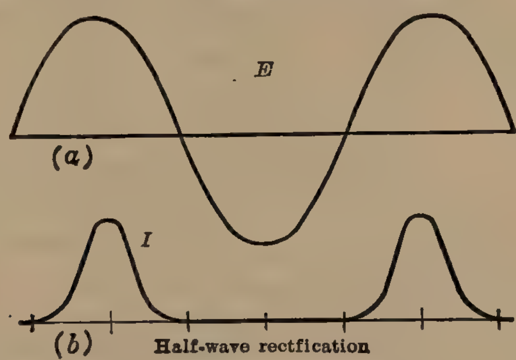


Fig. 405.—Half-wave rectification with Fleming valve.

225. Two-electrode Rectifier.—A single two-electrode tube, sometimes called a *kenotron*, can give only half-wave rectification. Both loops of voltage may be rectified by using two rectifiers, as shown in Fig. 406. The resulting current through the load is shown in Fig. 407. This is called *full-wave* rectification. If the load is of high resistance, the voltage impressed on it may be smoothed out, as shown by the full line in Fig. 408, by connecting a large condenser across the output circuit.

The filaments of rectifier tubes are usually operated from a low-voltage winding of the same alternating-current transformer which supplies the high voltage which is being rectified. In the tubes of small power the two units necessary for full-wave rectification are sealed into the same evacuated bulb. These full-wave rectifier tubes are largely used in the *B*-eliminators of alternating-current operated receiving sets. These units are provided with low-pass filters for smoothing out the voltage ripple. These filters consist usually of three large condensers in shunt to the load, and between each pair of condensers is connected a large

choke coil in the high-voltage lead. The cut-off frequency of this filter is placed at about 40 cycles per second.

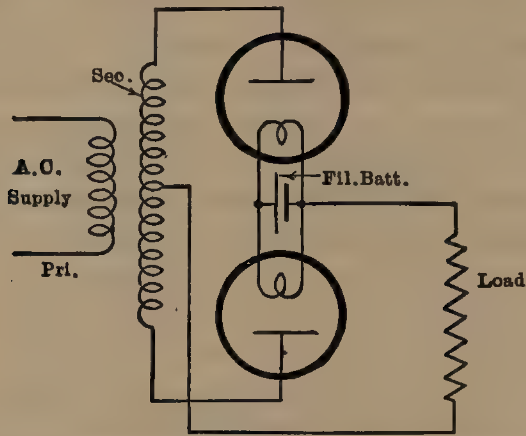


FIG. 406.—Connections which give full-wave rectification.

High-power rectifiers are used for the production of high-voltage direct current for high-voltage (2,000 to 15,000 volts),

FIG. 407.—Full-wave rectification with two rectifiers.

FIG. 408.—Full-wave voltage rectification with condenser across load.

three-electrode oscillators, X-ray tubes, and for obtaining high voltages for insulation-testing purposes.

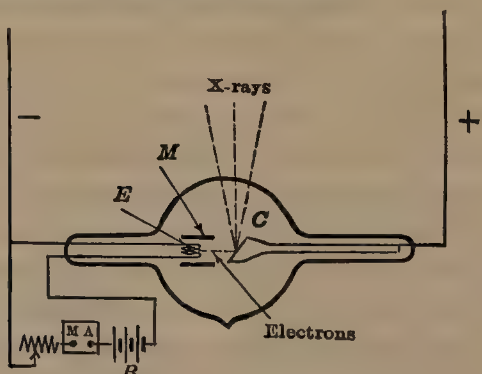


FIG. 409.—Coolidge X-ray tube.

226. X-ray Tubes.—The Coolidge hot-cathode X-ray tube (Fig. 409) is a special two-electrode tube, designed for the produc-

tion of X-rays. The filament E, heated by the low-voltage battery B, is concentrated and electrostatically shielded by the molybdenum tube M, so that the electrons are emitted in a fine pencil. The plate or anticathode C is a massive block of tungsten which serves as a target for the electrons. X-rays are given off by the target at the point where the electrons strike. The targets are frequently water cooled to allow the use of very high voltages and large currents, as, for example, 100 kv. and 0.5 amp.

It is interesting to notice that, on account of the high voltages used, X-ray tubes operate under the conditions for filament saturation. All other rectifier tubes operate under the conditions for space-charge saturation (Par. 222).

As is well known, X-rays have the property of penetrating substances which are impervious to ordinary light. They are used to a very large extent in medical work in the study of fractured bones, the roots of teeth; in industry, to locate flaws in castings; in chemistry, to determine the crystalline structure of substances; etc.

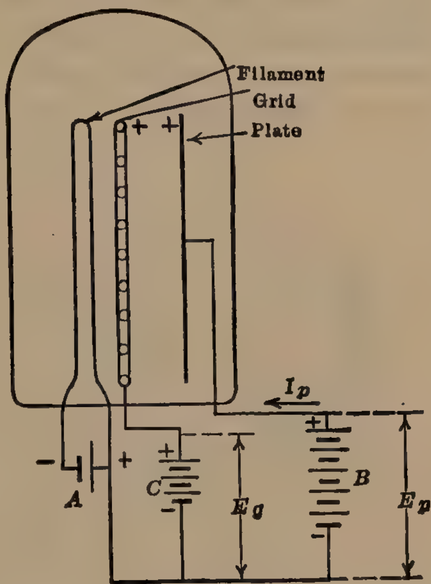


Fig. 410.—Three-electrode vacuum tube, grid positive.

227. Three-electrode Tube.—The addition of a third electrode to control the plate current was made by De Forest, in 1907, in a tube which he called the *audion*. His patent, issued in that year, became as fundamental as that of Fleming in electron-tube development. He placed a lattice or *grid*, as it is now called, between the filament and plate (Fig. 410). The electrons, in passing from filament to plate, must now pass through this grid. If the grid is positive with respect to the filament, it will assist the plate in drawing electrons from the filament and, hence, will increase the plate current. If the grid is negative with respect to the filament, it will act in the same manner as the space charge (Par. 220) and will repel negative charges or electrons

toward the filament and, hence, will decrease the plate current. The plate current is now, therefore, a function not only of the filament temperature T and the plate voltage E_p but also of the grid voltage E_g. As the grid is much nearer the filament than the plate, a given change in the grid voltage E_g will have a much greater effect on the plate current than a given change in the plate voltage E_p. Hence, a very small amount of energy applied to the grid will control a very much larger amount of energy passing from filament to plate. It is this characteristic of the tube which makes it so useful as an amplifier and as an oscillator.

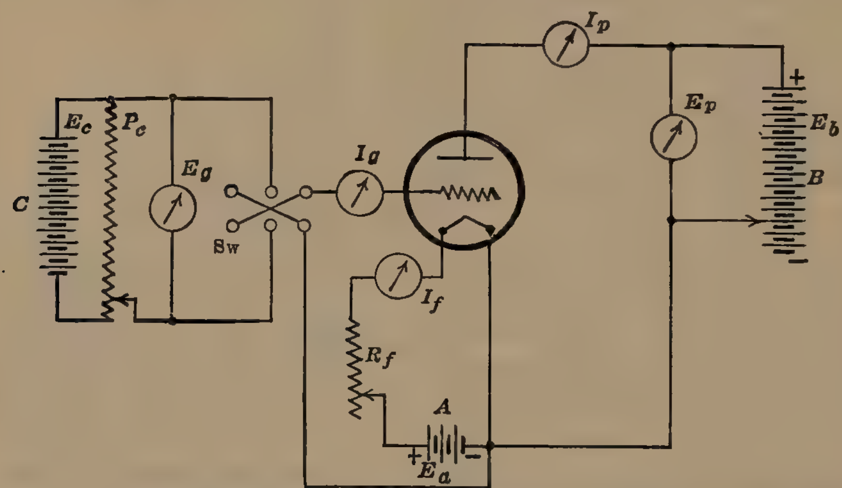


FIG. 411.—Connections for determining static characteristics of three-electrode tube.

228. Static Characteristics of Three-electrode Tube.—Figure 411 shows the connections which may be used to obtain the static or steady-current characteristics of the three-electrode tube. The voltage E_p applied to the plate by the B or plate battery depends on the type of tube being tested (see tables, pp. 477 and 484).

With too high plate voltage, the elements of the tube are overheated, and gas evolves from them, which ionizes and increases the plate current still further. For small overloads, the life of the filament is shortened in proportion to the amount of gas evolved. On large overloads, the glass of the tube softens or cracks, ruining the whole structure.

The voltage applied to the grid may be readily obtained from dry cells in series (C), a high-resistance drop wire P_c being used to vary the voltage, and a reversing switch *sw* to give the desired polarity. For receiving tubes, the range of grid voltage is ordinarily ± 10 volts. The plate current is of the order of 2 to 25 milliamp. The grid itself takes current of the order of 2 milliamp. Hence, sensitive ammeters are necessary.

If the filament current and the plate voltage are held constant at some value as a plate voltage E_1 (Fig. 412) and the grid voltage is varied, a curve *abc* is obtained. When the grid voltage is zero, current *ob* flows to the plate, since the plate itself attracts

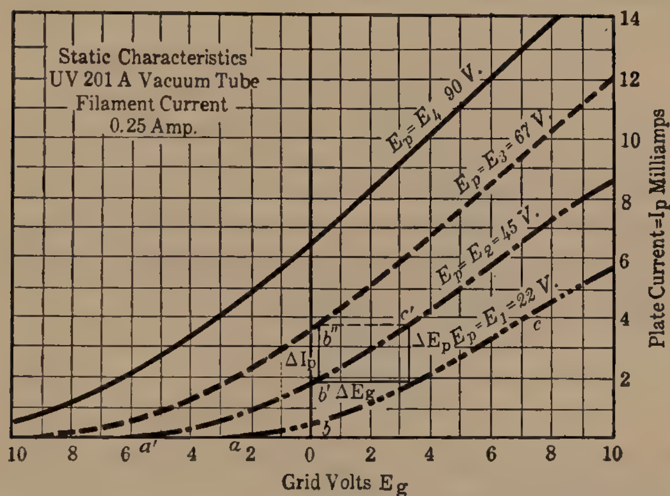


FIG. 412.—Static characteristics of U. V. 201 A vacuum tube.

electrons from the filament. In order to stop the flow of plate current, the grid potential must be negative and of a sufficient value *oa* to neutralize the effect of the plate voltage. The rate of increase of plate current decreases as the grid voltage is increased, due to filament saturation. If the plate voltage is increased to E_2, then, for a given grid voltage, more current will flow in the plate circuit, and curve $a'b'c'$ is obtained. Curves for still greater plate voltages E_3 and E_4 are also given.

Assume (Fig. 412) that when $E_p = E_2$, the grid voltage E_g increases by an amount ΔE_g. This increases the plate current by an amount $b'b''$ or ΔI_p. If the grid voltage be held constant as at b', the plate voltage must be increased ΔE_p or from E_2

to E_3 in order to increase the plate current by ΔI_p. The ratio of $\Delta E_p/\Delta E_g$ for constant plate current is the *amplification constant* and is denoted by μ.

$$\mu = \frac{\Delta E_p}{\Delta E_g} \Bigg| \; I_p \text{ constant.} \tag{149}$$

Since the plate-voltage scale is much larger than the grid-voltage scale, the amplification factor is larger than the ratio of the actual lengths of the lines $b''b'$ and $b''c'$. For receiving tubes, μ ranges from 3 to 20, with 8 as the most common value (see table on opposite page).

If a change of plate voltage ΔE_p produces a change of plate current ΔI_p, the factor

$$r_p = \frac{\Delta E_p}{\Delta I_p} \Bigg| \; E_g \text{ constant} \tag{150}$$

is called the *plate resistance*.

With the third combination of these three variables, the ratio of the change of plate current ΔI_p to the change of grid voltage ΔE_g is called the *mutual conductance*

$$\sigma = \frac{\Delta I_p}{\Delta E_g} \Bigg| \; E_p \text{ constant.} \tag{151}$$

On the opposite page are given data for the more common types of small receiving tubes.

CONSTANTS OF RECEIVING TUBES

Type	Filament		Plate		Grid	μ	r_p
	Volt	Ampere	Volt	m.-amp.	Volt		K-ohm
WD 11.......	1.1	0.25	22– 90	0.5–3	1– 5	6	15–20
UX 199......	3.3	0.06	22– 90	0.5–3	1– 6	6	15–25
UX 120......	3.3	0.13	45–135	2–7	4–22	3.3	7– 9
UX 200A....	5	0.25	22– 45	1– 2	0	20	30
UX 201A....	5	0.25	45–180	1– 3	1–13	8.5	10–20
UX 240......	5	0.25	135–180	0.2	1– 3	30	150
UX 226......	1.5	1.05	90–180	3– 8	6–13	8.2	7– 9
UY 227......	2.5	1.75	45– 90	2– 7	0	8	8–10
UX 112......	5	0.50	90–160	2– 8	6–11	8	5–9
UX 171......	5	0.50	90–180	11–20	16–40	3	2–3
UX 210......	7.5	1.25	250–425	12–20	18–35	7.5	5–6
UX 250......	7.5	1.25	250–450	28–55	45–84	3.8	2
UX 222......	3.3	0.13					
Screen grid...	...	....	135	1.5	1.5	300	850
Space charge grid........	...	....	180	0.3	1.5	80	150

The filaments of the tubes WD 11, UX 226, 112, 250 are oxide-coated ribbon. All others are thoriated tungsten wire, with the exception of UY 227, which has an oxide-coated cylinder heated by an inner insulated heater. The filaments of the first group, WD 11, UX 199, 120, are for dry-battery operation; of the fourth group, UX 226, UY 227, for alternating-current operation; of the fifth group, UX 112, 171, 210, 250, for either storage battery or alternating-current operation; of the other groups, for storage-battery operation.

Tubes UX 200A and UY 227 are detector tubes, the former having traces of calcium vapor. Tubes WD 11, UX 199, 201A are "all-purpose" tubes. Tube UX 240 is a high-amplification tube for use with resistance coupling. Tubes UX 120, 112, 171, 210, 250 have larger power output than the all-purpose tubes for supplying energy to a loud speaker. UX 222 is a four-electrode tube with two grids.

229. Amplification.—Since the amplification constant μ of a three-electrode tube is considerably greater than unity, the tube may be used to amplify small voltages. Figure 413 shows a

tube having a steady potential E_p connected in the plate circuit and another steady potential E_g in the grid circuit. E_g gives the grid a negative potential with respect to the filament. These voltages give a definite operating point P on the plate-current—

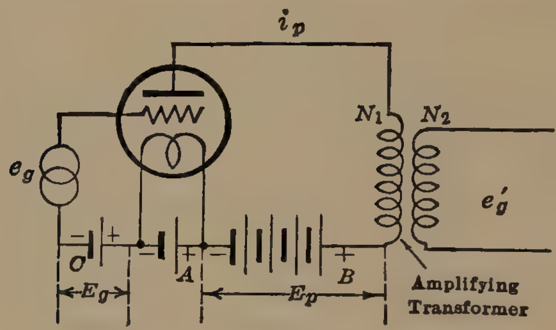


FIG. 413.—Vacuum tube as amplifier.

grid-voltage characteristic (Fig. 414). Now let a small alternating voltage e_g be introduced into the grid circuit. This variation of grid voltage must produce a variation of plate current. The actual plate current may be determined by projecting the e_g

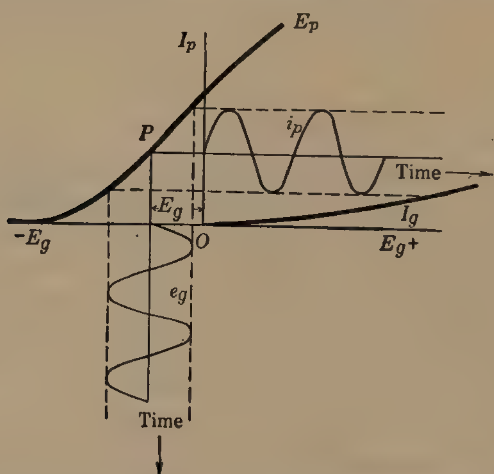


FIG. 414.—Amplification characteristics of vacuum tube.

curve vertically on the I_p–E_g curve. Thus, an alternating current i_p, superposed on the steady plate current, is produced in the plate circuit. This current may be considered as being produced by a voltage μe_g acting through the constant plate resistance r_p and the effective impedance of the transformer.

The steady voltages and currents in the plate and grid circuits have no effect in determining the alternating voltages and currents, except in so far as they define the portions of the static characteristics over which the tube operates. For distortionless reproduction, the voltages E_g and E_p should be so chosen that the alternating e.m.f. *is operating on the straight part of the characteristic* (Fig. 414) and also where E_g is negative, so that the grid current I_g is zero.

The total plate current flows through the primary N_1 of the amplifying transformer (Fig. 413). The steady component of this current has no effect on the secondary turns N_2, but the alternating component i_p causes an e.m.f. e_g' to be induced in the secondary turns N_2. If the turns of the transformer are properly chosen, e_g' may be from twenty to fifty times the e.m.f. e_g, thus effecting *amplification*. The e.m.f. e_g' may be impressed on the grid of a second tube, similar to this first tube, and a second stage of amplification obtained. Thus, several stages of amplification may be obtained, the number being limited by the tendency of the amplifier to feed back through capacitance to the grids in the first stages, causing the amplifier to oscillate at a frequency determined by the inductances and capacitances in the various circuits.

If audiofrequencies, 100 to 5,000 cycles per second, are amplified, audiofrequency (a.-f.) transformers, having laminated iron cores, are used. For radiofrequencies, of the order of 1,000 kilocycles per second, the use of iron is not permissible because of its high losses. Better results are obtained at radio frequencies, if a variable condenser is shunted across the transformer secondary and the entire circuit is tuned.

230. Regeneration.—It was shown, in Par. 229, that in the amplifier any change of the voltage applied to the grid causes not only a change of current in the plate circuit but also a change of energy in the plate circuit. This change of energy is several times greater than the change of energy applied to the grid. If the proper phase relation between the plate current and the voltage in the grid circuit is obtained, it is possible to feed a portion of this energy of the plate circuit *back* into the grid circuit and, hence, reinforce the effect of the grid. The grid, in turn, increases the plate current, which again reacts on the grid. This feed back of energy is called *regeneration*. The connections

for one method of regeneration are shown in Fig. 415. The grid is polarized negatively with a grid battery to a potential of E_g volts. An inductance L_g and a variable condenser C_g are connected in parallel between the grid and the negative terminal of the grid battery. A coil L_p, having mutual inductance M with L_g, and connected in the plate circuit, serves to couple inductively the plate and grid circuits.

An alternating voltage e_g introduced into the tuned grid circuit produces a plate current i_p in such a phase that, if the polarity of L_p is correct, the voltage which i_p induces in the tuned grid circuit is in phase with the original voltage e_g. The same effect may be produced by connecting a similar tuned circuit, consisting of inductance and capacitance in parallel with each other, in the plate circuit and for coupling between the plate and grid circuits depending upon capacitance from grid to plate of the tube itself. If the grid circuit is not tuned, as in an audioamplifier, the voltage induced back into the grid circuit is in quadrature with the impressed voltage e_g, and there is no regeneration.

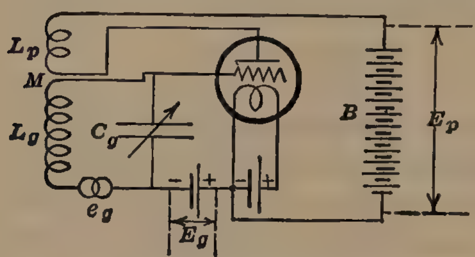


FIG. 415.—Connections for regeneration with tuned grid circuit.

The effect of regeneration is to introduce a *negative* resistance into the tuned circuit. That is, this tuned circuit now becomes a generator of energy, this energy being obtained from the plate or B battery. If the mutual inductance M between L_p and L_g be made sufficiently large, the total resistance of the tuned circuit may be reduced almost to zero, where the limit of regeneration is reached. At this limit of regeneration, the plate current has been found to be constant and independent of the magnitude of the impressed voltage e_g, as well as that of the initial resistance of the tuned circuit; its value is determined only by the characteristics of the tube. In practice, this maximum theoretical limit cannot be reached, since small disturbances, such as slight mechanical vibrations of the inductances, condensers, and of the tube itself, may cause the total resistance of the plate circuit to become negative momentarily and cause the tube to begin oscillating (see the next paragraph).

The maximum attainable plate current is then no longer constant but varies directly with the magnitude of the impressed voltage, if the voltage is small, and inversely with the resistance of the tuned circuit, if the voltage is large.

OSCILLATORS

231. Oscillation.—When the mutual inductance between the plate and grid circuit is increased to such a value that the resistance of the tuned circuit becomes zero or negative, sustained oscillations, independent of the impressed voltage e_g, are set up in the system. In fact, the impressed voltage e_g may be removed entirely without affecting the oscillations. These sustained oscillations will start even in the absence of any impressed voltage. For example, slight mechanical disturbances to parts of the system (see the previous paragraph), small electrical disturbances such as occur when the plate circuit is closed, etc., are sufficient to start the tube into oscillation. Under these conditions, the tube is said to be an *oscillator*. It behaves like an alternating-current generator, converting the energy of the plate battery into alternating-current energy in the tuned circuit. The frequency of the alternating current generated is practically equal to the natural frequency of the tuned circuit

$$f = \frac{1}{2\pi\sqrt{L \cdot C}} \tag{152}$$

where L and C are the inductance and capacitance of the tuned circuit (see Eq. (26), p. 41).

The type of oscillating circuit in which the tuned circuit is connected to the grid (Fig. 415) is that which is used in most receivers in which oscillating tubes are used, such as continuous-wave, carrier-frequency, and superheterodyne receivers.

As far as sustained oscillations are concerned, the tuned circuit may equally well be placed in the plate circuit, as shown in Fig. 416 (a), the tuned circuit being inductively coupled to the grid by the mutual inductance M. This type of circuit is used in most power oscillators, where the tube acts as an alternating-current generator. The grid polarizing battery C may also be replaced by a grid leak and grid condenser in parallel (Par. 237). The resistance R is not a resistance introduced purposely into the

oscillating circuit but is equal to the total inevitable resistance in the tuned circuit and the equivalent resistance of the load on the oscillator, such as the equivalent antenna resistance, etc.

For maximum tube output, there is a definite relation between L_p and C_p of the tuned circuit, this relation depending on the values of R and of the plate-circuit resistance. With sinusoidal current, the maximum output occurs when the maximum instantaneous voltage across the oscillating circuit is equal to the plate voltage E_p.

Expressed in terms of the constants of the tube and circuit,

$$\frac{L_p}{C_p} = Rr_p \tag{153}$$

where r_p is the average plate resistance of the oscillating tube.

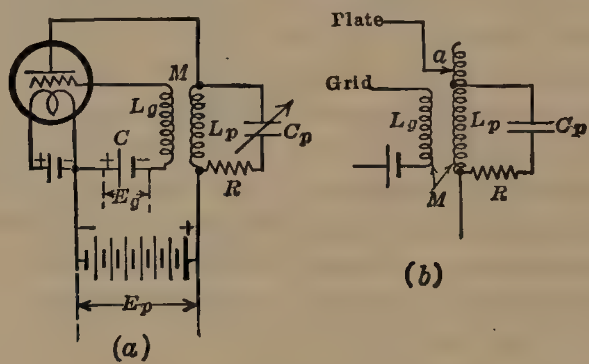


FIG. 416.—Vacuum-tube power oscillator.

It may happen that unavoidable conditions imposed by L_p, C_p, and the load prevent the oscillator from delivering more than a small portion of its total power capacity. This limitation may be overcome by connecting the plate tap to some point a (Fig. 416 (b)) on the winding of L_p, different from the point at which the condenser C_p is connected to the coil. It may be inside or outside the condenser tap.

The tuned circuit L_pC_p (Fig. 416 (a)) is at the plate potential E_p above ground. In order to bring it to ground potential for direct connection to an antenna and for safe operation generally, the parallel feed connection of Fig. 417 is used. C_b is a large blocking condenser which prevents the tuned circuit from short circuiting the plate battery E_p. L_b is a radiofrequency choke coil, which keeps radiofrequency current out of the plate battery.

The efficiencies of such oscillators when they deliver maximum output are between 50 and 80 per cent., dependent on operating conditions.

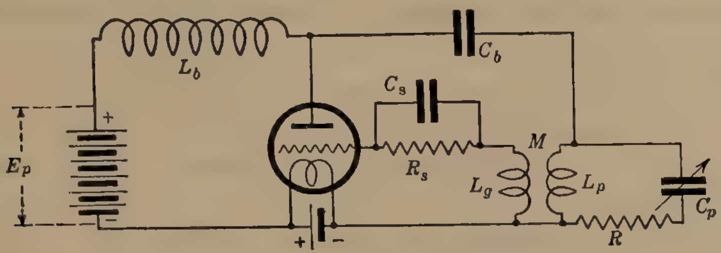


FIG. 417.—Parallel-feed oscillator.

232. Power Tubes.—All the tubes (see table p. 477), with the possible exception of U.X. 200A, will function as oscillators. Tubes of greater power are shown in Fig. 418.

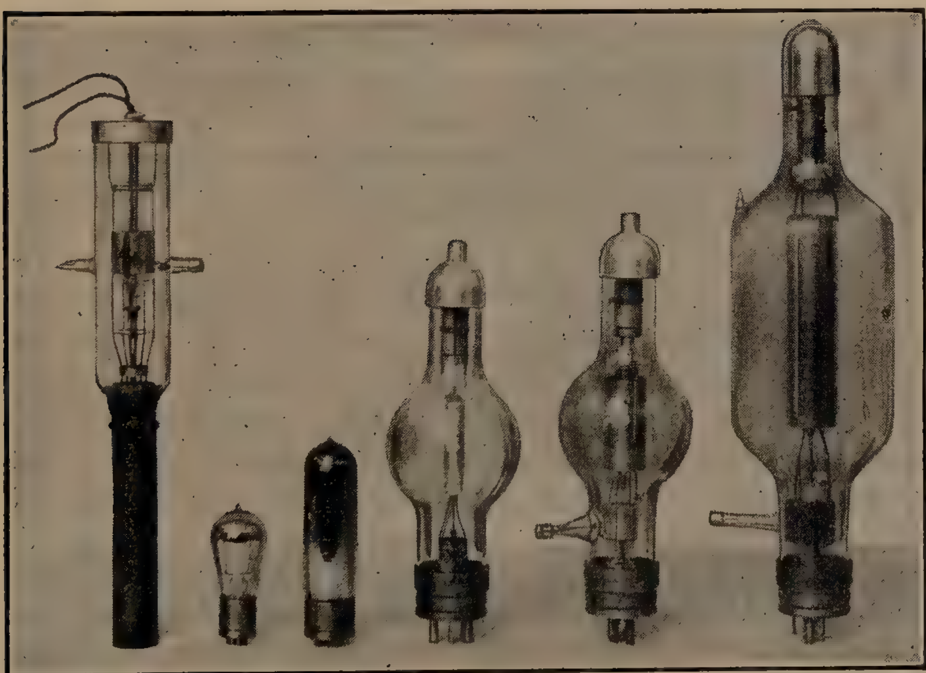


FIG. 418.—Pliotron power tubes, models (left to right 207, 202, 203, 204, 206, 208). (*Manufactured by General Electric Company.*)

As with most electrical machinery, the output of an oscillating tube is limited by the heating of its plate, due to its energy losses. Since this energy must be radiated out through the glass walls

of the evacuated tube, the limit of size of tubes of this type is between one and five kw. For large powers (UX 207 of Fig. 418), the plate is made of copper, sealed directly to the glass, and is water cooled. Below are given data for the more common types of power tubes.

CONSTANTS OF POWER TUBES

Type	Filament		Plate		μ	Output
	Volts	Amperes	Volts	Milliamp.		Watts
UX 210	7.5	1.25	350	70	7.5	7.5
UX 250	7.5	1.25	450	70	3.8	10
UV 203A	10	3.25	1,000	120	25	50
UV 204A	11	3.85	2,000	275	24	250
UV 851	11	15.5	2,000	1,550	20	1,000
UV 208	22	24.5	15,000	450	300	5,000
UV 207	22	52.0	15,000	1,800	40	20,000

The filament of the tube UX 250 is an oxide-coated ribbon; the filament of the tubes UV 208, 207 is pure tungsten wire; all others are thoriated tungsten wire. Tube UV 207 has a water-cooled plate.

MODULATION

233. Modulation.—Electrical communication over wires in its simplest form employs alternating currents of audiofrequencies only, either singly or in combination. These currents may be amplified (see Par. 229), but only a single communication can be conducted over a single effective circuit at one time. In order to open new channels of communication over any given effective wire circuit, *carrier* wire telephony and telegraphy are employed. Alternating currents having superaudio frequencies (3,000 to 33,000 cycles per second[1]) are used as carriers or vehicles for the audiofrequency currents. In radiotelephony and telegraphy, electromagnetic waves are used as carriers having frequencies of from 10,000 to 30,000,000 cycles per second. Of

[1] The ear may be sensitive to frequencies as high as 15,000 cycles per second, but most conversational frequencies do not exceed 2,500 cycles per second.

themselves, these superaudio frequencies could not transmit signals, being, for the most part, beyond the range of audibility of the ear, and they can transmit only very small amounts of power. By the superposition of audiofrequency currents on these carrier currents, however, it is possible to transmit several messages simultaneously over a given effective communication circuit. The superposition of an audiofrequency current on a carrier-frequency current is called *modulation.*

With the usual type of modulation, the original constant amplitude of the carrier-frequency alternating current is made to vary according to the amplitude of the superposed audio-frequency current. For example, in Fig. 419 (a) is shown the constant-frequency carrier current having a constant amplitude A and a frequency of a cycles per second. An audiofrequency current, having an amplitude B and a frequency of b cycles per second, is superposed on the carrier-frequency. The resulting current is shown in Fig. 419 (b). The superposed audiofrequency current is the *envelope* of both the positive and negative halves of this modulated carrier current.

The mathematical expression for this modulated current is

$$i = A(1 + m \sin 2\pi\, bt) \sin 2\pi at \tag{154}$$

where m is the degree of modulation, being the ratio of amplitudes of the audio and the radio currents

$$m = \frac{B}{A}. \tag{155}$$

It may be shown that the modulated carrier current (Fig. 419 (b)) actually consists of three sinusoidal currents, one having the frequency a, the frequency of the original carrier current; another having a frequency $(a - b)$, the *difference* between the carrier frequency and the audiofrequency; and a third having a frequency $(a + b)$, the *sum* of the carrier frequency and the audiofrequency. The currents having frequencies of $(a + b)$ and $(a - b)$ are called *side frequencies.* This is illustrated in Fig. 420, which shows a portion of the frequency spectrum, the abscissas being frequencies, and the ordinates being the amplitudes of the currents.

The amplitudes of these three currents are directly related to the amplitudes of the carrier- and audiofrequency currents,

as shown in Fig. 420. The degree of modulation is the ratio of the sum of the amplitudes of the two side frequencies to the amplitude of the carrier frequency. With the more complex

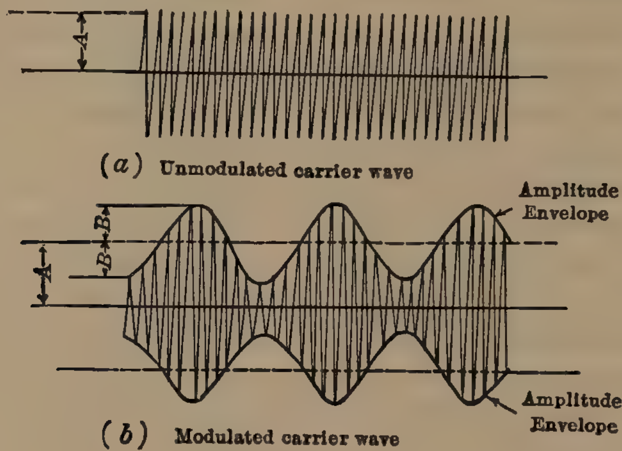


Fig. 419.—Modulated carrier current.

audiofrequency currents, such as would be produced by the voice, the resultant modulated current is quite complex, and the side frequencies widen out into *side bands*. The carrier frequency, however, is always sufficiently high so that the side-band

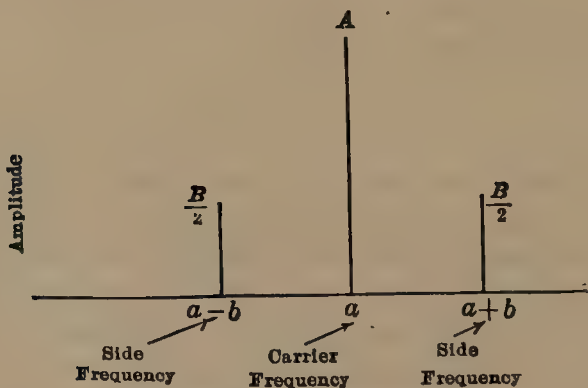


Fig. 420.—Frequency spectrum.

frequencies are near it in the frequency spectrum, and all are transmitted essentially as a single frequency.

234. Plate-circuit Modulation.—One method of modulation is to introduce into the plate circuit of a tube, oscillating at the carrier frequency, an additional voltage of audiofrequency,

whose peak value is somewhat less than the steady plate voltage E_p. This causes the output current I in the tuned circuit to have an amplitude envelope (Fig. 419 (*b*)) proportional to this audiofrequency voltage. The connections for this method of modulation are shown in Fig. 421. The tube oscillates at the carrier frequency due to the tuned circuit L_p and C_p, which is inductively coupled to the grid circuit. An audiofrequency transformer is introduced into the plate circuit at *b*. The primary current of this transformer is shown as coming from a microphone circuit consisting of a battery B in series with a telephone transmitter or microphone T. Ordinarily, there is not sufficient power in the microphone circuit to give sufficient modulation; hence, an amplifier between the microphone circuit and the audiofrequency transformer is necessary (see Fig. 413, p. 478). The secondary of the transformer introduces the modulation e.m.f. into the plate circuit. As the carrier- or radio-frequency current is unable to flow through the high inductance of the transformer secondary, a bypass condenser C' is necessary. If it were desired to broadcast with this circuit, an antenna would be inductively coupled to L_p, one end of the coupling winding being grounded, and the other connected to the antenna.

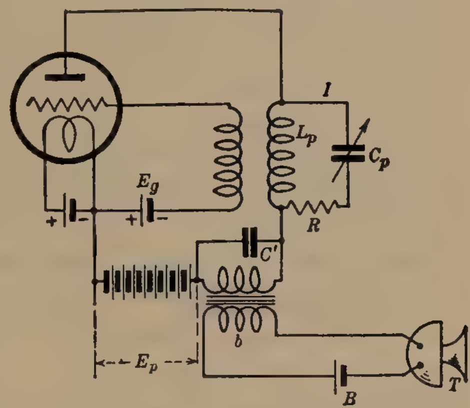


FIG. 421.—Connections for plate-circuit modulation.

DETECTION

A modulated high-frequency current, such as is shown in Fig. 419 (*b*), can have no effect on any ordinary sound-producing device, since such a device is unable to respond to such high frequencies. Neither can this high-frequency current produce any effect on the human ear, because its frequency is far beyond audible frequencies. It is, therefore, necessary to demodulate such currents in order that the receiving devices may be actuated by audiofrequency currents similar to those used for modulating. This process of demodulation is called *detection*.

235. Rectification with Two-electrode Tube.—Detection may be accomplished by any rectifying tube, such, for example, as the two-electrode tube (Fig. 422). The tube will eliminate the negative loops (Fig. 419 (*b*), p. 486), leaving a pulsating, unidirectional current (Fig. 423) made up of a unidirectional current,

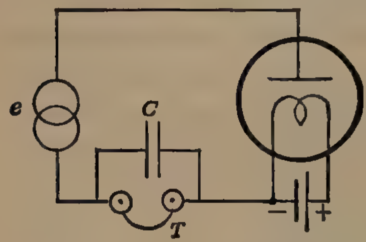


FIG. 422.—Two-electrode tube used as detector.

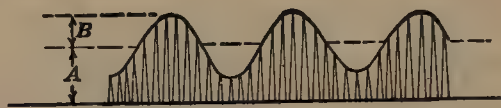


FIG. 423.—Rectified carrier wave.

an audiofrequency current, and a radiofrequency current. The unidirectional current and the audiofrequency current will flow through the telephones T, and the latter will reproduce in sound the initial audiofrequency current. The high-frequency component will be bypassed through the condenser C.

Although the two-electrode tube is a perfect rectifier, if used in this manner as a detector, it is very insensitive because of its excessively high resistance when operated at low voltage.

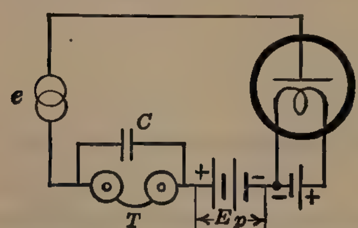


FIG. 424.—Two-electrode tube with polarizing voltage used as detector.

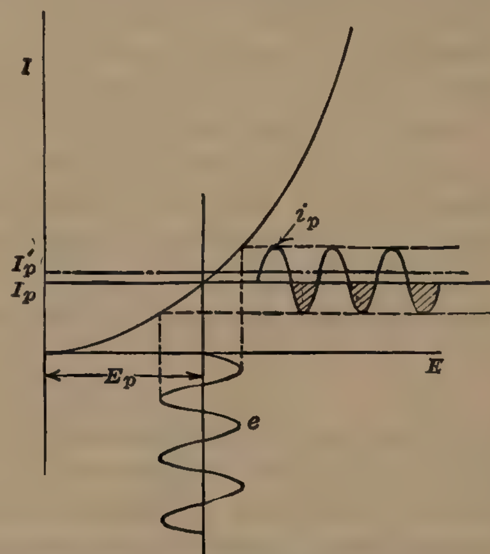


FIG. 425.—Detection with polarized two-electrode tube.

This may be seen in Fig. 425, where the current I for small values of voltage E is extremely small. This difficulty is, in part, overcome by inserting a positive polarizing voltage in series with the

tube (Fig. 424). Thus, in Fig. 425, the steady polarizing voltage E_p produces a steady current I_p in the tube circuit. Hence, an alternating e.m.f. e impressed on the tube is no longer perfectly rectified but produces an alternating current i_p. Owing to the curvature of the characteristic, this current i_p is dissymmetrical, the positive current being larger than the negative current. The negative current is shown shaded. Hence, the average current is increased from I_p to I_p', and thus the existence of the impressed e.m.f. is detected. The *change* in current, $I_p' - I_p$, is greater than the current which would flow for zero polarizing voltage and has its maximum value when the polarizing voltage corresponds to the point of greatest curvature of the characteristic. When the impressed voltage is modulated, this change in plate current will follow the variations of the modulating current.

236. Detection with Three-electrode Tube with Polarized Grid.—The three-electrode tube will detect in a manner very similar to the two-electrode tube with polarizing voltage, rectification depending on operating the tube at a point of curvature on its plate-current—grid-voltage characteristic.

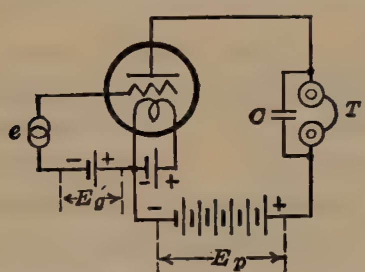


FIG. 426.—Three-electrode tube with polarized grid used as detector.

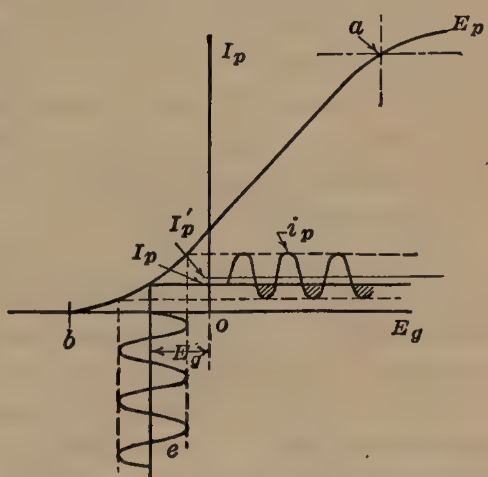


FIG. 427.—Detection with polarized three- electrode tube.

The connections for operating a tube as detector are shown in Fig. 426. The grid is polarized negatively with a voltage E_g by the grid or C battery to such a value as to cause the tube to operate on a point of curvature of the $I_p - E_g$ characteristic (Fig. 427). As with the two-electrode tube, a sinusoidal e.m.f. e impressed on the grid produces an alternating current i_p in the plate circuit, the axis of i_p being I_p. Owing to the curvature of the charac-

teristic, the negative portions of i_p, shown shaded, are less in magnitude than the positive portions, and the average current increases from I_p to I_p'. When the impressed voltage e is modulated, this change in plate current, $I_p' - I_p$, will follow the variations of the modulating current. The radiofrequency plate current I_p' is bypassed around the telephones through the condenser C (Fig. 426). For maximum sensitivity, the polarizing voltage E_g should be such that detection occurs at the point of maximum curvature of the characteristic.

Since the characteristic for positive values of E_g also has a curvature due to filament saturation, detection in a similar manner may occur at point a. At this point, the positive rather than the negative portion of the alternating component of plate current will be reduced in magnitude, and the average plate current will *decrease*. Detection with the grid polarized positively is not so efficient ordinarily as with the grid polarized negatively, since, when positive, the grid takes current. In detection, the e.m.f. e impressed on the grid is usually obtained through a transformer. Losses must inevitably occur in such transformers, reducing the sensitivity of detection. The detecting action in a three-electrode tube is greater than that of the two-electrode tube because of its lower effective plate resistance combined with its amplifying action.

Tubes similar to the UV-200, which contain slight traces of gas, are very sensitive detectors, because the presence of gas tends to neutralize the space charge and causes the $I_p - E_g$ characteristic to be unusually steep and so to have a large curvature at its lower bend. Since this action is due to the ionization of the residual gas, it is quite critical as regards the values of plate and grid voltages.

237. Detection with Three-electrode Tube with Grid Resistance.—The three-electrode tube also detects in a manner which is quite different from the foregoing and, in moden tubes, far more sensitive. The connections are shown in Fig. 428. A high resistance R_s of from one to five megohms is connected in series with and adjacent to the grid. This resistance is shunted by a small condenser C_s whose capacitance is between 50 and 200 $\mu\mu$f. The grid is polarized positively by the voltage E_g, so that a current I_g flows in the grid circuit, as shown in Fig. 429. This

current flowing through the high resistance R_s produces in it a voltage drop I_sR_s, so that the effective polarization of the grid is $E_g - I_gR_s$. The corresponding plate current is I_p. An alternating voltage e in the grid circuit will produce an alternating

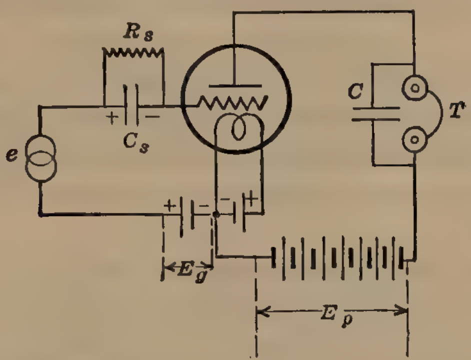


FIG. 428.—Three-electrode tube with grid resistance used as detector.

current i_g in the grid circuit, whose negative portions, shown shaded, are less in magnitude than its positive portions. Hence, the average grid current is increased from I_g to I_g'. This

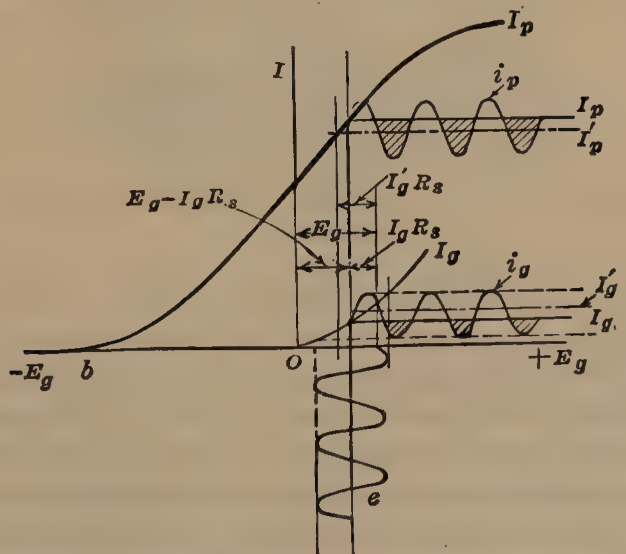


FIG. 429.—Detecting action with grid resistance.

decreases the polarization of the grid from $E_g - I_gR_s$ to $E_g - I_g'R_s$. The alternating component i_g is bypassed through the condenser C_s. The average plate current is decreased from I_p to I_p' with a superposed alternating current i_p. With reference to

I_p as an axis, the positive portions of i_p are less in magnitude than the negative portions shown shaded. When the impressed voltage e is modulated, this change in plate current, $I_p - I_p'$, will follow the variations of the modulating current. The radio-frequency plate current i_p is bypassed through the condenser C (Fig. 428).

The large curvature of the grid-current characteristic, the large slope of the plate-current characteristic, and the fact that the high resistance R_s may be made very large, all combine to make this type of detection the most sensitive of all the methods so far discussed.

238. Detection and Regeneration.—The two foregoing types of detection may be operated with a tuned circuit combined with

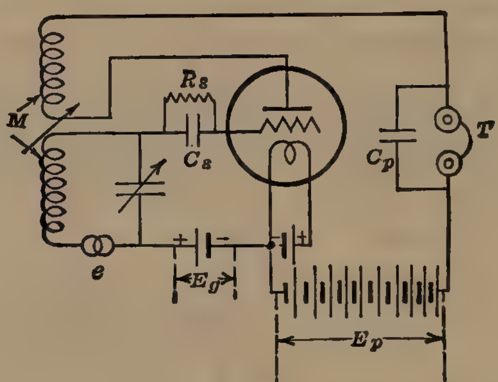


FIG. 430.—Three-electrode tube with grid resistance and regeneration used as detector.

regeneration. A very efficient circuit of this character, having a tuned grid circuit, grid resistance, and grid condenser, is shown in Fig. 430. The incoming signal e is detected, and a portion of the resulting energy of the plate circuit is fed back into the grid circuit through the coupling M. The condenser C_p shunts the high-frequency currents around the telephone receivers T.

239. Heterodyne or Beat Reception.—A high-frequency alternating current may have its frequency a lowered by superposing on it a second current of somewhat lower or higher frequency a'. The resulting current may be shown to be a modulated current having a frequency equal to the average of the two frequencies. Further, the amplitude *envelope* of this resultant frequency has itself a frequency $a - a'$, or the *difference* of the two impressed

frequencies. Figure 431 (a) shows the resulting current curve and the resulting amplitude envelope for the general case, that is, when the amplitudes of the two currents are unequal. Figure 431 (b) shows the resulting current curve and the resulting amplitude envelope when the amplitudes of the two currents are equal. In neither case is the envelope sinusoidal.

A detecting tube will separate the envelope frequency from the high frequency, thus giving a current having the envelope frequency of $a - a'$ cycles per second (see Par. 235). This frequency is called the *beat-note* frequency.

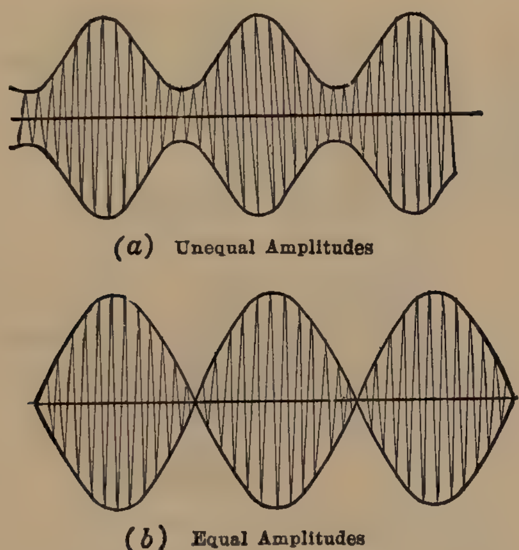


Fig. 431.—Beat-frequency envelopes

The superposed frequency may be obtained from an oscillating tube (Fig. 432), whose grid circuit is inductively coupled to the grid circuit of the detector through the mutual inductance M'. This method of reception is called *heterodyne reception* or *beat reception.*

In radiotelegraphy, where the high frequency or carrier current is modulated by the dots and dashes of the Morse code, the frequency of the beat note is made so low as to be audible, and the dots and dashes are heard at that frequency. In radiotelephony (or broadcasting), where the high-frequency current is modulated by speech or music, the frequency of the beat note (that is, the

amplitude *envelope* of the frequency $(a - a')$) is made so high as to be above audibility.

For example, if the incoming modulated frequency is 1,000,000 cycles per second and the superposed frequency is 970,000 cycles per second, the resulting frequency is

$$\frac{1{,}000{,}000 + 970{,}000}{2} = 985{,}000$$

cycles per second, and the frequency of the amplitude envelope, or the beat frequency, is 1,000,000 − 970,000, or 30,000 cycles per second.

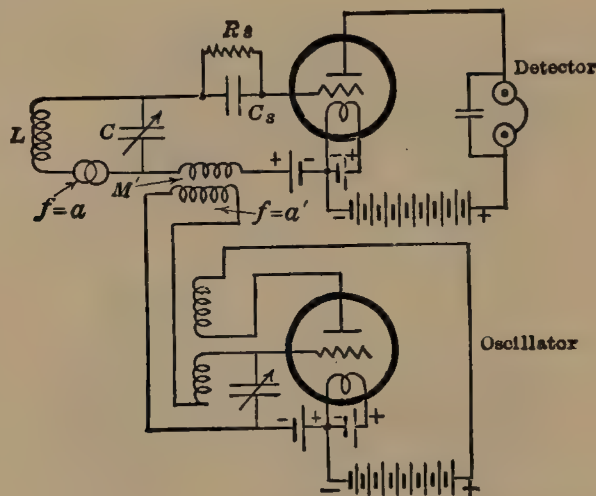


FIG. 432.—Separate heterodyne reception.

By means of the detector (Fig. 432), this amplitude envelope of beat frequency is converted into a carrier-frequency current of this same beat frequency and modulated by the original audio-frequency currents, such as those produced by speech and music. This current again must be detected, amplified, etc., in the ordinary manner. This is the principle of the superheterodyne receiver.

The functions of oscillator and detector may be combined in one tube. This method is called *self-heterodyne* or *autodyne* reception. The connections are exactly those of Fig. 430, with the omission of the grid polarizing battery. In the autodyne, the tube is already oscillating, which tends to increase its sensitivity, but this effect is frequently more than offset by the fact that the

grid circuit is tuned to the frequency of the oscillating current and not to that of the incoming signal. If these frequencies differ by very large amounts, further amplification is necessary to make autodyne reception equal to separate heterodyne reception.

It is possible to detect a speech-modulated, high-frequency current with either the separate heterodyne or the autodyne method by making the beat note of zero frequency. This greatly increases the detecting action, but serious distortion is likely to be introduced because of the difficulty of maintaining zero-beat frequency.

CHAPTER XV

ILLUMINATION AND PHOTOMETRY

Light is a form of radiant energy and is probably due to vibrations set up in the ether by luminous bodies. It has the property of producing the sensation of vision on the retina of the eye and so enables objects to be seen and distinguished.

Illumination means specifically the light incident on a surface or object, but in a broader sense it has come to signify that branch of engineering having to do with the distribution and utilization of light. The measurement of light and light distribution is called *photometry*.

240. Candlepower.—The brightness of a light source is called its *luminous* intensity. The luminous intensity of a body is measured in terms of the light intensity in a horizontal direction given by a standard candle and is called *candlepower*. Candlepower is denoted by I.[1] That is, if a light source, such as an incandescent lamp, were replaced by 14 standard candles without altering either the total light emitted or its distribution, the incandescent lamp would have a luminous intensity in a horizontal direction of 14 candlepower (cp.).

Candles of standard dimensions, burning under standard conditions, have in the past been used as standards of luminous intensity. Owing to the difficulty of reproducing such a standard with a sufficiently high degree of precision and owing to the variation of its luminous intensity with atmospheric conditions, etc., the candle has not proved an acceptable reference standard, particularly at the present time when a high degree of precision in light measurements is necessary.

No perfectly satisfactory standard of luminous intensity has as yet been devised. At present, the Bureau of Standards maintains incandescent lamps which constitute a standard of

[1] Photometry symbols will be found often to duplicate electrical symbols. For example, I = candlepower, and in electrical units I = current. Photometric and electric units are not of the same character.

luminous intensity at some known voltage. These lamps are constant for a considerable time if used only occasionally. By means of these lamps, secondary incandescent-lamp standards may be calibrated and used.

241. Unit Solid Angle or Steradian.—In order to understand the fundamentals of light emission and distribution, it is necessary to know what is meant by *solid angle. A unit solid angle is the angle at the center subtended by a unit area on the surface of a sphere which has a unit radius.*

Figure 433 shows a sphere whose radius is one ft. An area of one sq. ft. on its surface subtends a conical solid angle at the center. This angle is a unit solid angle, sometimes called the *steradian.*

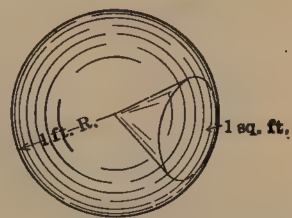


FIG. 433.—Unit solid angle.

As the area of the surface of a sphere is equal to $4\pi r^2$, there must be 4π units of solid angle about the center of a sphere. This may be seen by letting $r = 1$.

If any area on the surface of a sphere be divided by the square of the radius, the result is the solid angle that this area subtends at the center.

Example.—A certain sphere is 2.5 ft. in diameter. How many unit solid angles does an area of 3 sq. ft. on its surface subtend at its center?

The number of unit solid angles

$$\frac{3}{(1.25)^2} = 1.92 \text{ steradians.} \quad \textit{Ans.}$$

242. Luminous Flux: Lumen.—Light may be considered as a flux which emanates from a luminous source in the same way that magnetic flux emanates from a magnetic pole. The amount of illumination emitted by a luminous source may be considered as being the total light *flux* emanating from that source.

Figure 434 shows a candle placed at the center of a sphere whose radius is one ft. Assume that this candle emits light uniformly in all directions, the intensity being equal to that in the horizontal plane or one cp. (A candle of this type is never met with in practice but is given here merely for purposes of illustration. The ordinary standard candle emits an intensity of one cp. in the horizontal plane only, the intensity in other direc-

tions being much less than one cp.) Let B be a unit solid angle at the center subtended by an area of 1 sq. ft. on the surface of the sphere. A certain amount of light flux will be confined by this unit solid angle, and, as light flux is emitted radially in straight lines, no flux enters or leaves the solid angle through its sides.

The light confined by this unit solid angle and coming from such a standard candle is the unit of light flux and is called the *lumen*. The number of lumens is denoted by F.

As there are 4π units of solid angle at the center of a sphere, it is evident that each standard candle would emit 4π lumens if its light intensity were the same in every direction and equal to the horizontal intensity. The difference between candlepower and luminous flux should be clearly understood. The candlepower is *intensity* of light emission and may vary in different directions. On the other hand, luminous flux represents the *total* light emitted in any given region.

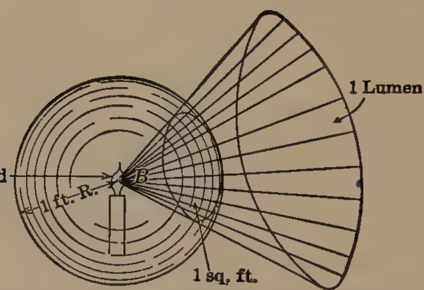


Fig. 434.—One lumen, the unit of light flux.

In the past, incandescent lamps have been rated on their mean horizontal candlepower, because, in the carbon lamps, which were then the only type in general use, the shape of the filament and its distribution in the bulb were practically the same in all lamps. All lamps, therefore, had light-distribution curves of the same general form. That is, the ratio of mean spherical candlepower to mean horizontal candlepower was practically constant in the lamps then in use.

With the advent of new types of lamps, the disposition of the filaments became quite different in the various lamps, and the mean horizontal candlepower was no longer a measure of the total light output of a lamp. Hence, it would seem more rational to rate such lamps according to the total light flux which they emit rather than by their intensity of illumination in one plane. At the present time, therefore, the tendency is to rate lamps in lumens rather than by their mean horizontal candlepower.

The mean spherical candlepower, which is the average of the candlepower emitted in all directions, is also a measure of the total light flux emitted by a luminous source.

A luminous source which has a luminous intensity of one cp. in every direction has a mean spherical candlepower equal to 1.0 and emits 4π lumens. *Therefore, the number of lumens emitted by a light source is equal to 4π times the mean spherical candlepower.*

Example.—An incandescent lamp has a mean spherical candlepower of 20. How many lumens does it emit?

$$F = 4\pi\, 20 = 251.4 \text{ lumens.} \quad \textit{Ans.}$$

243. Illumination.—Illumination is the amount of light flux or the number of lumens falling on a unit area. This corresponds to flux density in magnetism. It will be remembered that *flux density* is defined as the number of magnetic lines passing normally through a unit area (see Vol. I, p. 8, Par. 13). The unit of illumination is the *foot-candle* and corresponds to one lumen per square foot, the square foot being taken normal to the direction of the light flux. It is denoted by the symbol E, where $E = F/A$. A is the area of the surface taken normal to the direction of the light flux. For example, in Fig. 434, one lumen is included by the solid angle B. If the sphere be thought of as hollow and having a radius of one ft., a square foot on its surface intercepts one lumen, and the light flux is perpendicular to the surface at every point. As the illumination is assumed to be equal in all directions, therefore, the illumination at every point on the inside wall of this sphere is one ft.-candle. Such uniform distribution of light seldom occurs in practice.

A sphere having a radius of 2 ft. has a surface area four times as great as a sphere having a radius of 1 ft. With a fixed luminous source at the center, both spheres intercept the same total light flux. The light *intensity* at the surface of the 2-ft. sphere is one-fourth the light intensity at the surface of the 1-ft. sphere. *Therefore, to obtain the illumination* in foot-candles *on a surface* which is normal to the direction of the light flux, divide the candlepower of the light source by the *square* of the distance in feet from the light source to the surface illuminated.

Example.—A light has an intensity of 25 cp. downward in a vertical direction. What is the illumination in foot-candles on a horizontal table 4 ft. below this light?

$$E = \frac{25}{(4)^2} = \frac{25}{16} = 1.56 \text{ ft.-candles.} \quad \textit{Ans.}$$

244. Law of Inverse Squares.—Figure 435 shows that portion of the light emitted by a certain source which is included within a given solid angle. Let A_1 be a perfectly transparent surface at a distance D_1 from the source. Let A_2 be a similar surface at a distance D_2 from the source. By geometry, the areas A_1 and

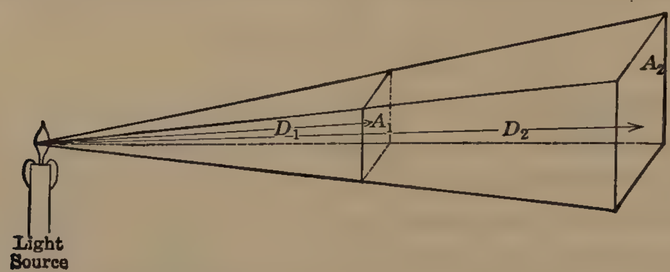


FIG. 435.—Variation of light intensity with distance from source.

A_2 are proportional to the *squares* of their distances from the apex of the cone or pyramid. That is,

$$\frac{A_1}{A_2} = \frac{D_1^2}{D_2^2}.$$

The light flux passing through A_1 is equal to the light flux passing through A_2, as none of the light flux passes out through the sides of the solid angle. If the light flux passing through A_1 and A_2 is the same, then the density of the light flux or the lumens per square foot must be inversely as the areas. *Therefore, the intensity of illumination from a point source varies inversely as the square of the distance from the source.*

Let E_1 be the illumination on surface A_1, and E_2 the illumination on surface A_2. Then

$$\frac{E_1}{E_2} = \frac{D_2^2}{D_1^2}. \tag{156}$$

The above law of inverse squares also follows from the discussion in Par. 242. The law is strictly true only when the light source is a point. It is impossible to obtain a point source in practice, but with the usual light sources, no great error is

introduced in assuming a point source, unless the illuminated areas under consideration are very close to the source. With mercury-tube lamps, Moore tubes, neon tubes, and lamps having certain types of reflectors, the law of inverse squares must be applied with great caution.

Example.—A drawing board directly under an incandescent lamp and 4 ft. distant has an average illumination of 3 ft. candles. What is the illumination on the drawing board when the lamp is raised 2 ft.?

$$\frac{E_2}{E_1} = \frac{E_2}{3} = \frac{(4)^2}{(6)^2} = \frac{16}{36}$$

$$E_2 = \frac{48}{36} = 1.333 \text{ ft.-candles.} \quad \textit{Ans.}$$

245. Absorption; Brightness.—When light falls on a surface or object, a certain amount of the light is either absorbed or transmitted and the rest is reflected. No substance reflects all the light which it receives, although highly polished surfaces reflect a very large proportion, as is shown in the table which follows. A white surface reflects a high percentage of the light falling on it and reflects all colors equally well. A pure black surface should reflect no light at all, but practically all black surfaces reflect a certain amount of light. When illuminated with white light, the color of an object as seen by reflected light is determined by its ability to absorb, transmit, and reflect the various colors of the spectrum. For example, a green object as seen by reflected light has the property of reflecting green and of absorbing or transmitting practically all other colors. Hence, the object appears green by reflected light.

The color of an object as seen by *transmitted* light is frequently different from its color as seen by *reflected* light. For example, the color of thin gold leaf as seen by reflected light is yellow, whereas its color as seen by transmitted light is greenish.

The unit of *brightness* in the metric system is the *lambert*, which is 1 lumen per square centimeter. Brightness may also be measured in candles per square inch. These units are used when the brightness of a luminous source, such as an incandescent filament, is under consideration.

The brightness of a surface is the number of lumens per unit area which the surface *emits* in the direction of the normal. Let it be expressed by E'. E' is always less than the illumination

E, as the surface absorbs some light. The ratio E'/E is called the *coefficient of reflection.* It is the ratio of the light emitted to the light received and is aways less than unity. Below are given the coefficients of reflection for various well-known surfaces.

Coefficient of Reflection[1]

	Per cent. reflection
New aluminum bronze (unprotected)	54
Polished brass	60
Baked white enamel (paint)	72
Matt surface, porcelain enamel	79
Silvered mirror	83
White matt surface paper (smooth)	57
Light-buff surface paper (smooth)	45
Embossed gilt paper	43
Light-blue paper	12

[1] "Standard Handbook," Sec. 14.

246. Light Distribution.—Light from sources such as incandescent lamps, arc lamps, etc., varies in intensity in different directions. The distribution depends not only on the light source itself but also on the reflectors, refractors, and fixtures which are used with the source. In most light sources, the light-giving element is so designed that the horizontal intensity is nearly the same in all directions. This is particularly true of incandescent lamps and arc lamps.

The intensity in vertical planes passing through the axis of the lamp varies considerably in different directions. This is illustrated in Fig. 436, which shows the distribution in one vertical plane of the light from an incandescent tungsten-filament lamp without and with a reflector. The intensity of the light in the vertically upward direction (180°) is small, owing to the presence of the base and socket and to the disposition of the filament. The intensity of the light in a vertically downward direction is also small.

These distribution curves are particularly useful in determining the suitability of a lamp for any particular purpose. The distribution may be modified by shades, reflectors, etc. as shown in Fig. 436. As will be shown later, the area of these

distribution curves is not proportional to the total light flux emitted by the lamp.

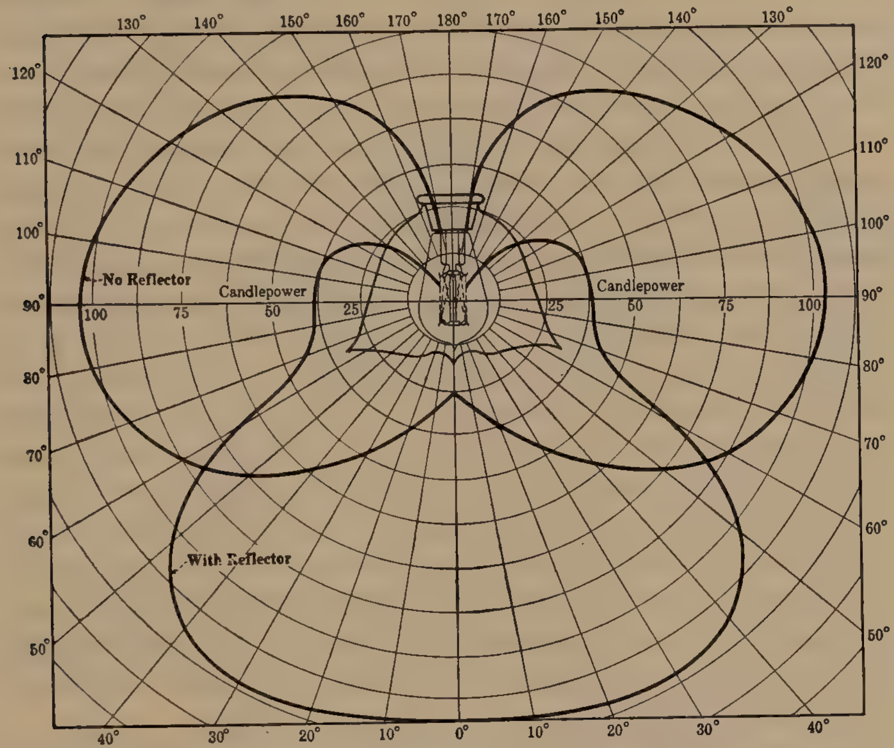


FIG. 436.—Distribution of light intensity about a Mazda-B lamp without and with reflector.

247. Light Sources; Incandescence; Luminescence.—Light is emitted by sources under two conditions—incandescence and luminescence. Incandescence is produced by heating a substance to a high temperature, as in the incandescent lamp, the carbon arc, and the Welsbach gas mantle. The amount of light emitted by a substance increases very rapidly with its temperature. In fact, the light emitted increases somewhere from the eighth to the twelfth power of its absolute temperature. Hence, a small increase in the temperature of an illuminant results in a very large increase in the light which it emits. The light becomes more nearly white as the temperature rises. This principle should be kept in mind, because it is the reason for the different efficiencies which are obtained with different types of lamps.

Light emitted by incandescent bodies is accompanied by high temperature and a corresponding dissipation of considerable energy as heat. On the other hand, a luminescent substance may give out light at moderate temperatures. Examples of luminescent light sources are Moore and neon tubes, the mercury arc, the flaming and luminous arcs, all of which will be discussed later. The firefly is an excellent example of a luminescent source.

248. Carbon-filament Lamp.—The requirements of an incandescent filament are that it shall be highly refractory, that is, that it shall be able to withstand high temperatures without rapid deterioration, and that it shall be mechanically strong at these high temperatures. Also, the electrical resistivity of the filament material should be so high that the length of the filament for a given resistance will not be excessive. For many years, the carbon filament was the only one that proved satisfactory. It consists of a filament of carbonized cellulose, bent into horse shoe form and operated in a vacuum. A later development was the "GEM" lamp in which the carbon is "metallized," that is, it is flashed in a gas rich in hydrocarbons. This gives the carbon filament a metallic appearance and a positive temperature coefficient, such as metals have, and permits the filament to operate at a higher temperature.

The purpose of a vacuum in a lamp is twofold. A vacuum prevents chemical action on the filament and is an excellent heat insulator.

The treated carbon-filament lamp has an efficiency of about 3.0 watts per mean horizontal candlepower, and the metallized filament an efficiency of about 2.5 watts per mean horizontal candlepower.

The objection to the carbon-filament lamp is its low efficiency. Remembering that the amount of light emitted by an incandescent source increases from the eighth to the twelfth power of its absolute temperature, carbon-filament lamps can be made to have very high efficiency by operating them at high temperatures. This increased efficiency, however, is accompanied by rapid evaporation of the filament, resulting in a very short life. Carbon-filament lamps are, therefore, operated at such a temperature that their life is from 700 to 1,000 hr. Their candlepower may become so reduced after long use that it is more

economical to discard the lamps than to continue their use at this low candlepower.

249. Tantalum Lamp.—The metal tantalum was next used as a lamp filament. Because of the high temperature at which tantalum can safely operate, tantalum lamps are able to develop an efficiency of 2.0 watts per mean horizontal candlepower and at the same time have an average life of about 600 hr. One peculiarity of the lamp is that, when alternating current is used, the filament offsets badly. Owing to the high efficiency reached by the tungsten lamp, the tantalum lamp has been absolutely superseded.

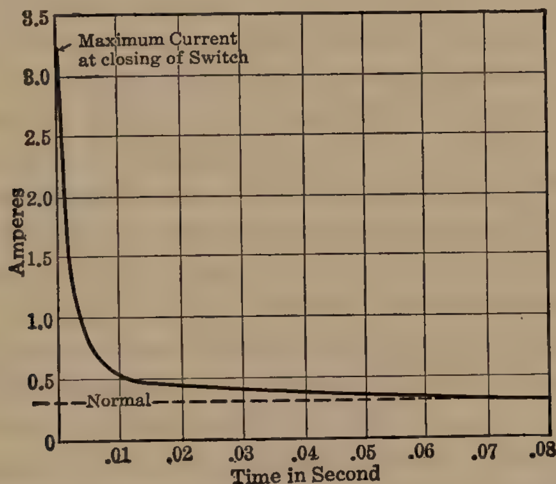


FIG. 437.—Current variation in tungsten lamp when switched in circuit.

250. Tungsten Type-A and Type-B Mazda Lamps.—Tungsten is a metal of high resistivity and is capable of withstanding very high temperatures. It is, therefore, well adapted for use as the filament in incandescent lamps. In the early days, it was not possible to draw it through a die as ordinary wire is drawn. Powdered tungsten was mixed with an organic binder, such as cellulose, and forced through a die. The binder was then driven out by heating, leaving a porous and pitted tungsten filament which was very fragile. This type of filament was used in the Mazda-A lamp. In 1911, the process of drawing tungsten wire was perfected, so that the present-day filaments are rugged. Lamps using this drawn-wire filament in a vacuum are called *Mazda-B* lamps.

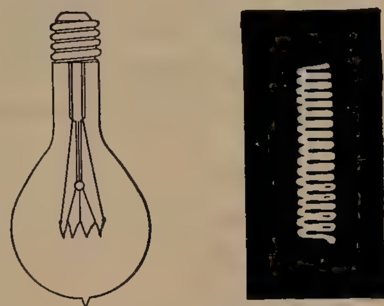
Type-B tungsten lamps have an efficiency of about 1.2 watts per mean horizontal candlepower. In the large sizes, 150 watts and above, the efficiency may be as high as 0.90 watt per mean horizontal candlepower (see Fig. 436). The reason for this high efficiency is the very high temperature (about 2150° C.) at which it is possible to operate the tungsten without too rapid deterioration. The life of tungsten lamps is longer than that of most other lamps, the guaranteed life being about 1,000 hr.

Tungsten has a positive temperature coefficient so that its resistance at operating temperatures is several times that when cold. This results in a high initial current when the lamp is first switched in circuit. This is called *overshooting.* The relation of current to time is shown in Fig. 437. Fuses in tungsten-lamp circuits have been known to blow as a result of this first rush of current, although the fuses were of ample capacity to take care of the steady operating current of the lamp.

251. Gas-filled Lamps, Type C.—Remembering that the light emitted by an incandescent body increases as the eighth to the twelfth power of its absolute temperature, it is obvious that there is still opportunity for increasing the efficiency of tungsten lamps if the temperature of the filament can be safely increased. The factor which limits the operating temperature of the type-A and type-B filaments is the volatilization or evaporation of the filament itself. This volatilization has two effects. The evaporation removes useful material from the filament, resulting in a reduction of efficiency and in an ultimate burning out of the lamp. The tungsten vapor condenses on the bulb, blackening it and so cutting down the useful light by absorption. This last factor has, in part, been remedied by introducing some chemical called a *getter* into the bulb, which tends to keep the tungsten deposit semitransparent.

The gas-filled or type-C Mazda lamp (Fig. 438 (*a*)) is based on the principle of vapor pressure. The higher the pressure the higher the temperature at which water and other substances evaporate. Water will boil at a very low temperature in a rarefied atmosphere, whereas under pressures greater than atmospheric its boiling temperature may become quite high. The same rule applies to tungsten. In a vacuum, the tungsten evaporates quite readily, which results in a more rapid deterioration

of the filament. In the Mazda-C lamps, the bulb is filled with an inert gas, such as nitrogen or argon. This gas does not enter into chemical reactions with the filament, and yet it causes sufficient pressure to increase materially the evaporation temperature of the filament. Due to the higher temperature of evaporation, the filament may be operated at a higher temperature without rapid deterioration (2580° C.). This gives not only increased efficiency but also a whiter light.



(a) (b)

Fig. 438.—Gas-filled lamp (300 watt) and section of filament.

The gas sweeping by the filament has one very undesirable effect which is not present in a vacuum lamp. It carries heat away from the filament rapidly by convection. This cooling of the filament tends to decrease the lamp efficiency. To minimize this effect, the filament, instead of being a straight wire zigzagged back and forth on supports, as it is in the Mazda-B lamp, is wound in the form of a very fine helix with the turns very close together, as shown in Fig. 438 (*b*). This keeps the filament in compact form and reduces the convection losses.

Below is given a table of efficiencies for gas-filled lamps. It will be noted that the larger sizes have the higher efficiencies, due to their having larger filaments and, hence, less proportionate convection losses.

Rating—Gas-filled Multiple Lamps

Watts	Mean horizontal candlepower	Watts per mean horizontal candlepower	Lumens per watt
75	88	0.85	11.5
100	120	0.83	12.6
200	267	0.75	14.0
500	714	0.70	16.1
750	1,154	0.65	17.1
1,000	1,667	0.60	18.0

By using a bulb having a blue tint, the gas-filled lamp gives a light closely resembling daylight.

252. Effect of Voltage Variation on Incandescent Lamps.—As the voltage on commercial lighting systems will vary more or less from time to time, it is important to understand the effect of this voltage variation on the operation of incandescent lamps. An increased voltage results in a higher operating temperature of the filament and, hence, a higher efficiency. This is accompanied, however, by a decreased life of the lamp.

With an untreated carbon filament, the effect of increasing the voltage is to *decrease* the resistance, as carbon has a nega-

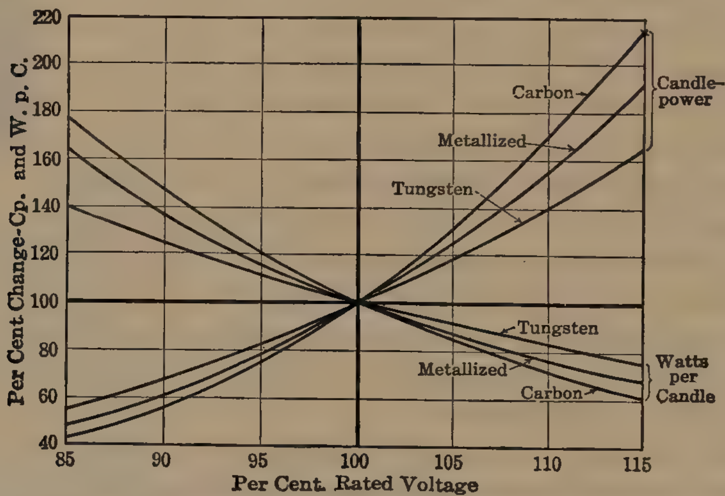


FIG. 439.—Effect of voltage change on candlepower and watts per candle.

tive temperature coefficient. Hence, the power taken by the filament increases even more rapidly than the voltage squared. This makes the carbon lamp more sensitive to voltage changes than it otherwise would be. On the other hand, the metallized carbon filament and the tantalum and tungsten filaments all have positive temperature coefficients. An increased voltage is accompanied by increased resistance, which tends to prevent the lamp's taking more power. These last three types of lamps are less sensitive, therefore, to voltage changes than is the carbon lamp. Figure 439 shows the variation of candlepower and efficiency with percentage of normal voltage for carbon and for tungsten lamps.

ARC LAMPS

253. Carbon Arcs.—The arc lamp was the first successful electric lighting unit. Its principle is the heating of the tips of carbons, or other electrodes, to incandescence by means of an electric current. This is illustrated by Fig. 440. Two carbon rods are connected in series across the lighting mains, a resistance R being in series with the rods. If the carbon tips are first touched together, the heat developed at the point of contact produces a hot vapor which immediately becomes conducting. If the carbons now be drawn apart, more vapor will form, and this hot vapor becomes a conductor of electric current. A large

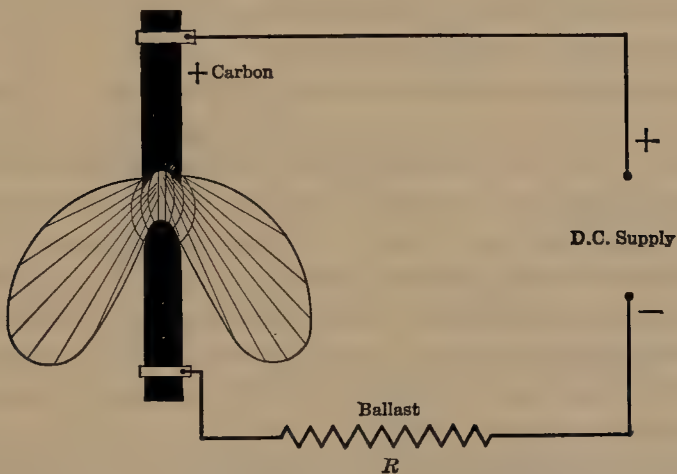


FIG. 440.—Direct-current arc and candlepower distribution curve.

amount of heat energy is developed in a very small space, resulting in an extremely high temperature. This heats the carbon tips to incandescence and results in their emitting an intense white light. Because of its extremely high temperature, the arc is a very efficient illuminant.

The resistance of the arc itself varies nearly inversely as its cross-section. If the carbons were connected directly across the line, a slight increase of current would result in a greater cross-section of arc, which would reduce its resistance. The arc would then take more current, resulting in a still lesser resistance, etc., which would ultimately produce such an extremely low value of resistance as to be practically a short circuit. To prevent this

instability in multiple lamps, a series resistance R, called the *ballast,* is necessary. The power loss in the ballast reduces the over-all efficiency of the lamp. In a 110-volt lamp, the drop across the ballast is about 50 volts.

Figure 440 shows the early type of open, direct-current arc. A crater, formed in the positive carbon, becomes filled with molten carbon. This electrode is at a higher temperature than the negative one. Most of the light comes not from the arc itself but from this *hot* incandescent positive crater, so that the upper carbon should always be the positive one. Figure 440 indicates the light-distribution curve of this type of lamp. It will be noted that most of the light is thrown downward. The positive carbon is consumed more rapidly than the negative and so requires more frequent renewals.

If alternating current be substituted for direct current, it will be found that both carbons consume equally and have the same shape of crater. Hence, the light is not directed downward, and a reflector would be needed. The alternating-current multiple arc has one advantage over the direct-current arc in that it has an inductive ballast which consumes no appreciable power, although it does lower the power factor.

In the *enclosed arc,* the arc is surrounded by a small opalescent globe which prevents the free access of oxygen and increases the hours per trim from 8 to 100.

The open arc, especially the direct-current type, is used to a limited extent for microphotographic work, for searchlights, and for projection purposes, as for stereopticans, moving-picture machines, etc. The high intensity of the arc, combined with its being concentrated, makes it very desirable for these purposes. The Mazda-C tungsten lamp has, however, practically displaced the arc for projection purposes. The foregoing types of arc lamps are practically obsolete as far as general illumination is concerned, although they may be used for street lighting in isolated instances.

254. Flame Arcs.—Approximately 85 per cent. of the light in the direct-current open arc comes from the incandescent crater of the positive carbon. In the flame arc, the carbons are impregnated with salts, as of calcium, strontium, and titanium, so that the arc becomes luminescent, due to the effects of these salts. Hence, nearly all the light comes from the arc

itself. Because of its disagreeable odor and the relatively few burning hours per trim, this lamp is practically not used in this country.

255. Metallic-electrode or Magnetite Lamp.—The metallic-electrode arc lamp or luminous arc or *magnetite lamp*, as it is often called, differs from other arc lamps in that it employs metallic electrodes. As in the flame arc, the light is derived from the arc itself, being due to the luminescence of the vapor which comes from the cathode or negative electrode. A lamp of this type is shown in Fig. 448 (p. 522). The upper electrode or anode is positive and consists of a large cylinder of solid copper. The copper, being an excellent conductor of heat, tends to keep moderately cool. The lower electrode or cathode is negative and consists of iron oxide containing titanium oxide to give a white light. Other ingredients are added to give the electrode desirable burning characteristics.

The arc stream consists of negatively charged luminescent gas particles which originate at the negative electrode. The iron oxides and the titanium of the negative electrode give a white light. Being negatively charged, the gas particles of the arc stream are repelled by the negative electrode and are attracted by the positive electrode. The arc stream moves, therefore, from negative to positive. The copper must always be positive. If it becomes negative, the arc will then consist of luminescent copper vapor and be green in color. In connecting a lamp in circuit, care must be taken to insure correct polarity.

As the electrodes are comparatively cool under operating conditions, there is not sufficient heat to maintain the arc if the electric power is interrupted even for an instant. Hence, outside the question of the greenish arc resulting from the copper's operating as cathode, this type of lamp cannot be used with alternating current.

The feed mechanism is slightly different in principle from that of other types of arc lamps. Were the ordinary type of feed used, the hot metal of the cathode or negative electrode would weld to the copper and "freeze" when the current was turned off. The mechanism is so designed, therefore, that the feed is intermittent, the arc being maintained by restriking. Also, when the lamp is out of circuit, its electrodes are not in

contact but are separated by a gap. When current flows, a starting magnet brings the lower electrode (Fig. 448) into contact with the upper electrode, striking a sharp blow. The control mechanism then permits the lower electrode to drop, drawing out the arc. As the lower electrode is being consumed, the voltage across it rises. When the voltage becomes sufficiently high, the starting magnets cause the lower electrode to strike the upper electrode another sharp blow, after which it drops, bringing about a shorter arc once more.

The copper anode, which is comparatively cool, is consumed very slowly and needs to be replaced only at long intervals. The magnetite electrodes, however, need to be replaced at frequent intervals. The 4.4-amp. lamp will burn from 160 to 200 hr. per trim. The 6.6-amp. lamp burns about 125 hr. per trim. Magnetite lamps have a high efficiency. The 4.4-amp. lamps have an efficiency of approximately 0.70 watt per mean spherical candlepower. The 6.6-amp. lamps have an efficiency of 0.45 watt per mean spherical candlepower. The intense white light of this type of lamp makes it very attractive, particularly when the lamp is mounted on ornamental poles. In addition to general street lighting, it is used to a considerable extent for "white way" and boulevard lighting (see Fig. 455, p. 529).

256. Mercury-arc Lamp; Moore Tube; Neon Lights; Nernst Lamp.—The mercury-arc lamp consists of a long tube containing metallic mercury. The pressure in the tube is very low. When an arc is formed in the tube, the mercury is vaporized and gives a greenish-blue light. This light is due almost wholly to the luminescence of the mercury vapor, the temperature of the vapor being only 250 to 300° C. The lamp owes its high efficiency to the low temperature at which it is able to give its light. Most of the light emitted comes from the blue-violet end of the spectrum, there being little red. This absence of red rays makes people appear ghastly when viewed under this light, an objectionable feature in its use. Red light is fatiguing to the eye and contributes little to the visibility of objects. From the physiological standpoint, therefore, the mercury-vapor lamp is excellent for drafting rooms and for other places where fine detail is concerned. Very satisfactory results are obtained if tungsten lamps are used in conjunction with the mercury-vapor

lamps, thus doing away with the ghastly appearance of the workers.

The mercury-vapor lamp has an efficiency of about 0.75 watt per mean spherical candlepower. It is fundamentally a direct-current lamp, but with a compensator it can be used on alternating-current circuits. The connections when used with direct current are shown in Fig. 441.

A higher-pressure mercury-vapor lamp has been developed operating at a higher temperature. The low melting point of glass necessitates the use of quartz tubes with this type of lamp.

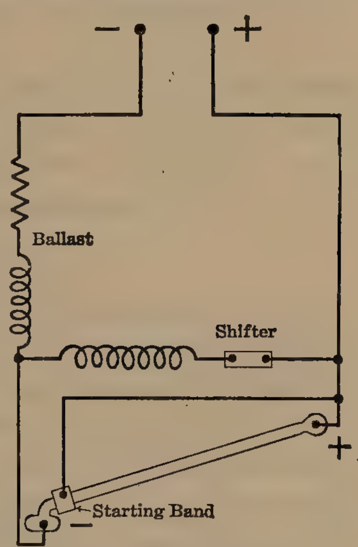


FIG. 441.—Connections of mercury-vapor lamp.

In the Moore tube, the light is obtained from a luminescent gas at a comparatively low temperature. This luminescence is produced by high-voltage electric discharge taking place in vacuum tubes. These tubes are sometimes 200 ft. long and are bulky per candlepower. They have an advantage in that the color can be controlled by means of the gas used in the tube.

If carbon dioxide be used, for example, a light closely approaching daylight is obtained. This characteristic, together with its uniform distribution, makes it an excellent lamp for color matching, as with textiles. Its efficiency is about 2.5 watts per candlepower. The power factor is low.

The neon tube is a slight modification of the Moore tube. Neon at low pressure is introduced into glass tubing which has been very highly evacuated. An electric current of about 7 or 8 milliamp. at high voltage causes the neon to give a brilliant orange-red light which is very penetrating. The tubes are ordinarily 25 ft. long and are used to a large extent for sign lighting and for outlining parts of buildings, etc. With direct-current supply, the tube requires an induction coil; with alternating-current supply, it requires a step-up transformer.

The Nernst lamp operates on the principle that porcelain, at a red heat, is a conductor of electricity. This lamp has a heating coil which makes a porcelain filament conducting. The lamp gives a substantially white light and gives off no odor or dirt. It does not require trimming. Its maintenance is high. Tungsten lamps have driven it from the lighting field. Its efficiency is about 1.3 watts per mean hemispherical candlepower.

As a maximum, the foregoing lamps emit as light energy from 2 to 5 per cent. of the electrical energy supplied to them. The remainder of the energy is liberated as heat. On the other hand, the firefly has a light-giving efficiency of 96 per cent. It emits very little heat energy. Although great improvements have been made in the field of illuminants during the past few years, there is still a tremendous opportunity for further advances.

PHOTOMETRY

257. Photometry.—Photometry is the measurement of light. Light measurements are nearly all made by direct comparison. The chief source of inaccuracy in making such comparisons is the question of color. Unless the lamps under comparison are of almost exactly the same color, only an approximate photometric balance can be obtained. This balance varies with different persons, owing to the effect of different colors upon the eye.

258. Bunsen Photometer.—The Bunsen photometer is the simplest type of photometric measuring device and is illustrated in Fig. 442. Assume that it is desired to measure the candlepower of the incandescent lamp L, using the candle C as a standard for comparison. The two lights are placed some 10 to 20 ft. apart, and a movable screen S is placed between them. The screen S consists of a piece of paper or parchment with a grease spot in the center. The grease spot on the screen is translucent and so allows light to pass. If viewed from the side which is illuminated, the spot will appear darker than the rest of the screen, owing to the fact that light passes through the translucent grease spot more readily than through the surrounding part of the screen.

On the other hand, if this same screen be viewed from the non-illuminated side, the grease spot will appear brighter than the

rest of the screen, since it is more translucent than the rest of the screen. Now, if both sides of the screen receive equal illumination, the grease spot will look the same in comparison with the surrounding portion of the disc, when viewed from either side. When this occurs, a photometrical balance is obtained.

In order that the observer may view both sides of the screen simultaneously, two mirrors M (Fig. 442) are set at an angle and reflect the light in the manner shown by the dotted lines.

The screen S is moved until the screen looks the same on both sides, and the distances l and l_1 are read. Let E be the

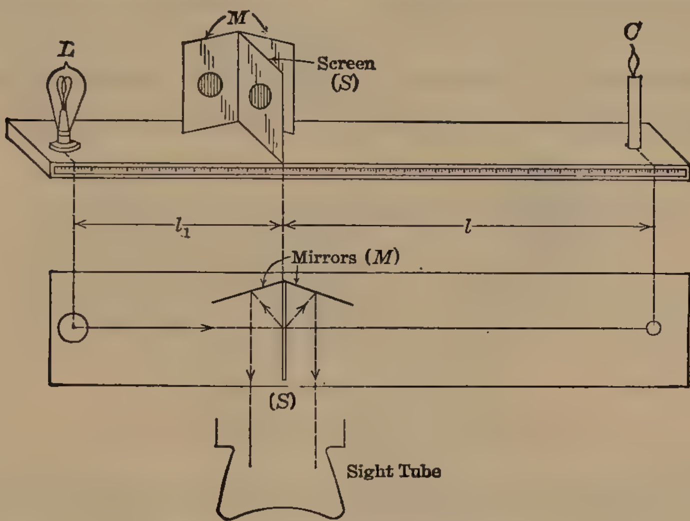


FIG. 442.—Bunsen photometer.

candlepower of the candle or standard and E_1 the candlepower of the test lamp. Remembering that, for point sources, light intensity varies inversely as the square of the distance,

$$\frac{E_1}{E} = \frac{l_1^2}{l^2}. \tag{157}$$

The candlepower of the test lamp

$$E_1 = E\frac{l_1^2}{l^2}. \tag{158}$$

If a standard candle is used, $E = 1.0$.

If the two lights have different color, the two sides of the screen will never appear alike, and only an approximate balance

can be obtained. The position of balance is, to a considerable extent, determined by the personal equation of the observer.

Because of the unreliability of candles, standard incandescent lamps are used. These may be obtained from the Bureau of Standards. Such lamps, when used at the voltage at which they are calibrated, are very accurate standards. An arrow on the lamp indicates the position in which it was calibrated. It is customary to use the standard lamp only to calibrate a working standard so that the candlepower of the ultimate standard will not change due to its being in too constant use.

As the candlepower of lamps is very sensitive to changes in voltage, the connections are often made as shown in Fig. 443. Both lights are fed through an adjustable resistance R_2. An adjustable resistance R is in series with the standard lamp, and

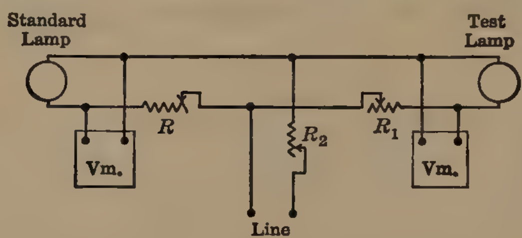


FIG. 443.—Connections for photometric test.

another adjustable resistance R_1 is in series with the test lamp. A voltmeter is in parallel with each lamp. Both lamps are brought to the desired values of voltage by adjusting roughly R_2 and then by separate adjustments of R and R_1. Any fluctuations of line voltage can be taken care of by the resistance R_2, which affects both lamps. Further, any unnoticed change of line voltage affects both lamps approximately to the same degree if the lamps have similar voltage-candlepower characteristics.

259. Lummer-Brodhun Comparison Photometer.—The Lummer-Brodhun comparison screen presents two clearly defined elliptical areas, as shown in Fig. 444. When a photometric balance is obtained, the two ellipses merge into one if the lights have the same color. The operation of this screen is as follows: S_1, S_2 is a white, opaque screen. The light coming from one source falls on the side S_1, and that coming from the other source falls on the side S_2. The brightness of S_1 and of S_2

depends on the intensity of the source which illuminates each. The light from S_1 and S_2 is reflected by the plane mirrors M_1 and M_2 to the total reflecting prisms P_1 and P_2. The hypotenuses of the two prisms are in contact over a circular area only. The light striking this area of contact can pass through; other light is totally reflected. That is, only the central beam, shown dotted, from the mirror M_1, passes through to the eyepiece. The remainder of the light is turned away. On the other hand, the central beam from M_2 shown by a solid line, passes through the center circle and is absorbed by the walls of the box. The remaining light is reflected to the eyepiece. The observer sees two distinct ellipses if the photometer is out of balance. (The circle of contact of the two prisms appears as an ellipse to the observer, because he is viewing the circle at an angle.) When the

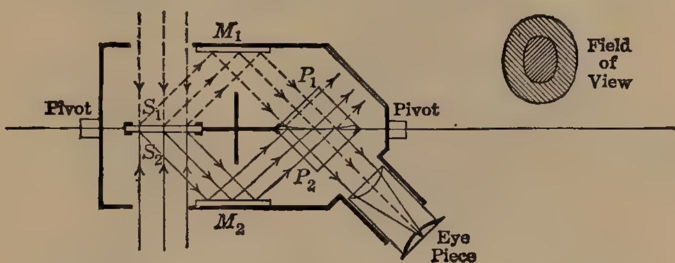


FIG. 444.—Lummer-Brodhun comparison photometer.

transmitted light from M_1 (center dotted line) is equal in intensity to the reflected light from M_2 (two outside solid lines), the two ellipses have the same appearance. That is, when the two ellipses blend, the same illumination is coming from each source, and the photometer is in balance. To eliminate any errors due to differences in the two sides of the screen S_1 and S_2, the photometer screen should be reversed.

260. Lummer-Brodhun Contrast Photometer.—The Lummer-Brodhun contrast photometer is the most sensitive of all the photometers and is also more independent of color than any other. Balances cannot be made so rapidly with it, however, as with the comparison type.

The principle of operation is shown in Fig. 445. There are two prisms in contact (Fig. 445 (*a*)), as with the comparison type. The portion *e* of one prism is etched to a trapezoidal shape. The glass at *a* and *c* is etched away, leaving a trapezoidal contact area

at b. G and G_1 are two glass screens which reduce the transmitted light by 7 to 8 per cent. Assume that the photometer is out of balance and that the intensity S_1 is greater than S. The appearance of the field of view is shown in (b) (Fig. 445). The light S coming through the contact areas d and f will produce in the eyepiece the darkened semicircle S; the light S coming through the screen G and then through the trapezoidal contact area b will produce the darkened trapezoid S, 7 or 8 per cent. darker than the semicircle S as shown in (b) (Fig. 445). All other light will be totally reflected away from the field of view, as shown by the beam g.

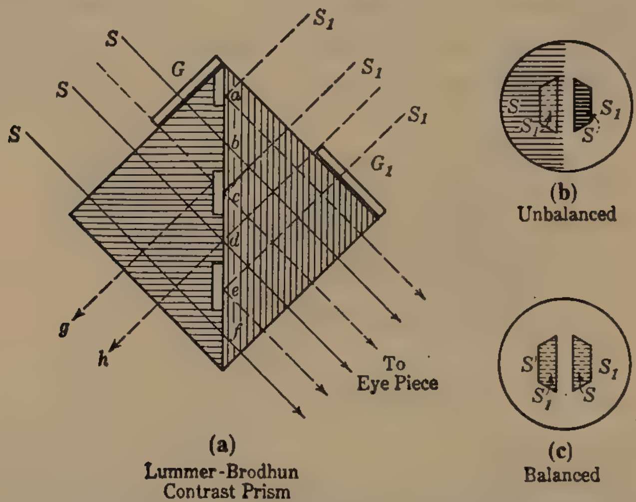


FIG. 445.—Lummer-Brodhun contrast photometer.

The light coming from S_1 and meeting surfaces a and c is totally reflected to the eyepiece and gives a light semicircle S_1 (Fig. 445 (b)); the light S_1 coming through the screen G_1 and meeting the trapezoidal area e is totally reflected to the eyepiece and produces a trapezoid S_1 lighter than its background, the semicircle S, but 7 or 8 per cent. darker than the semicircle S_1.

Figure 445 (c) shows the appearance of the field of view when the photometer is in balance. The semicircles S and S_1 are matched to equal brightness, and the dividing line between them disappears; the trapezoids S and S_1 are of equal brightness but are 7 or 8 per cent. darker than the rest of the screen. Each trapezoid is, therefore, equally contrasted against its own background.

261. Sharp-Millar Portable Photometer.—A portable photometer is necessary for making such photometric measurements as cannot be made in the laboratory, as on arc lamps in service, street-lighting units, store lighting, etc.

Portable photometers involve the same principles as laboratory photometers. The difference lies in the compactness and in the ease of manipulation.

The Sharp-Millar photometer is typical of the portable type. A plan of the instrument is shown in Fig. 446. T is a tube which can swing through 180°. At the elbow R is either a mirror or a white diffusing surface. The light entering the tube is reflected at right angles by R and is directed toward P, a Lummer-Brodhun

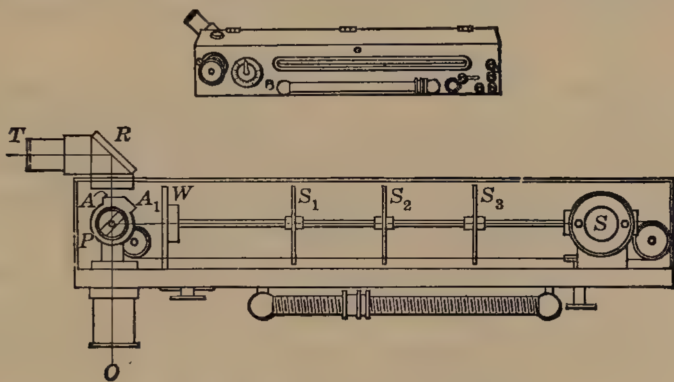


FIG. 446.—Sharp-Millar photometer.

comparison screen. This light is balanced against the brightness of a screen W, illuminated by a 6-volt tungsten lamp S, which is standardized. The viewing aperture is at O. The screens S_1, S_2, S_3 prevent stray light from the lamp S from falling on the window W. The balance is obtained by moving the lamp S, which varies the illumination on the window W. By reading the position of the lamp on a scale when balance is obtained, the candlepower can be determined. If illumination is being measured, a white, translucent glass, called a *test plate*, is placed at T, and the mirror used at R. The brightness of T is a measure of the total illumination falling upon it.

On the other hand, if the candlepower of a lamp is being measured, extraneous light must be excluded. The test plate at T is removed, therefore, and the mirror at R is reversed, which substitutes for R the white diffusing surface. The only light

which enters the tube is that coming from the source toward which the tube is directed, the extraneous light being cut off by the sides of the tube.

If the light to be measured is too bright for obtaining a balance, either screen A or A_1 may be interposed between R and the photometer head, thus reducing the light in a known ratio. On the other hand, if the measured light is found to be too dim, these screens may be turned so as to lie between W and the photometer head, thus reducing the light from the standard and making a balance possible.

The standard lamp S is supplied by a 6-volt storage battery. Its candlepower is controlled by a rheostat. The correct adjustment may be determined by an ammeter connected in series with the lamp.

262. Rousseau Diagram.—The mean horizontal candlepower of an incandescent lamp may be determined by rotating the lamp and at the same time measuring its candlepower in a horizontal direction. The speed of rotation must be sufficiently high so that flicker will not prevent the obtaining of an accurate balance. A speed too high distorts the filament by centrifugal force. The rotator is usually so designed that the lamp can be turned through any vertical angle between 0 and 180°. If the candlepower is measured at various angles between 0 and 180° and the corresponding values of candlepower are laid off on the radii of polar coordinate paper, a curve similar to that shown in Fig. 447 is obtained. As would be expected, an incandescent lamp in the ordinary position emits but little light in the upward vertical direction, owing to the presence of the socket and base. But little light is thrown vertically downward because of the small length of filament exposed in that direction.

The mean spherical candlepower of the lamp is obtained by the Rousseau diagram as follows:

A circle mnm' of some convenient diameter is drawn about the polar candlepower diagram, as shown in Fig. 447. The ends of the various radii are projected on a straight line $a'b'$. Perpendiculars are dropped from these projection points to ab, which is parallel to $a'b'$. The distances od, of, etc., are laid off from ab on these perpendiculars, as shown at $o'd'$, $o'f'$. The *average* height of the curve $gf'e'$ to scale gives the mean spherical

candlepower of the lamp. This average height may be obtained by measuring the area *abge'a* and dividing by the base *ab*, the proper scale being used, or by taking the average of several *equally spaced* perpendiculars.

The output of the lamp in lumens is obtained by multiplying the mean spherical candlepower by 4π (12.57).

The *spherical reduction factor* of a lamp is the ratio of the mean spherical candlepower to the mean horizontal candlepower. It is usually less than 1.0. This factor depends on the shape and disposition of the filament and also on the reflector, if one be used. For a given type of lamp, the spherical reduction factor is

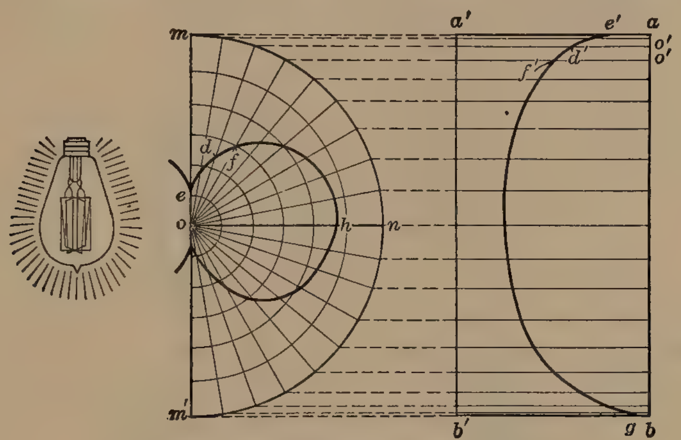


FIG. 447.—Rousseau diagram.

nearly a constant quantity. Knowing this factor for a given type of lamp, the mean spherical candlepower and the luminous output of a lamp can be readily calculated after making one measurement of the mean horizontal candlepower. Below are the spherical reduction factors of a few standard lamps.

SPHERICAL REDUCTION FACTORS[1]

Treated carbon, oval filament	0.82
Metallized carbon (GEM) oval filament	0.82
Tantalum	0.97
Mazda, 60 watts	0.78
Gas Filled	0.80 to 0.90

[1] "Standard Handbook," Sec. 14.

Example.—In the polar diagram of Fig. 447, 1 in. radial distance equals 20 cp. The diameter of the circle and the length of the line *ab* are 4 in. The area of *abge'a* is 8 sq. in. The lamp takes 60 watts, and the length of

the line *oh* is 2.5 in. Determine: (*a*) the mean spherical candlepower; (*b*) the output of the lamp in lumens; (*c*) the mean horizontal candlepower; (*d*) the efficiency of the lamp in watts per mean horizontal and per mean spherical candlepower; (*e*) the spherical reduction factor of the lamp.

(*a*) $\text{m.s.cp.} = \frac{8 \times 20}{4} = 40$ cp. *Ans.*

(*b*) $40 \times 12.57 = 502.8$ lumens. *Ans.*

(*c*) The mean horizontal candlepower = the length of the line $oh \times 20 = 2.5 \times 20 = 50$ cp. *Ans.*

(*d*) $60/50 = 1.20$ watts per mean horizontal candlepower. *Ans.*
$60/40 = 1.5$ watts per mean spherical candlepower. *Ans.*

(*e*) The spherical reduction factor $= 40/50 = 0.80$. *Ans.*

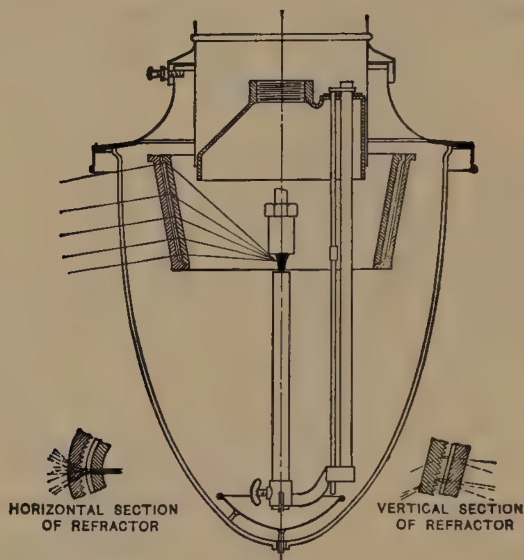


Fig. 448.—Magnetite arc lamp with prismatic refractor.

263. Reflectors.—It is impossible, except in a general way, to design a lamp so that it will distribute its illumination in a desired manner. The distribution of light from light sources is readily controlled, however, by reflectors, the type of reflector being determined by the existing conditions. In "white-way" illumination, for example, as used in the business districts, it is desirable that considerable light be delivered upward so as to illuminate the fronts of the buildings. Ornamental fixtures similar to that shown in Fig. 455 (p. 529) can be used. On the other hand, light sent upward in the residential districts is either lost or is undesirable from the point of view of the residents, because such light comes in at their windows. Hence, some type of

fixture with a reflector, or with a prismatic refractor similar to that shown in Fig. 448, should be used. The prismatic refractor is very efficient. It consists of a glass cylinder in which prisms are cut, as shown by the cross-section (Fig. 448). The prisms bend the light beams downward by refraction, as shown. Often times, a second refractor is used in which the prisms are cut vertically, as shown in Fig. 448 (horizontal section). This breaks up the light in the horizontal planes and eliminates glare. Figure 454 shows a roadway illuminated by the use of prismatic refractors. Figure 436, p. 503 shows the effect on the light-distribution curve of placing a reflector on a lamp used for interior illumination. The upward light is redirected downward where it is more useful.

The type of reflector to be used is determined almost entirely by the conditions and the illumination desired. Such reflectors may absorb from 20 to 25 per cent. of the light emitted by the lamp, but they increase the light in the direction in which it is utilized.

264. Interior Illumination.—Interior illumination is a very big subject and requires considerable space for even a moderately comprehensive treatment. It is more or less complicated by the number of lamps which it may be necessary to use, by the reflection from ceilings, walls, etc. The more involved problems require the services of an illuminating engineer who has had considerable experience in the art of illumination. A few fundamental principles, however, underly the general problem, and these will be treated briefly.

Interior illuminants are confined practically to the vacuum Mazda lamp and the gas-filled lamp, although carbon lamps are unfortunately still used in some instances. The purposes to be accomplished by interior illumination are to provide sufficient light for working, reading, writing, etc., and to distribute this light properly without glare. A lamp with a clear globe may give sufficient light and distribute it properly, but the glare of the bare filament may be objectionable. Further, the artistic aspect of illumination should be kept in mind. The fixtures should be pleasing, the proper light thrown on pictures, decorations, etc.

Reading and writing require an illumination of from 3 to 4 ft.-candles, whereas drafting and detail work may require as

many as 8 ft.-candles. This is the illumination necessary on a plane about 30 in. above the floor. In an office, or drafting room, it is desirable that this light be provided entirely by overhead

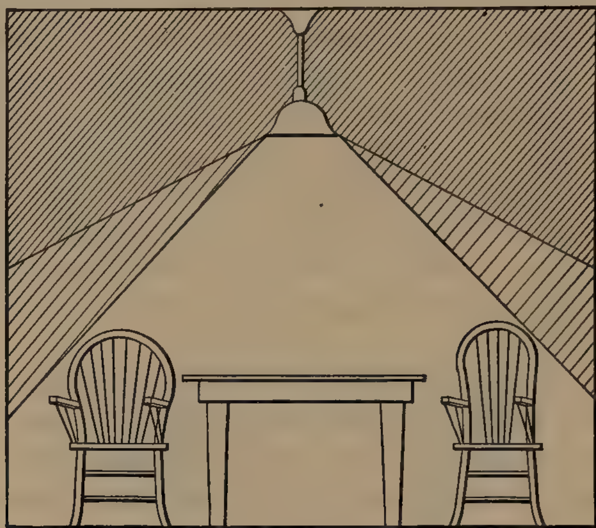
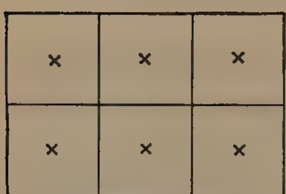


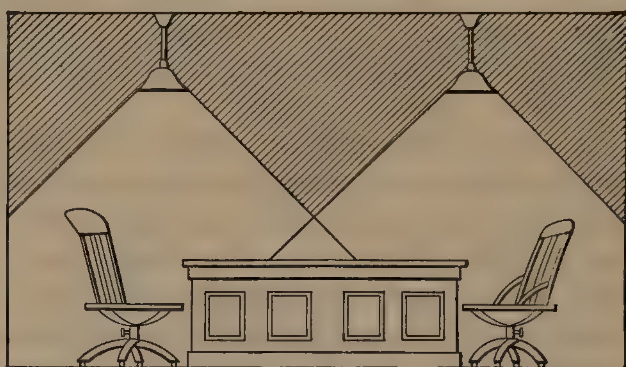
FIG. 449.—Single lamp in room.

sources. Where a single desk requires a higher intensity of illumination than the rest of the room, desk lamps may be provided.

When lighting an interior, a single ceiling light in the center may suffice if the room is not too large. In this case, the reflector



(a) Spacing of lamps in large room



(b) Reflector distribution in large room

FIG. 450.—Distribution of lighting units and lamps in large room.

should distribute the light in a manner similar to that indicated in Fig. 449, so that the walls will receive some of the light. If a large room is to be illuminated, a single center light may be

insufficient. Such a room should be divided approximately in squares, as shown in Fig. 450 (*a*), and a light placed at the center of each square. Under these conditions, the reflector should distribute the light in the manner indicated in Fig. 450 (*b*). This secures a more uniform lighting of the room. The position of the fixtures should be such, however, that little or no specular reflection from the table top reaches the eye.

Indirect lighting is used to a considerable extent. An opaque inverted bowl is suspended from the ceiling, as shown in Fig. 451. The light is first directed to the ceiling and is then diffused

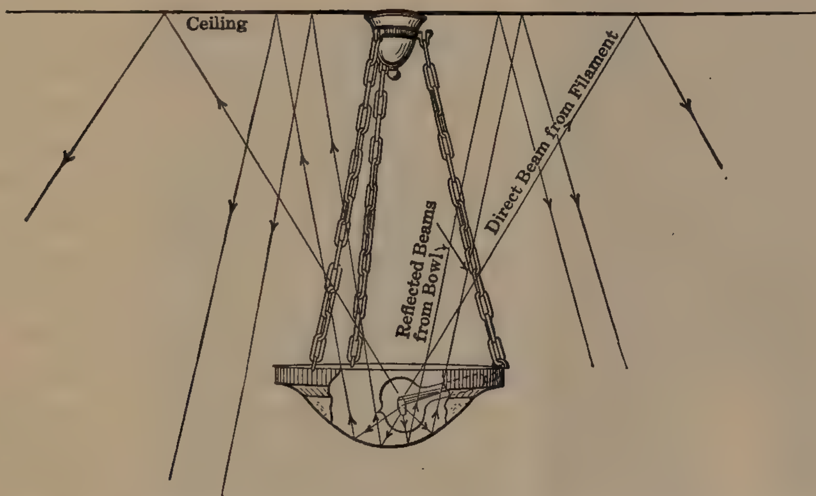


FIG. 451.—Reflection and illumination from indirect-lighting source.

throughout the room. This method of lighting conceals the source and insures a fairly uniform distribution. The method is inefficient, and the complete hiding of the source of light is considered objectionable by many persons.

An improvement over the indirect is the semi-indirect system. The opaque bowl is replaced by a translucent bowl, allowing a considerable portion of the light to be transmitted through the fixture. As in the indirect system, a large amount of light is directed to the ceiling and then reflected through the room. This system is more efficient than the indirect and is more pleasing because the source of light is not entirely concealed. Both systems require clean, light-colored ceilings for their most effective use.

The "Filterlite" (Holophane Company) shown in Fig. 452 is a semi-indirect bowl with the top enclosed with glass which is prismatic *inside*. This prevents dust from entering the bowl (dust is readily removed from the outside), and the prismatic dispersion eliminates shadows of the fixtures on the ceiling.

Fig. 452.—"Filterlite"—a semi-indirect fixture.

Factory[1] and shop lighting is a field in itself. Individual machines, where fine work is being done, may require individual lamps. These lamps should be provided with reflectors which concentrate the light on the work. Overhead belting militates against good illumination, and this fact constitutes a good argument for individual and group drives. Cranes often necessitate lamps' being placed much higher than is efficient from the standpoint of illumination. Where units are placed high above the floor, large units are more efficient than small ones. The cost of lighting a factory is a very small percentage of the total cost of operation. Good illumination, moreover, results in increased production, more accurate work, and fewer accidents. Particular care should be taken to provide good illumination in factories, mills, etc.

265. Street Illumination.—Street illumination differs materially from interior illumination. Interior illumination must be such that small details can be clearly seen. This requires intensities of from 1.25 to 8.0 ft.-candles. When this high intensity is obtained, there is no difficulty whatever in seeing and recognizing objects. On the other hand, the purpose of street lighting is not to show details but to enable one to see and recognize

[1] See *Elec. World*, Vol. 70, 1917. Various articles on illumination by Prof. C. E. Clewell.

objects and persons. Obviously, this must be accomplished with intensities very much less than those used in interior illumination. In a room, the ceilings, walls, paper, etc., are nearly always light in color, and considerable light is obtained by reflection from them. In street lighting, on the other hand, the objects illuminated are the street and sidewalk surfaces, trees, etc., all of which are dark in tone, and any light not falling directly on them is lost. The light reflected from buildings is so small that it need not be seriously considered. The light which goes upward and into trees is lost.

FIG. 453.—Road objects recognized by silhouette.

Formerly, the idea of good street lighting was to imitate daylight as closely as possible by securing uniform illumination. Those responsible for the design and placing of street-lighting units directed their efforts toward obtaining this result, and the greater the uniformity the more successful the installation was considered. This method of illumination has one fault that was not appreciated at first. With the dim illumination which necessarily accompanies street lighting, objects are seen mostly by *shadow* and *silhouette*. This is illustrated by Fig. 453, in which the automobile is recognized almost entirely by its silhouette. With uniform lighting, obtained by using a large

number of small units placed close together, shadows and silhouette are almost entirely eliminated. Depressions and objects in the road become much more difficult to distinguish. This is particularly true when the object and the road have the same general color.

Uniform lighting is considered good engineering in the great white ways which have become popular in recent years. The conditions here are, however, quite different from those of average

FIG. 454.—Night illumination, Nahant Road, Mass., with General Electric luminous arc lamps equipped with prismatic refractors. (Illustrates specular reflection.)

street lighting. The intensity of illumination is very high. Such white ways are almost always situated in the down-town business streets where the light from the lamps is supplemented by illumination from shop windows, illuminated signs, and display lighting. Objects and persons are recognizable by very slight differences in color or in outline, because of the high intensity of illumination which exists. This is analogous to interior illumination.

Another feature of road illumination has only recently become recognized. Automobiles give oiled, tarred roads a glazed sur-

face. When illuminated at night, light is reflected from the roadway to the observer as from a mirror. It is the same phenomenon as the moon shining on the water. Figures 453 and 454 show examples of this phenomenon, which is called *specular reflection*. Just as in the case of a floating object's coming into the reflected moonlight on the water, so an object on the road is clearly silhouetted against this "specular reflection" from the road. This is well illustrated by Figs. 453 and 454, where even the very slight road ripples can be clearly seen. This reflection is better obtained by *using a few large units, spaced some distance apart*, than by using a large number of small units placed close together, as in attempting to obtain uniform illumination.

Fig. 455.—King Co. standard equipped with General Electric Novalux lighting units.

The use of automobiles has added to the problems of street illumination. Owing to the high speed of this type of vehicle, improper illumination may lead to many accidents. A very common cause of such accidents is the improper location of the lighting units at curves, etc. Care should be taken so to locate the units as to eliminate the glare in the drivers' eyes and at the same time make clearly visible any object approaching in the opposite direction.

At the present time, much attention is also being given to the artistic features of street-lighting units. In many instances, crude poles, mast arms, etc., have been replaced by ornamental fixtures. This is well illustrated in the unit of Georgian design shown in Fig. 455. Its delicate and artistic design as well as its excellent proportions should be noted.

266. Floodlighting.—Many spectacular and pleasing effects have resulted from floodlighting. This type of illumination is obtained by projecting the light on the building or object to be illuminated by means of properly located projectors. Flood-

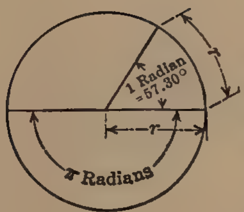
lighting is commonly applied to the illumination of public buildings, statues, etc., (the Statue of Liberty, and the Capital at Washington in particular). It was extensively used at the Pan-American Exposition in San Francisco, in 1915. In flood-

FIG. 456.—Illumination of Public Library, Lynn, Mass., with flood-lighting projectors.

lighting, it is necessary not only to illuminate the object but the projectors must be so placed that architectural details are brought out by relief and shadow. This is well illustrated by Fig. 456 showing the Public Library at Lynn, Mass.

APPENDIX A

Circular Measure—The Radian



The *radian* is a circular angle subtended by an arc equal in length to the radius of its circle, as shown in the figure. The circle has a radius of r units, and the radian is subtended by an arc whose length is r units.

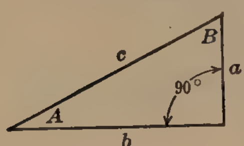
As the circumference of a circle is $2\pi r$ units, there must be 2π or 6.283 radians in 360°. Therefore, 1 radian equals $360°/2\pi = 57.30°$. It follows that $180° = \pi$ radians.

Angular velocity is often expressed in radians per second, and the accepted symbol is ω (omega). In every revolution, a rotating quantity completes 2π radians. If the rotating quantity makes n revolutions per second, its angular velocity $\omega = 2\pi n$ radians per second.

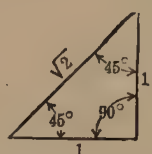
APPENDIX B

Trigonometry—Simple Functions

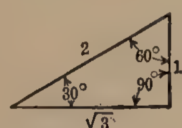
1. The sine (sin) of an angle = opposite side/hypotenuse
2. The cosine (cos) of an angle = adjacent side/hypotenuse
3. The tangent (tan) of an angle = opposite side/adjacent side
4. The cotangent (cot) = 1/tan = adjacent side/opposite side
5. The secant (sec) = 1/cos = hypotenuse/adjacent side
6. The cosecant (cosec) = 1/sin = hypotenuse/opposite side



7. $\sin A = a/c$
8. $\cos A = b/c$
9. $\tan A = a/b$
10. $\cot A = b/a = 1/\tan A$
11. $\sec A = c/b = 1/\cos A$
12. $\operatorname{cosec} A = c/a = 1/\sin A$

13. 

Ratio of sides in a right isosceles triangle

14. 

Ratio of sides in a 30—60° right triangle

15. $\sin B = b/c = \cos A = \cos(90° - B)$, since $A = 90° - B$
16. $\cos B = a/c = \sin A = \sin(90° - B)$

17. $\frac{\sin A}{\cos A} = \frac{a/c}{b/c} = \frac{a}{b} = \tan A$

18. $\sin 30° = 0.5$

19. $\cos 30° = \sqrt{3}/2 = 0.866$

20. $\sin 60° = \sqrt{3}/2 = 0.866$

21. $\cos 60° = 0.5$

22. $\tan 30° = 1/\sqrt{3} = 0.577$

23. $\tan 60° = \sqrt{3} = 1.732$

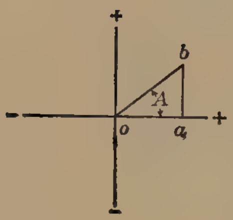
24. $\sin 45° = \cos 45° = 1/\sqrt{2} = 0.707$

25. $\tan 45° = 1.0$

APPENDIX C

Functions of Angles greater than 90°

(*ob* the radius vector is always positive)

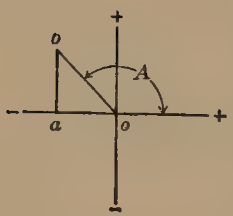


FIRST QUADRANT

$\sin A = \dfrac{+ab}{+ob}$ sin is (+)

$\cos A = \dfrac{+oa}{+ob}$ cos is (+)

$\tan A = \dfrac{+ab}{+oa}$ tan is (+)

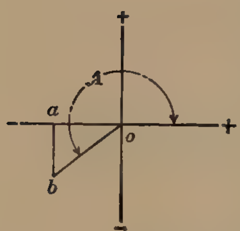


SECOND QUADRANT

$\sin A = \dfrac{+ab}{+ob}$ sin is (+)

$\cos A = \dfrac{-oa}{+ob}$ cos is (−)

$\tan A = \dfrac{+ab}{-oa}$ tan is (−)

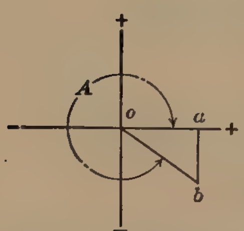


THIRD QUADRANT

$\sin A = \dfrac{-ab}{+ob}$ sin is (−)

$\cos A = \dfrac{-oa}{+ob}$ cos is (−)

$\tan A = \dfrac{-ab}{-oa}$ tan is (+)



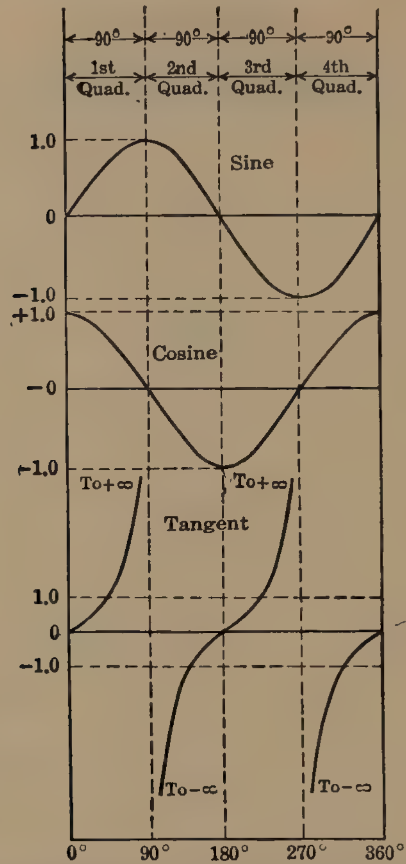
FOURTH QUADRANT

$\sin A = \dfrac{-ab}{+ob}$ sin is (−)

$\cos A = \dfrac{+oa}{+ob}$ cos is (+)

$\tan A = \dfrac{-ab}{+oa}$ tan is (−)

(Also, see "Graphic Representation of Trigonometric Functions," following page.)

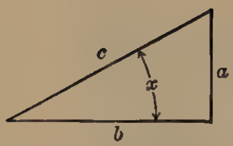


Graphic Representation of Trigonometric Functions.

26. $\sin (90° + x) = \cos x$
27. $\cos (90° + x) = -\sin x$
28. $\tan (90° + x) = -\cot x$
29. $\sin (180° - x) = \sin x$
30. $\cos (180° - x) = -\cos x$
31. $\tan (180° - x) = -\tan x$

APPENDIX D

Simple Trigonometric Formulas

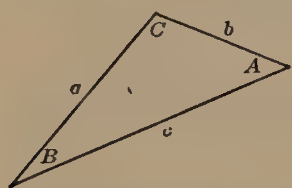


$$a^2 + b^2 = c^2$$

$$\frac{a^2}{c^2} + \frac{b^2}{c^2} = \frac{c^2}{c^2} = 1$$

since $\sin x = \frac{a}{c}$ $\qquad \cos x = \frac{b}{c}$

32. $\therefore \sin^2 x + \cos^2 x = 1$
33. $\sec^2 x = 1 + \tan^2 x$
34. $\sin (x + y) = \sin x \cos y + \cos x \sin y$
35. $\sin (x - y) = \sin x \cos y - \cos x \sin y$
36. $\cos (x + y) = \cos x \cos y - \sin x \sin y$
37. $\cos (x - y) = \cos x \cos y + \sin x \sin y$
38. $\tan (x + y) = \dfrac{\tan x + \tan y}{1 - \tan x \tan y}$
39. $\tan (x - y) = \dfrac{\tan x - \tan y}{1 + \tan x \tan y}$
40. $\sin 2x = 2 \sin x \cos x$
41. $\cos 2x = \cos^2 x - \sin^2 x$
42. $\tan 2x = \dfrac{2 \tan x}{1 - \tan^2 x}$
43.



Law of Sines.—In *any* triangle

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of Cosines.—In any triangle the square of any side is equal to the sum of the squares of the other two sides minus twice the product of these two sides into the cosine of their included angle.

That is:

44. $a^2 = b^2 + c^2 - 2bc \cos A$
45. $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$
46. $\cos B = \dfrac{c^2 + a^2 - b^2}{2ca}$
47. $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

APPENDIX E

MATHEMATICAL TABLES

Natural Sines and Cosines

NOTE.—For cosines use right-hand column of degrees and lower line of tenths

Deg.	°0.0	°0.1	°0.2	°0.3	°0.4	°0.5	°0.6	°0.7	°0.8	°0.9	
0°	0.0000	0.0017	0.0035	0.0052	0.0070	0.0087	0.0105	0.0122	0.0140	0.0157	89
1	0.0175	0.0192	0.0209	0.0227	0.0244	0.0262	0.0279	0.0297	0.0314	0.0332	88
2	0.0349	0.0366	0.0384	0.0401	0.0419	0.0436	0.0454	0.0471	0.0488	0.0506	87
3	0.0523	0.0541	0.0558	0.0576	0.0593	0.0610	0.0628	0.0645	0.0663	0.0680	86
4	0.0698	0.0715	0.0732	0.0750	0.0767	0.0785	0.0802	0.0819	0.0837	0.0854	85
5	0.0872	0.0889	0.0906	0.0924	0.0941	0.0958	0.0976	0.0993	0.1011	0.1028	84
6	0.1045	0.1063	0.1080	0.1097	0.1115	0.1132	0.1149	0.1167	0.1184	0.1201	83
7	0.1219	0.1236	0.1253	0.1271	0.1288	0.1305	0.1323	0.1340	0.1357	0.1374	82
8	0.1392	0.1409	0.1426	0.1444	0.1461	0.1478	0.1495	0.1513	0.1530	0.1547	81
9	0.1564	0.1582	0.1599	0.1616	0.1633	0.1650	0.1668	0.1685	0.1702	0.1719	80°
10°	0.1736	0.1754	0.1771	0.1788	0.1805	0.1822	0.1840	0.1857	0.1874	0.1891	79
11	0.1908	0.1925	0.1942	0.1959	0.1977	0.1994	0.2011	0.2028	0.2045	0.2062	78
12	0.2079	0.2096	0.2113	0.2130	0.2147	0.2164	0.2181	0.2198	0.2215	0.2232	77
13	0.2250	0.2267	0.2284	0.2300	0.2317	0.2334	0.2351	0.2368	0.2385	0.2402	76
14	0.2419	0.2436	0.2453	0.2470	0.2487	0.2504	0.2521	0.2538	0.2554	0.2571	75
15	0.2588	0.2605	0.2622	0.2639	0.2656	0.2672	0.2689	0.2706	0.2723	0.2740	74
16	0.2756	0.2773	0.2790	0.2807	0.2823	0.2840	0.2857	0.2874	0.2890	0.2907	73
17	0.2924	0.2940	0.2957	0.2974	0.2990	0.3007	0.3024	0.3040	0.3057	0.3074	72
18	0.3090	0.3107	0.3123	0.3140	0.3156	0.3173	0.3190	0.3206	0.3223	0.3239	71
19	0.3256	0.3272	0.3289	0.3305	0.3322	0.3338	0.3355	0.3371	0.3387	0.3404	70°
20°	0.3420	0.3437	0.3453	0.3469	0.3486	0.3502	0.3518	0.3535	0.3551	0.3567	69
21	0.3584	0.3600	0.3616	0.3633	0.3649	0.3665	0.3681	0.3697	0.3714	0.3730	68
22	0.3746	0.3762	0.3778	0.3795	0.3811	0.3827	0.3843	0.3859	0.3875	0.3891	67
23	0.3907	0.3923	0.3939	0.3955	0.3971	0.3987	0.4003	0.4019	0.4035	0.4051	66
24	0.4067	0.4083	0.4099	0.4115	0.4131	0.4147	0.4163	0.4179	0.4195	0.4210	65
25	0.4226	0.4242	0.4258	0.4274	0.4289	0.4305	0.4321	0.4337	0.4352	0.4368	64
26	0.4384	0.4399	0.4415	0.4431	0.4446	0.4462	0.4478	0.4493	0.4509	0.4524	63
27	0.4540	0.4555	0.4571	0.4586	0.4602	0.4617	0.4633	0.4648	0.4664	0.4679	62
28	0.4695	0.4710	0.4726	0.4741	0.4756	0.4772	0.4787	0.4802	0.4818	0.4833	61
29	0.4848	0.4863	0.4879	0.4894	0.4909	0.4924	0.4939	0.4955	0.4970	0.4985	60°
30°	0.5000	0.5015	0.5030	0.5045	0.5060	0.5075	0.5090	0.5105	0.5120	0.5135	59
31	0.5150	0.5165	0.5180	0.5195	0.5210	0.5225	0.5240	0.5255	0.5270	0.5284	58
32	0.5299	0.5314	0.5329	0.5344	0.5358	0.5373	0.5388	0.5402	0.5417	0.5432	57
33	0.5446	0.5461	0.5476	0.5490	0.5505	0.5519	0.5534	0.5548	0.5563	0.5577	56
34	0.5592	0.5606	0.5621	0.5635	0.5650	0.5664	0.5678	0.5693	0.5707	0.5721	55
35	0.5736	0.5750	0.5764	0.5779	0.5793	0.5807	0.5821	0.5835	0.5850	0.5864	54
36	0.5878	0.5892	0.5906	0.5920	0.5934	0.5948	0.5962	0.5976	0.5990	0.6004	53
37	0.6018	0.6032	0.6046	0.6060	0.6074	0.6088	0.6101	0.6115	0.6129	0.6143	52
38	0.6157	0.6170	0.6184	0.6198	0.6211	0.6225	0.6239	0.6252	0.6266	0.6280	51
39	0.6293	0.6307	0.6320	0.6334	0.6347	0.6361	0.6374	0.6388	0.6401	0.6414	50°
40°	0.6428	0.6441	0.6455	0.6468	0.6481	0.6494	0.6508	0.6521	0.6534	0.6547	49
41	0.6561	0.6574	0.6587	0.6600	0.6613	0.6626	0.6639	0.6652	0.6665	0.6678	48
42	0.6691	0.6704	0.6717	0.6730	0.6743	0.6756	0.6769	0.6782	0.6794	0.6807	47
43	0.6820	0.6833	0.6845	0.6858	0.6871	0.6884	0.6896	0.6909	0.6921	0.6934	46
44	0.6947	0.6959	0.6972	0.6984	0.6997	0.7009	0.7022	0.7034	0.7046	0.7059	45
	°1.0	°0.9	°0.8	°0.7	°0.6	°0.5	°0.4	°0.3	°0.2	°0.1	Deg.

Natural Sines and Cosines.—*Concluded*

Deg.	°0.0	°0.1	°0.2	°0.3	°0.4	°0.5	°0.6	°0.7	°0.8	°0.9	
45	0.7071	0.7083	0.7096	0.7108	0.7120	0.7133	0.7145	0.7157	0.7169	0.7181	44
46	0.7193	0.7206	0.7218	0.7230	0.7242	0.7254	0.7266	0.7278	0.7290	0.7302	43
47	0.7314	0.7325	0.7337	0.7349	0.7361	0.7373	0.7385	0.7396	0.7408	0.7420	42
48	0.7431	0.7443	0.7455	0.7466	0.7478	0.7490	0.7501	0.7513	0.7524	0.7536	41
49	0.7547	0.7559	0.7570	0.7581	0.7593	0.7604	0.7615	0.7627	0.7638	0.7649	40°
50°	0.7660	0.7672	0.7683	0.7694	0.7705	0.7716	9.7727	0.7738	0.7749	0.7760	39
51	0.7771	0.7782	0.7793	0.7804	0.7815	0.7826	0.7837	0.7848	0.7859	0.7869	38
52	0.7880	0.7891	0.7902	0.7912	0.7923	0.7934	0.7944	0.7955	0.7965	0.7976	37
53	0.7986	0.7997	0.8007	0.8018	0.8028	0.8039	0.8049	0.8059	0.8070	0.8080	36
54	0.8090	0.8100	0.8111	0.8121	0.8131	0.8141	0.8151	0.8161	0.8171	0.8181	35
55	0.8192	0.8202	0.8211	0.8221	0.8231	0.8241	0.8251	0.8261	0.8271	0.8281	34
56	0.8290	0.8300	0.8310	0.8320	0.8329	0.8339	0.8348	0.8358	0.8368	0.8377	33
57	0.8387	0.8396	0.8406	0.8415	0.8425	0.8434	0.8443	0.8453	0.8462	0.8471	32
58	0.8480	0.8490	0.8499	0.8508	0.8517	0.8526	0.8536	0.8545	0.8554	0.8563	31
59	0.8572	0.8581	0.8590	0.8599	0.8607	0.8616	0.8625	0.8634	0.8643	0.8652	30°
60°	0.8660	0.8669	0.8678	0.8686	0.8695	0.8704	0.8712	0.8721	0.8729	0.8738	29
61	0.8746	0.8755	0.8763	0.8771	0.8780	0.8788	0.8796	0.8805	0.8813	0.8821	28
62	0.8829	0.8838	0.8846	0.8854	0.8862	0.8870	0.8878	0.8886	0.8894	0.8902	27
63	0.8910	0.8918	0.8926	0.8934	0.8942	0.8949	0.8957	0.8965	0.8973	0.8980	26
64	0.8988	0.8996	0.9003	0.9011	0.9018	0.9026	0.9033	0.9041	0.9048	0.9056	25
65	0.9063	0.9070	0.9078	0.9085	0.9092	0.9100	0.9107	0.9114	0.9121	0.9128	24
66	0.9135	0.9143	0.9150	0.9157	0.9164	0.9171	0.9178	0.9184	0.9191	0.9198	23
67	0.9205	0.9212	0.9219	0.9225	0.9232	0.9239	0.9245	0.9252	0.9259	0.9265	22
68	0.9272	0.9278	0.9285	0.9291	0.9298	0.9304	0.9311	0.9317	0.9323	0.9330	21
69	0.9336	0.9342	0.9348	0.9354	0.9361	0.9367	0.9373	0.9379	0.9385	0.9391	20°
70°	0.9397	0.9403	0.9409	0.9415	0.9421	0.9426	0.9432	0.9438	0.9444	0.9449	19
71	0.9455	0.9461	0.9466	0.9472	0.9478	0.9483	0.9489	0.9494	0.9500	0.9505	18
72	0.9511	0.9516	0.9521	0.9527	0.9532	0.9537	0.9542	0.9548	0.9553	0.9558	17
73	0.9563	0.9568	0.9573	0.9578	0.9583	0.9588	0.9593	0.9598	0.9603	0.9608	16
74	0.9613	0.9617	0.9622	0.9627	0.9632	0.9636	0.9641	0.9646	0.9650	0.9655	15
75	0.9659	0.9664	0.9668	0.9673	0.9677	0.9681	0.9686	0.9690	0.9694	0.9699	14
76	0.9703	0.9707	0.9711	0.9715	0.9720	0.9724	0.9728	0.9732	0.9736	0.9740	13
77	0.9744	0.9748	0.9751	0.9755	0.9759	0.9763	0.9767	0.9770	0.9774	0.9778	12
78	0.9781	0.9785	0.9789	0.9792	0.9796	0.9799	0.9803	0.9806	0.9810	0.9813	11
79	0.9816	0.9820	0.9823	0.9826	0.9829	0.9833	0.9836	0.9839	0.9842	0.9845	10°
80°	0.9848	0.9851	0.9854	0.9857	0.9860	0.9863	0.9866	0.9869	0.9871	0.9874	9
81	0.9877	0.9880	0.9882	0.9885	0.9888	0.9890	0.9893	0.9895	0.9898	0.9900	8
82	0.9903	0.9905	0.9907	0.9910	0.9912	0.9914	0.9917	0.9919	0.9921	0.9923	7
83	0.9925	0.9928	0.9930	0.9932	0.9934	0.9936	0.9938	0.9940	0.9942	0.9943	6
84	0.9945	0.9947	0.9949	0.9951	0.9952	0.9954	0.9956	0.9957	0.9959	0.9960	5
85	0.9962	0.9963	0.9965	0.9966	0.9968	0.9969	0.9971	0.9972	0.9973	0.9974	4
86	0.9976	0.9977	0.9978	0.9979	0.9980	0.9981	0.9982	0.9983	0.9984	0.9985	3
87	0.9986	0.9987	0.9988	0.9989	0.9990	0.9990	0.9991	0.9992	0.9993	0.9993	2
88	0.9994	0.9995	0.9995	0.9996	0.9996	0.9997	0.9997	0.9997	0.9998	0.9998	1
89	0.9998	0.9999	0.9999	0.9999	0.9999	1.000	1.000	1.000	1.000	1.000	0°
	°1.0	°0.9	°0.8	°0.7	°0.6	°0.5	°0.4	°0.3	°0.2	°0.1	Deg.

APPENDIX F

Natural Tangents and Cotangents

NOTE.—For cotangents use right-hand column of degrees and lower line of tenths

Deg.	°0.0	°0.1	°0.2	°0.3	°0.4	°0.5	°0.6	°0.7	°0.8	°0.9	
0°	0.0000	0.0017	0.0035	0.0052	0.0070	0.0087	0.0105	0.0122	0.0140	0.0157	89
1	0.0175	0.0192	0.0209	0.0227	0.0244	0.0262	0.0279	0.0297	0.0314	0.0332	88
2	0.0349	0.0367	0.0384	0.0402	0.0419	0.0437	0.0454	0.0472	0.0489	0.0507	87
3	0.0524	0.0542	0.0559	0.0577	0.0594	0.0612	0.0629	0.0647	0.0664	0.0682	86
4	0.0699	0.0717	0.0734	0.0752	0.0769	0.0787	0.0805	0.0822	0.0840	0.0857	85
5	0.0875	0.0892	0.0910	0.0928	0.0945	0.0963	0.0981	0.0998	0.1016	0.1033	84
6	0.1051	0.1069	0.1086	0.1104	0.1122	0.1139	0.1157	0.1175	0.1192	0.1210	83
7	0.1228	0.1246	0.1263	0.1281	0.1299	0.1317	0.1334	0.1352	0.1370	0.1388	82
8	0.1405	0.1423	0.1441	0.1459	0.1477	0.1495	0.1512	0.1530	0.1548	0.1566	81
9	0.1584	0.1602	0.1620	0.1638	0.1655	0.1673	0.1691	0.1709	0.1727	0.1745	80°
10°	0.1763	0.1781	0.1799	0.1817	0.1835	0.1853	0.1871	0.1890	0.1908	0.1926	79
11	0.1944	0.1962	0.1980	0.1998	0.2016	0.2035	0.2053	0.2071	0.2089	0.2107	78
12	0.2126	0.2144	0.2162	0.2180	0.2199	0.2217	0.2235	0.2254	0.2272	0.2290	77
13	0.2309	0.2327	0.2345	0.2364	0.2382	0.2401	0.2419	0.2438	0.2456	0.2475	76
14	0.2493	0.2512	0.2530	0.2549	0.2568	0.2586	0.2605	0.2623	0.2642	0.2661	75
15	0.2679	0.2698	0.2717	0.2736	0.2754	0.2773	0.2792	0.2811	0.2830	0.2849	74
16	0.2867	0.2886	0.2905	0.2924	0.2943	0.2962	0.2981	0.3000	0.3019	0.3038	73
17	0.3057	0.3076	0.3096	0.3115	0.3134	0.3153	0.3172	0.3191	0.3211	0.3230	72
18	0.3249	0.3269	0.3288	0.3307	0.3327	0.3346	0.3365	0.3385	0.3404	0.3424	71
19	0.3443	0.3463	0.3482	0.3502	0.3522	0.3541	0.3561	0.3581	0.3600	0.3620	70°
20°	0.3640	0.3659	0.3679	0.3699	0.3719	0.3739	0.3759	0.3779	0.3799	0.3819	69
21	0.3839	0.3859	0.3879	0.3899	0.3919	0.3939	0.3959	0.3979	0.4000	0.4020	68
22	0.4040	0.4061	0.4081	0.4101	0.4122	0.4142	0.4163	0.4183	0.4204	0.4224	67
23	0.4245	0.4265	0.4286	0.4307	0.4327	0.4348	0.4369	0.4390	0.4411	0.4431	66
24	0.4452	0.4473	0.4494	0.4515	0.4536	0.4557	0.4578	0.4599	0.4621	0.4642	65
25	0.4663	0.4684	0.4706	0.4727	0.4748	0.4770	0.4791	0.4813	0.4834	0.4856	64
26	0.4877	0.4899	0.4921	0.4942	0.4964	0.4986	0.5008	0.5029	0.5051	0.5073	63
27	0.5095	0.5117	0.5139	0.5161	0.5184	0.5206	0.5228	0.5250	0.5272	0.5295	62
28	0.5317	0.5340	0.5362	0.5384	0.5407	0.5430	0.5452	0.5475	0.5498	0.5520	61
29	0.5543	0.5566	0.5589	0.5612	0.5635	0.5658	0.5681	0.5704	0.5727	0.5750	60°
30°	0.5774	0.5797	0.5820	0.5844	0.5867	0.5890	0.5914	0.5938	0.5961	0.5985	59
31	0.6009	0.6032	0.6056	0.6080	0.6104	0.6128	0.6152	0.6176	0.6200	0.6224	58
32	0.6249	0.6273	0.6297	0.6322	0.6346	0.6371	0.6395	0.6420	0.6445	0.6469	57
33	0.6494	0.6519	0.6544	0.6569	0.6594	0.6619	0.6644	0.6669	0.6694	0.6720	56
34	0.6745	0.6771	0.6796	0.6822	0.6847	0.6873	0.6899	0.6924	0.6950	0.6976	55
35	0.7002	0.7028	0.7054	0.7080	0.7107	0.7133	0.7159	0.7186	0.7212	0.7239	54
36	0.7265	0.7292	0.7319	0.7346	0.7373	0.7400	0.7427	0.7454	0.7481	0.7508	53
37	0.7536	0.7563	0.7590	0.7618	0.7646	0.7673	0.7701	0.7729	0.7757	0.7785	52
38	0.7813	0.7841	0.7869	0.7898	0.7926	0.7954	0.7983	0.8012	0.8040	0.8069	51
39	0.8098	0.8127	0.8156	0.8185	0.8214	0.8243	0.8273	0.8302	0.8332	0.8361	50°
40°	0.8391	0.8421	0.8451	0.8481	0.8511	0.8541	0.8571	0.8601	0.8632	0.8662	49
41	0.8693	0.8724	0.8754	0.8785	0.8816	0.8847	0.8878	0.8910	0.8941	0.8972	48
42	0.9004	0.9036	0.9067	0.9099	0.9131	0.9163	0.9195	0.9228	0.9260	0.9293	47
43	0.9325	0.9358	0.9391	0.9424	0.9457	0.9490	0.9523	0.9556	0.9590	0.9623	46
44	0.9657	0.9691	0.9725	0.9759	0.9793	0.9827	0.9861	0.9896	0.9930	0.9965	45
	°1.0	°0.9	°0.8	°0.7	°0.6	°0.5	°0.4	°0.3	°0.2	°0.1	Deg.

Natural Tangents and Cotangents—*Concluded*

Deg.	°0.0	°0.1	°0.2	°0.3	°0.4	°0.5	°0.6	°0.7	°0.8	°0.9	
45	1.0000	1.0035	1.0070	1.0105	1.0141	1.0176	1.0212	1.0247	1.0283	1.0319	44
46	1.0355	1.0392	1.0428	1.0464	1.0501	1.0538	1.0575	1.0612	1.0649	1.0686	43
47	1.0724	1.0761	1.0799	1.0837	1.0875	1.0913	1.0951	1.0990	1.1028	1.1067	42
48	1.1106	1.1145	1.1184	1.1224	1.1263	1.1303	1.1343	1.1383	1.1423	1.1463	41
49	1.1504	1.1544	1.1585	1.1626	1.1667	1.1708	1.1750	1.1792	1.1833	1.1875	40°
50°	1.1918	1.1960	1.2002	1.2045	1.2088	1.2131	1.2174	1.2218	1.2261	1.2305	39
51	1.2349	1.2393	1.2437	1.2482	1.2527	1.2572	1.2617	1.2662	1.2708	1.2753	38
52	1.2799	1.2846	1.2892	1.2938	1.3985	1.3032	1.3079	1.3127	1.3175	1.3222	37
53	1.3270	1.3319	1.3367	1.3416	1.3465	1.3514	1.3564	1.3613	1.3663	1.3713	36
54	1.3764	1.3814	1.3865	1.3916	1.3968	1.4019	1.4071	1.4124	1.4176	1.4229	35
55	1.4281	1.4335	1.4388	1.4442	1.4496	1.4550	1.4605	1.4659	1.4715	1.4770	34
56	1.4826	1.4882	1.4938	1.4994	1.5051	1.5108	1.5166	1.5224	1.5282	1.5340	33
57	1.5399	1.5458	1.5517	1.5577	1.5637	1.5697	1.5757	1.5818	1.5880	1.5941	32
58	1.6003	1.6066	1.6128	1.6191	1.6255	1.6319	1.6383	1.6447	1.6512	1.6577	31
59	1.6643	1.6709	1.6775	1.6842	1.6909	1.6977	1.7045	1.7113	1.7182	1.7251	30°
60°	1.7321	1.7391	1.7461	1.7532	1.7603	1.7675	1.7747	1.7820	1.7893	1.7966	29
61	1.8040	1.8115	1.8190	1.8265	1.8341	1.8418	1.8495	1.8572	1.8650	1.8728	28
62	1.8807	1.8887	1.8967	1.9047	1.9128	1.9210	1.9292	1.9375	1.9458	1.9542	27
63	1.9626	1.9711	1.9797	1.9883	1.9970	2.0057	2.0145	2.0233	2.0323	2.0413	26
64	2.0503	2.0594	2.0686	2.0778	2.0872	2.0965	2.1060	2.1155	2.1251	2.1348	25
65	2.1445	2.1543	2.1642	2.1742	2.1842	2.1943	2.2045	2.2148	2.2251	2.2355	24
66	2.2460	2.2566	2.2673	2.2781	3.2889	2.2998	2.3109	2.3220	2.3332	2.3445	23
67	2.3559	2.3673	2.3789	2.3906	2.4023	2.4142	2.4262	2.4383	2.4504	2.4627	22
68	2.4751	2.4876	2.5002	2.5129	2.5257	2.5386	2.5517	2.5649	2.5782	2.5916	21
69	2.6051	2.6187	2.6325	2.6464	2.6605	2.6746	2.6889	2.7034	2.7179	2.7326	20°
70°	2.7475	2.7625	2.7776	2.7929	2.8083	2.8239	2.8397	2.8556	2.8716	2.8878	19
71	2.9042	2.9208	2.9375	2.9544	2.9714	2.9887	3.0061	3.0237	2.0415	3.0595	18
72	3.0777	3.0961	3.1146	3.1334	3.1524	2.1716	3.1910	3.2106	3.2305	3.2506	17
73	3.2709	3.2914	3.3122	3.3332	3.3544	3.3759	3.3977	3.4197	3.4420	3.4646	16
74	3.4874	3.5105	3.5339	3.5576	3.5816	3.6059	3.6305	3.6554	3.6806	3.7062	15
75	3.7321	3.7583	3.7848	3.8118	3.8391	3.8667	3.8947	3.9232	3.9520	3.9812	14
76	4.0108	4.0408	4.0713	4.1022	4.1335	4.1653	4.1976	4.2303	4.2635	4.2972	13
77	4.3315	4.3662	4.4015	4.4374	4.4737	4.5107	4.5483	4.5864	4.6252	4.6646	12
78	4.7046	4.7453	4.7867	4.8288	4.8716	4.9152	4.9594	5.0045	5.0504	5.0970	11
79	5.1446	5.1929	5.2422	5.2924	5.3435	5.3955	5.4486	5.5026	5.5578	5.6140	10°
80°	5.6713	5.7297	5.7894	5.8502	5.9124	5.9758	6.0405	6.1066	6.1742	6.2432	9
81	6.3138	6.3859	6.4596	6.5350	6.6122	6.6912	6.7720	6.8548	6.9395	7.0264	8
82	7.1154	7.2066	7.3002	7.3962	7.4947	7.5958	7.6996	7.8062	7.9158	8.0285	7
83	8.1443	8.2636	8.3863	8.5126	8.6427	8.7769	8.9152	9.0579	9.2052	9.3572	6
84	9.5144	9.677	9.845	10.02	10.20	10.39	10.58	10.78	10.99	11.20	5
85	11.43	11.66	11.91	12.16	12.43	12.71	13.00	13.30	13.62	13.95	4
86	14.30	14.67	15.06	15.46	15.89	16.35	16.83	17.34	17.89	18.46	3
87	19.08	19.74	20.45	21.20	22.02	22.90	23.86	24.90	26.03	27.27	2
88	28.64	30.14	31.82	33.69	35.80	38.19	40.92	44.07	47.74	52.08	1
89	57.29	63.66	71.62	81.85	95.49	114.6	143.2	191.0	286.5	573.0	0°
	°1.0	°0.9	°0.8	°0.7	°0.6	°0.5	°0.4	°0.3	°0.2	°0.1	Deg.

APPENDIX G

Logarithms of Numbers

N	0	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396

Logarithms of Numbers.—*Concluded*

N	0	1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745
75	8751	8456	8762	8768	8774	8779	8785	8791	8797	8802
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996

APPENDIX H

Resistance of Copper Wire, Ohms per Mile 25°C. (77°F.)

Size, cir. mils A.W.G.	Number of wires	Outside diam., mils	Ohms per mile
STRANDED			
500,000	37	814	0.1130
450,000	37	772	0.1267
400,000	37	728	0.1426
350,000	37	681	0.1626
300,000	37	630	0.1900
250,000	37	575	0.2278
0000	19	528	0.2690
000	19	470	0.339
00	19	418	0.428
0	19	373	0.538
1	19	332	0.681
2	7	292	0.856
3	7	260	1.083
4	7	232	1.367
SOLID			
0000		460	0.264
000		410	0.333
00		365	0.420
0		325	0.528
1		289	0.665
2		258	0.839
3		229	1.061
4		204	1.335

For more detailed tables, see Vol. I, pp. 49 and 50.

Resistance of Aluminum Cable Steel Reinforced (A.C.S.R.) Bare

Aluminum area		Copper equivalent circular mils	Diameter, inches		Ohms per mile, 20° C.
Circular mils	Square inches		Complete cable	Steel core	
1,590,000	1.249	1,000,000	1.544	0.515	0.0576
1,431,000	1.124	900,000	1.465	0.488	0.0639
1,272,000	0.999	800,000	1.382	0.461	0.0713
1,113,000	0.874	700,000	1.292	0.431	0.0818
1,033,500	0.812	650,000	1.246	0.415	0.0882
954,000	0.749	600,000	1.196	0.399	0.0956
874,500	0.687	550,000	1.146	0.382	0.1040
795,000	0.624	500,000	1.093	0.364	0.1146
715,500	0.562	450,000	1.036	0.345	0.1272
636,000	0.500	400,000	0.977	0.326	0.1431
*556,000	0.437	350,000	0.953	0.409	0.1642
*500,000	0.393	314,500	0.904	0.387	0.1832

Aluminum, 54 strands; steel, 7 strands.
* Aluminum, 30 strands.

APPENDIX I

Inductive Reactance per Single Conductor, Ohms per Mile[1]

Size cir. mils A.W.G.	60 cycles per sec.												
	Spacing, in.												
	12	24	36	48	60	72	84	96	108	120	132	144	156
Stranded													
500,000	0.451	0.535	0.584	0.619	0.647	0.669	0.688	0.703	0.718	0.730	0.742	0.752	0.762
450,000	0.458	0.541	0.591	0.625	0.653	0.675	0.693	0.709	0.724	0.736	0.748	0.758	0.767
400,000	0.464	0.548	0.598	0.632	0.660	0.682	0.700	0.716	0.731	0.743	0.755	0.765	0.775
350,000	0.472	0.556	0.606	0.640	0.668	0.690	0.708	0.724	0.739	0.751	0.763	0.774	0.783
300,000	0.482	0.566	0.615	0.650	0.677	0.699	0.718	0.734	0.748	0.760	0.772	0.783	0.792
250,000	0.493	0.577	0.626	0.661	0.688	0.711	0.729	0.745	0.759	0.772	0.783	0.794	0.804
0000	0.503	0.587	0.636	0.672	0.698	0.722	0.739	0.755	0.770	0.782	0.793	0.804	0.814
000	0.517	0.601	0.650	0.685	0.713	0.735	0.754	0.769	0.784	0.796	0.808	0.818	0.828
00	0.531	0.615	0.664	0.699	0.726	0.748	0.767	0.782	0.798	0.810	0.822	0.832	0.842
0	0.546	0.629	0.678	0.714	0.740	0.762	0.781	0.797	0.812	0.824	0.836	0.846	0.856
Solid													
0000	0.510	0.594	0.642	0.677	0.704	0.726	0.746	0.762	0.776	0.788	0.800	0.810	0.820
000	0.524	0.608	0.656	0.692	0.718	0.740	0.760	0.776	0.790	0.802	0.814	0.824	0.834
00	0.538	0.622	0.670	0.706	0.732	0.754	0.774	0.790	0.804	0.816	0.828	0.838	0.848
0	0.552	0.636	0.684	0.720	0.746	0.768	0.788	0.804	0.818	0.830	0.842	0.852	0.862
1	0.566	0.649	0.698	0.734	0.760	0.782	0.802	0.818	0.832	0.844	0.856	0.866	0.876
2	0.580	0.664	0.712	0.748	0.774	0.796	0.816	0.832	0.846	0.858	0.870	0.880	0.890
3	0.594	0.678	0.726	0.762	0.788	0.810	0.829	0.846	0.860	0.872	0.884	0.894	0.904
4	0.608	0.692	0.740	0.776	0.803	0.824	0.843	0.860	0.874	0.886	0.898	0.908	0.918
5	0.622	0.706	0.754	0.790	0.817	0.838	0.858	0.874	0.888	0.900	0.912	0.922	0.932
6	0.636	0.720	0.768	0.804	0.831	0.853	0.872	0.888	0.902	0.915	0.926	0.936	0.946

[1]From formula $x = 2\pi f\left(80 + 741.1 \log \frac{D}{r}\right)10^{-6}$.

APPENDIX J

Charging Current per Single Wire, Amperes per Mile per 100,000 Volts from Phase Wire to Neutral[1]

STRANDED

Size cir. mils A.W.G.	60 cycles per sec.												
	Spacing, in.												
	12	24	36	48	60	72	84	96	108	120	132	144	156
500,000	1.01	0.830	0.755	0.709	0.676	0.653	0.634	0.615	0.605	0.593	0.582	0.574	0.567
450,000	0.989	0.819	0.746	0.700	0.668	0.644	0.626	0.609	0.599	0.588	0.577	0.569	0.561
400,000	0.[illegible]70	0.808	0.734	0.691	0.661	0.636	0.618	0.602	0.593	0.582	0.572	0.562	0.555
350,000	0.951	0.796	0.725	0.681	0.651	0.628	0.610	0.595	0.585	0.575	0.565	0.556	0.548
300,000	0.932	0.780	0.711	0.671	0.641	0.619	0.601	0.588	0.577	0.568	0.557	0.549	0.541
250,000	0.904	0.765	0.698	0.659	0.631	0.610	0.592	0.581	0.568	0.559	0.550	0.542	0.534
0000	0.887	0.749	0.686	0.646	0.620	0.600	0.583	0.572	0.560	0.549	0.542	0.534	0.527
000	0.857	0.730	0.671	0.633	0.608	0.590	0.572	0.562	0.550	0.539	0.532	0.524	0.520
00	0.836	0.712	0.656	0.620	0.595	0.579	0.562	0.550	0.539	0.529	0.523	0.514	0.510
0	0.813	0.695	0.640	0.608	0.583	0.568	0.551	0.539	0.529	0.521	0.514	0.505	0.502
SOLID													
0000	0.852	0.725	0.667	0.631	0.606	0.587	0.571	0.559	0.548	0.539	0.531	0.523	0.517
000	0.828	0.708	0.652	0.618	0.593	0.575	0.560	0.548	0.538	0.529	0.521	0.514	0.508
00	0.805	0.691	0.638	0.605	0.582	0.564	0.550	0.538	0.528	0.519	0.512	0.505	0.499
0	0.783	0.675	0.624	0.593	0.570	0.553	0.540	0.528	0.519	0.510	0.503	0.497	0.491
1	0.763	0.660	0.611	0.581	0.559	0.543	0.530	0.519	0.510	0.502	0.495	0.488	0.483
2	0.743	0.645	0.598	0.569	0.549	0.533	0.520	0.510	0.501	0.493	0.486	0.480	0.475
3	0.725	0.631	0.586	0.558	0.539	0.523	0.511	0.501	0.492	0.485	0.478	0.472	0.467
4	0.707	0.618	0.575	0.548	0.529	0.514	0.502	0.492	0.484	0.477	0.470	0.465	0.460
5	0.690	0.605	0.564	0.538	0.519	0.505	0.494	0.484	0.476	0.469	0.463	0.458	0.453
6	0.674	0.592	0.553	0.528	0.510	0.497	0.485	0.476	0.468	0.462	0.456	0.450	0.446

[1]From formula $I = \dfrac{2\pi f \times 38.83 \times 10^{-9} E}{\log_{10} \dfrac{D}{r}}$

QUESTIONS ON CHAPTER I

1. State briefly some of the advantages of direct over alternating current for industrial purposes. Which type of power supply is best adapted to traction purposes? Why is direct current necessary for electrolytic work?

2. In spite of the many advantages of direct current, why is a large percentage of power at the present time generated as alternating current? Name some secondary reasons for generating power as alternating current.

3. How does the weight of transmission conductor vary with the transmission voltage? Give reasons why it may be more economical to generate power in large quantities and transmit it over expensive transmission systems rather than to generate it at the point of use.

4. What is meant by a *sine wave?* Discuss the wave form of commercial alternators. Why are sine waves assumed in making alternating-current calculations?

5. Describe a graphical method of producing a sine wave. Show how such a wave may be plotted by the use of sine tables.

6. Through how many space-degrees must a coil, rotating in a bipolar field, turn before one cycle is completed? Under these conditions, what is the relation of the space-degree to the electrical space-degree? What is meant by an *alternation?*

7. In a four-pole machine, through how many space-degrees must a coil turn before a cycle is completed? Why? In this case, what is the relation between electrical space-degrees and space-degrees? How fast in r.p.s. must such a coil rotate in order to produce a frequency of 60 cycles per second? in r.p.m.?

8. What are two advantages of the higher frequencies for commercial generation and utilization? Name two distinct disadvantages of the higher frequencies.

9. Why is either 50 or 60 cycles per second usually chosen as the system frequency when a power company supplies both lighting and power loads? Under what conditions is 25 cycles used? What is the advantage of this frequency over 60 cycles?

10. What is the average value of an alternating-current wave over one complete cycle? Upon what basis is the value of an alternating-current ampere determined? Define an *alternating-current ampere.*

11. How does the heating effect of a current vary with the current? How does the squared current sine wave compare with the original current sine wave as regards frequency, maximum value, and its axis of symmetry? What is the ratio of the maximum to the effective value of a sine wave? What is the ratio of effective to average for a half-cycle, and what is this ratio called?

12. Compare 1 ohm resistance for alternating current with 1 ohm resistance for direct current. How is an *alternating-current volt* defined?

13. State a simple trigonometric expression which gives the variation with time of a sine-wave current.

From this expression, derive the trigonometric expression for the current squared wave. Analyze this wave with particular reference to its frequency and axis of symmetry.

14. Define a *scalar quantity;* a *vector quantity.* How are vectors represented? How are they added? What is meant by the *parallelogram of forces?* The *triangle of forces?* How are vectors subtracted from one another?

15. What is meant by a current and a voltage's being in phase with each other? In what terms is phase difference expressed? A certain wave crosses the zero axis in a positive direction to the right of another wave. Is the first wave lagging or leading the other? Explain.

16. If two current waves are plotted, how can the sum of the currents be determined? If two currents are in phase, how is their sum found?

17. Show that the sum of two current waves is not necessarily equal to their algebraic sum.

18. Demonstrate the method of producing a sine wave by means of a rotating vector. How is the value of the wave determined at any instant? What is the relation of the speed of the rotating vector to the circuit frequency?

19. If two current waves differ in phase by a certain angle, what is the relation existing between the vectors which produce these waves? Illustrate with sketches.

20. Describe the most fundamental method of adding two currents, showing how the resultant current is determined.

21. What relation exists between the resultant wave and the vector sum of the rotating vectors? Show that this offers a ready method for adding alternating currents or voltages. Why may vectors representing effective values be used as well as vectors representing maximum values?

PROBLEMS ON CHAPTER I

1. An alternating current has an effective value of 35.35 amp., making its maximum value 50 amp. Draw this wave to scale by the method of Fig. 2 (p. 4), and also construct this wave from a table of sines (see p. 536). Indicate the effective and average values of this wave.

2. Find the instantaneous values of the current in Prob. 1 for angles of 30, 60, 75, 270, and 290 degrees using a table of sines. If the frequency of this wave is 50 cycles per second, to what values of time do the above angles correspond, assuming that the time is zero when the wave crosses the axis in a positive direction?

3. An alternating voltage, following a sine law, has a maximum value of 162.6 volts and a frequency of 60 cycles per second. What is the value of this voltage 0.001, 0.004, and 0.01 sec. after crossing the zero axis in a

positive direction? Determine the angles corresponding to each of these values of time.

4. A 60-cycle alternating current has a maximum instantaneous value of 28.28 amp. It crosses the zero axis in a positive direction when time is zero. (*a*) What time has elapsed when the current first reaches a value of 9.0 amp.? (*b*) When the current, after having gone through its maximum positive value, reaches a value of 24.0 amp.?

5. What is the value of the current (Prob. 4) when the time is $\frac{1}{75}$ sec.? At what value of time does the current first reach a negative value of 15 amp.?

6. A 50-cycle waterwheel generator has a speed of 500 r.p.m. How many poles has this generator? How many electrical space-degrees correspond to 1 actual space-degree?

7. What is the speed in r.p.m. of a 60-cycle, 10-pole, turbo-driven alternator? How many electrical space-degrees correspond to 1 actual space-degree?

8. What is the frequency of a 750-r.p.m., six-pole alternator? Of a 75-r.p.m., 96-pole, engine-driven alternator?

9. A current is given by $i = 2.4 \sin 377t$. Determine: (*a*) its maximum value; (*b*) its effective value; (*c*) its frequency; (*d*) the number of radians through which its vector has gone when $t = 0.008$ sec.; (*e*) the number of degrees through which it has gone; (*f*) the value of the current at this instant.

10. Plot the squared values of the current wave of Prob. 1, using a much smaller scale for the squared values. (*a*) What is the frequency of the squared wave? (*b*) What distance in amperes is its axis above the original axis? (*c*) Determine graphically the average of this squared wave. (*d*) What is the square root of this average value? (*e*) What relation exists between (*d*) and the maximum value of the current wave?

11. (*a*) Determine the average value of the current wave (Prob. 9) over a half-cycle. (*b*) Find the ratio of this average value to the effective value determined in 9(*b*).

12. Determine the average value of the wave (Prob. 10) over a half-cycle. Compare this average value with the effective value determined in (*d*).

13. A direct current of 3.4 amp. flows through a 14-ohm, non-inductive resistance. (*a*) What is the maximum value of an alternating current which will produce heat at the same rate in this resistance? (*b*) What is the maximum rate at which this alternating current dissipates energy? (*c*) At what average rate does it dissipate energy?

14. A certain direct-current series-arc system is not guaranteed to operate safely at any voltage in excess of 10,000 volts between wires. This system is later supplied with alternating current by a constant-current transformer. What is the effective value of the highest alternating e.m.f. that can be safely used?

15. Derive the trigonometric expression for the squared current wave (Prob. 9), expressing this wave as a cosine function of its frequency doubled. What is the distance in amperes of its axis of symmetry above the axis of

reference? What is the value of the ordinate of the squared curve when the time is equal to $\frac{1}{360}$ sec.? $\frac{3}{320}$ sec.?

16. A current is expressed by $i = 15 \sin 251.3\, t$. Express the squared value of this current as a cosine function of the frequency doubled. Give the maximum value of this squared current wave and the distance in amperes of its axis of symmetry above the axis of reference. At what value of time does this wave first reach a value of 64 $\overline{\text{amp.}}^2$? 120 $\overline{\text{amp.}}^2$? 200 $\overline{\text{amp.}}^2$? For what values of time in the initial cycle does this squared wave reach values of 112.5 $\overline{\text{amp.}}^2$?

17. Sketch a current and a voltage wave having effective values of 14 amp. and 120 volts, respectively, the current lagging the voltage by 45°. If the voltage wave is passing through zero in a positive direction at 1:00 o'clock, at which time will the current wave be going through zero in a positive direction? The frequency is 60 cycles per second.

18. Plot two waves corresponding to two 50-cycle currents having effective values of 40 and 30 amp., respectively, these two currents differing in phase by 90°, the 30-amp. current lagging. Add the ordinates of these two waves point by point and plot the resulting wave. (*a*) What is its maximum value? (*b*) What is its effective value? (*c*) What result is obtained by adding two current vectors of 40 and 30 amp. laid off at right angles? (*d*) What is the angle between the resultant wave and the 40-amp. wave?

19. The radius vector (see Par. 12) corresponding to the 40-amp. wave (Prob. 18) makes an angle of 0° with the axis when the time $t = 0$. (*a*) Through how many degrees must it advance in order that the value of the 40-amp. wave may be equal to the negative value of the 30-amp. wave when $t = 0$. (*b*) What is the corresponding value of time under these conditions? (*c*) What are the values of current?

20. (*a*) Through how many degrees must the radius vector of the 40-amp. wave advance (Prob. 19) in order that the positive values of both currents may be equal? (*b*) What is the corresponding value of time? (*c*) What is the value of the currents under these conditions?

21. Repeat Prob. 18 when the two waves differ in phase by 60°, the 40-amp. wave lagging.

22. Two alternators *A* and *B* are connected to the same 2,300-volt bus-bars (Fig. 22*A*). Alternator *A* is delivering 115 amp., and alternator *B* is delivering 200 amp. The current delivered by alternator *A* leads that delivered by *B* by an angle of 20°. (*a*) What total current are these two alternators delivering to the bus-bars? (*b*) Determine the angle by which this resultant current lags that delivered by alternator *A*.

23. When the total current on the system (Prob. 22) is 400 amp., alternator *A* delivers 200 amp., and this current leads the 400-amp. current by 12°. Determine the current delivered by *B* and the phase angle which it makes with the current delivered by *A*.

24. Two generator coils generate 300 and 400 volts, respectively. These two voltages differ in phase by 120°, the 300 volts leading. If connected in series, what is the voltage across their open ends?

25. If the 400-volt coil is reversed, the two voltages now differ in phase by 60°. Find the resultant voltage.

26. If both coils (Prob. 24) generate 300 volts, find the resultant voltage.

27. Repeat Prob. 26 except that one of the voltage coils is reversed, making the two voltages differ in phase by 60°.

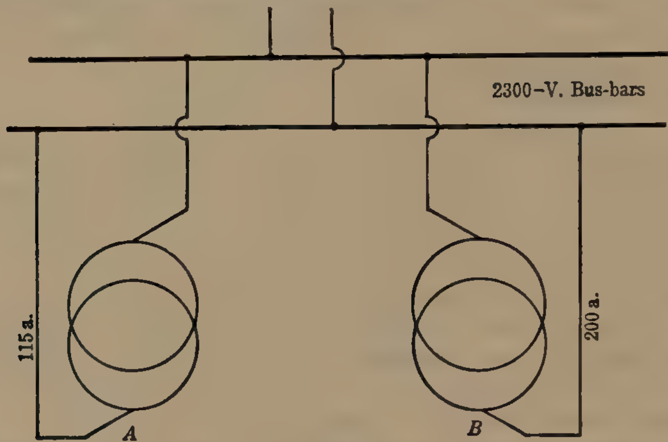


FIG. 22A.

QUESTIONS ON CHAPTER II

1. How may the power in an alternating-current circuit be determined at any instant? What is the general character of the power curve when the voltage and the current are in phase (illustrate by sketch)? What is the maximum value of this curve in watts? the average value?

2. In what way does the power curve for a circuit in which the current lags 90° behind the voltage differ from that in which the current is in phase with the voltage? What is the *average* power in such a circuit?

3. What is the *average* power of a circuit in which the current *leads* the voltage by 90°?

4. When the current and the voltage differ in phase by an angle which is less than 90° but greater than zero, what is the general character of the power curve? Make a sketch. What is meant by *power factor?* To what ratio is power factor equal.

5. When an alternating voltage is impressed across a resistance, what phase relation exists between this voltage and the resulting current? How may the value of the current be calculated, knowing the voltage and the resistance? How is the power in such a circuit determined?

6. What is the effect of inductance on the building up of a current in a circuit across which a steady direct-current voltage is impressed? What occurs when the current attempts to die out in an inductive circuit?

7. State how the current may be prevented from reaching its Ohm's-law value in an inductive circuit.

8. What effect does inductance in an alternating-current circuit have upon: (*a*) the phase angle between the current and the voltage; (*b*) the

magnitude of the current? What is the effect of frequency upon the magnitude of the current? What is *inductive reactance?*

9. What, in general, is the effect of capacitance upón the flow of current in any electric circuit? How does capacitance affect: (*a*) the phase angle between the current and the voltage in an alternating-current circuit; (*b*) the magnitude of the current?

10. What is the effect of frequency upon the magnitude of the current in a condensive circuit? What is *condensive reactance?* What is the value of the average power taken by a perfect condenser?

11. What is the phase relation existing between the current and the voltage across the resistance in a circuit containing resistance and inductance in series? between the current and the voltage across the inductance? How is the line voltage obtained?

12. What is meant by *impedance?* How may the angle between the line voltage and the current be determined? What relation does this angle bear to the power in the circuit?

13. What is the phase relation existing between the current and the voltage across the resistance in a circuit containing resistance and capacitance in series? between the current and the voltage across the capacitance? What phase relation exists between the line voltage and the current?

14. Sketch the vector diagram of a circuit containing resistance, inductance, and capacitance all in series. How may the circuit phase angle be found?

15. What is meant by *resonance* in an alternating-current circuit? What phase relation exists between the line voltage and the line current? What is the numerical relation existing between the inductive voltage and the condensive voltage? Show that, with both inductance and capacitance in a series circuit, the voltage across each can be several times the line voltage.

16. Show with sketches the effect on the current of changing the frequency in a series circuit having fixed values of inductance and capacitance. Also, show the effect of changing the relation of inductance to capacitance in the circuit when the resonant frequency is kept constant.

17. In practice, why is parallel grouping of resistances, inductances, etc., more common than series grouping? How may the resultant of several currents be found? In what way does parallel resonance differ from series resonance, especially with regard to the value of the current? In what way are the two similar?

18. Explain why the alternating-current resistance of an iron-cored impedance coil differs from the direct-current resistance. How may the resistance of the coil be taken into consideration when the impedance coil is connected in circuit with resistance, etc?

19. How may the phase relations existing in a series circuit having two component voltages be determined when the voltages across the various parts of the circuit are known? Make a sketch. How may the power and the power factor of all parts of the circuit be determined?

20. Explain why a circuit in which the line voltage and three component voltages are all known in magnitude only cannot be determined unless one more factor be known. What additional information makes the circuit relations determinable?

21. In what way is the polygon of currents similar to the polygon of voltages? In what way do the two polygons differ?

22. Define *effective* resistance. How is it determined?

23. What is meant by *energy current?* What relation does it bear to the power? What is *quadrature current*, and what relation does it bear to the power? Why is quadrature current usually undesirable?

PROBLEMS ON CHAPTER II

28. A certain lamp load consists of 40 60-watt lamps, each lamp having a resistance of 220 ohms. Compare the power taken by this load when connected across 115-volts direct current and across 115-volts alternating current.

29. An electric flatiron takes 420 watts when connected across 110-volt direct-current mains. How much power does it take from 115-volt, 60-cycle mains? The heating element is practically a pure resistance.

30. At no load, a transformer takes 6.8 amp. at 2,200 volts, and the power factor is 0.072. What power does it take at no load?

31. At full load, the transformer of Prob. 30 takes 182 amp. at 2,200 volts, and the power is 365 kw. What is its full-load power factor?

32. A single-phase motor takes 22 amp. at 220 volts, and the power factor is 0.76. What power does it take?

33. A 220-volt, single-phase generator is rated at 50 kv-a. at a power factor of 0.75. What is its rated current?

34. A coil has an inductance of 0.22 henry, and its resistance is negligible. What current does it take when connected across 120-volt, 60-cycle mains? When connected across 120-volt, 25-cycle mains?

35. A coil which has an inductance of 0.064 henry and negligible resistance takes 10.95 amp. from 220-volt mains. What is the frequency? What current would this coil take from 220-volt, 60-cycle mains?

36. A reactance coil, whose resistance is negligible, takes 4.2 amp. from 115-volt, 60-cycle mains. What is its inductance?

37. A condenser has a capacitance of 18 μf. What current does it take from 110-volt, 60-cycle mains? from 110-volt, 133-cycle mains?

38. It is desired to obtain from 220-volt, 60-cycle mains a current of 40 amp. leading its voltage by 90° by means of a condenser. What should be its capacitance?

39. A telephone condenser has a capacitance of 2 μf. What is its reactance at 1,000 cycles? What current does it take when an e.m.f. of 5 volts at 796 cycles is impressed across it?

40. It is desired to obtain 25 amp. at 2,200 volts, 60 cycles, by means of a static condenser. (Static condensers are used on power systems to correct power factor.) (*a*) How much capacitance is necessary? (*b*) What is the kilovolt-ampere rating of this condenser?

41. What must be the capacitance of a static condenser to give the same kilovolt-amperes as the condenser of Prob. 40 but at 2,200 volts, 25 cycles?

42. A condenser when connected across 120 volts, 60 cycles, takes 6 amp. What current does it take when the capacitance and frequency are both doubled?

43. An inductance when connected across 120 volts, 60 cycles, takes 6 amp. What current does it take when the inductance and frequency are both doubled? Compare result with that of Prob. 42.

44. An electric circuit having an inductive reactance of 6 ohms at 60 cycles and a resistance of 8 ohms takes 10 amp. Determine: (*a*) the impedance of the circuit; (*b*) the voltage across the resistance; (*c*) the voltage across the reactance; (*d*) the circuit voltage; (*e*) the power taken by the circuit; (*f*) the angle between the current and voltage; (*g*) the power factor of the circuit; (*h*) the inductance.

45. An inductance of 0.022 henry and a resistance of 7 ohms are connected in series across 220-volt, 60-cycle mains. Determine: (*a*) the reactance; (*b*) the impedance; (*c*) the current; (*d*) the power; (*e*) the phase angle; (*f*) the power factor; (*g*) the voltage across the resistance; (*h*) the voltage across the reactance. (*i*) Draw a vector diagram.

46. Repeat Prob. 45 with the inductance halved.

47. When an inductance of 0.04 henry and an unknown resistance are connected in series across 100-volt, 25-cycle mains, the current is 12 amp. What is (*a*) the value of the unknown resistance? (*b*) the power factor of the circuit?

48. The primary of a No. 46 telephone induction coil has an effective resistance of 40 ohms and an inductance of 0.15 henry. (*a*) What is its impedance at 1,000 cycles? (*b*) What current does it take when 6 volts at 1,000 cycles are impressed across it? (*c*) How much power is dissipated in the coil?

49. An inductive circuit takes 3.2 amp. from 220-volt, 60-cycle mains, and the power factor of the circuit is 0.68. Determine: (*a*) the power; (*b*) the resistance; (*c*) the reactance; (*d*) the inductance.

50. The current in an inductive circuit is 4.5 amp. at 25 cycles. The circuit takes 240 watts, and the power factor is 0.74. Determine: (*a*) the circuit voltage; (*b*) the inductance in the circuit. (*c*) Draw a vector diagram.

51. A condenser whose capacitance is 40 μf. is connected in series with a resistance of 35 ohms across a 220-volt, 50-cycle circuit. Determine: (*a*) the current; (*b*) the power factor; (*c*) the power.

52. A current of 1.5 amp. at 60 cycles flows in a circuit consisting of capacitance and resistance in series. The voltage across the resistance is 42 volts, and that across the condenser is 102 volts. Determine: (*a*) the circuit voltage; (*b*) the power; (*c*) the power factor. (*d*) Draw a vector diagram.

53. A circuit consisting of a capacitance and a resistance in series takes 40 watts at a power factor of 0.28 from 100-volt, 50-cycle mains. Determine: (*a*) the current; (*b*) the power-factor angle; (*c*) the resistance; (*d*) the impedance; (*e*) the capacitance.

54. A circuit consisting of a 30-μf. condenser and an adjustable resistance in series is connected across 120-volt, 60-cycle mains. To what value must the resistance be adjusted in order that the circuit may take 42 watts? (Two values of resistance will satisfy this condition.)

55. A 50-cycle current of 0.8 amp. flows in a circuit consisting of an adjustable resistance and a 35.4-μf. condenser in series. To what value should the resistance be adjusted in order that the voltage across the condenser shall be 0.6 the voltage across the circuit? Determine the power.

56. What is the power factor of a series circuit which has a resistance of 4 ohms and a reactance of 3 ohms? What current flows if the voltage is 60 volts, and how much power does the circuit take?

57. Repeat Prob. 56, making the inductive reactance twice the value given.

58. A series circuit consisting of 12-ohms resistance, 20-ohms inductive reactance, and 30-ohms condensive reactance is connected across 220-volt, 60-cycle mains. Find: (*a*) the current; (*b*) the voltage across each part of the circuit; (*c*) the power taken by the circuit; (*d*) the power factor and power-factor angle of the circuit. (*e*) Draw a vector diagram of the circuit to scale.

59. A potential difference of 220 volts at 50 cycles is impressed across a circuit having 60-ohms resistance, 20-μf. capacitance, and 0.3-henry inductance in series. Determine: (*a*) the current; (*b*) the voltage across the resistance; the inductance; the capacitance; (*c*) the total power; (*d*) the power factor and power-factor angle. (*e*) Draw to scale a vector diagram for this circuit.

60. A resistance of 15 ohms, an inductance of 0.25 henry, and a 40-μf. condenser are connected in series across 200-volt, 60-cycle mains. Determine: (*a*) the current; (*b*) the power; (*c*) the voltage across the resistance; the inductance; the capacitance; (*d*) the circuit power factor and power-factor angle. (*e*) Draw a vector diagram.

61. At what frequency would the voltage across the resistance and the voltage across the inductance (Prob. 60) be equal?

62. A series circuit consists of a resistance of 8 ohms, an inductive reactance of 24 ohms, and an adjustable condenser. To what two values can the capacitive reactance be adjusted in order that the circuit may take 620 watts from 120-volt, 25-cycle mains?

63. A series circuit is connected across 120-volt, 25-cycle mains. The capacitance is 50μ*f*., and the inductance is 0.5 henry. To what values should the resistance be adjusted in order that the circuit may take 120 watts?

64. If the inductance of Prob. 60 were adjustable, for what value would the current be a maximum? Find the current, the power, the power factor, and the voltage across each part of the circuit under these conditions.

65. A circuit contains a resistance of 12 ohms, an inductance of 0.352 henry, and a variable condenser in series. If the frequency is 60 cycles, for what value of the capacitance will the current be a maximum? If the line

voltage is 48 volts, what are the current, power, power factor, and the voltages across the resistance, the inductance, and the capacitance?

66. A telephone receiver has an effective resistance of 75 ohms and an impedance of 300 ohms at 800 cycles. What value of capacitance connected in series will make the phase angle between the applied voltage and current equal to zero at 800 cycles? What power is dissipated in the receiver when the applied voltage is 2.4 volts? What is the voltage across the receiver and the condenser under these conditions?

67. Plot curves of current and frequency in a circuit having 20 ohms resistance and an adjustable condenser for the following values of inductance: 0.01, 0.04, 0.1, 0.5 henry. In each case, the circuit is tuned for 50 cycles. The impressed voltage is maintained constant at 120 volts. In each case, note the sharpness of the tuning.

68. A resistance of 42 ohms and an inductance of 0.15 henry are connected in parallel across 120-volt, 50-cycle mains. Determine: (*a*) the total current; (*b*) the power factor; (*c*) the power-factor angle. Draw a vector diagram.

69. A circuit consisting of a resistance of 40 ohms and an unknown condenser are connected in parallel across 120-volt, 60-cycle mains. The circuit takes a current of 3.6 amp. Determine the capacitance of the unknown condenser.

70. Determine the current taken by the circuit of Prob. 68 when a 45-μf. condenser is connected across the mains. Also, determine the power-factor angle and the power factor of the circuit.

71. A telephone induction coil having an effective resistance of 2.8 ohms and an inductance of 0.0047 henry is in parallel with a 2-μf. condenser of negligible resistance. With 5 volts at a frequency of 1,000 cycles across the combination, determine: (*a*) the current taken by the induction coil; (*b*) the current taken by the condenser; (*c*) the total current; (*d*) the power consumed by the entire circuit; (*e*) the phase angle between current and voltage.

72. What value of capacitance, connected across the circuit of Prob. 68, will cause the circuit to be in anti-resonance?

73. An inductance of 0.05 henry, a resistance of 14 ohms, and an unknown capacitance are connected in parallel across a 100-volt circuit. The current taken by the inductance is 10.6 amp., and the current taken by the capacitance is adjusted until it, also, equals 10.6 amp. Determine the frequency and the value of the capacitance.

74. A parallel circuit consists of an inductance of 0.254 henry having negligible resistance in parallel with a condenser of 10-μf. capacitance. For a constant applied voltage of 100 volts plot a curve of current and frequency as the frequency is varied from zero to 500 cycles. What is the current when the frequency is infinite?

75. A 2-μf. condenser, a 200-ohm resistance, and an inductance of 0.0127 henry are connected in parallel. If the voltage is maintained constant at 10 volts, for what value of frequency will the current be a minimum? Determine this value of current. What is the impedance of this circuit at 800 cycles?

76. A non-inductive resistance and an impedance coil are connected in series across 115-volt, 60-cycle mains. The circuit takes 6.4 amp. A voltmeter when connected across the resistance reads 84 volts; when connected across the impedance, it reads 64 volts. Determine: (*a*) the impedance of the impedance coil; (*b*) the circuit power factor; (*c*) the power-factor angle of the impedance coil; (*d*) the resistance of the impedance coil; (*e*) the inductance of the impedance coil; (*f*) the power taken by the entire circuit. Draw a vector diagram.

77. In order to measure the power taken by a small 115-volt, 60-cycle induction motor, it is connected in series with a non-inductive resistance across 220-volt, 60-cycle mains. The resistance is adjusted until the voltage across the motor is 115 volts. The current is measured and found to be 3.4 amp.; the voltage across the resistance is found to be 130 volts. Determine: (*a*) the power factor of the entire circuit; (*b*) the power factor of the motor; (*c*) the power taken by the motor; (*d*) the total power of the circuit.

78. A non-inductive resistance and an impedance coil are connected in series across 120-volt, 60-cycle mains. The voltage across the resistance is found to be 85 volts, and that across the impedance, 64 volts. If the current is 4.6 amp., determine: (*a*) the circuit power; (*b*) the circuit power factor; (*c*) the impedance-coil power factor; (*d*) the inductance of the impedance coil.

79. A series circuit consists of a resistance, an impedance coil, and a condenser having negligible loss. This circuit is connected across 115-volt, 60-cycle mains, and the current is 3.4 amp. The voltage across the resistance is measured and found to be 92 volts; that across the condenser, 70 volts; and that across the impedance coil, 58 volts. Determine: (*a*) the power-factor angle and the power factor of the entire circuit; (*b*) the power factor of the impedance coil; (*c*) the resistance of the impedance coil; (*d*) the inductance of the impedance coil.

80. A series alternating-current circuit, consisting of a resistance, an impedance coil, and a condenser having negligible leakage, takes 340 watts at 120 volts, 60 cycles, and 4.0-amp. lagging current. The voltage across the resistance is 75 volts, and that across the capacitance is 90 volts. Find: (*a*) the power consumed in the resistance; (*b*) the power consumed in the impedance coil; (*c*) the voltage across the impedance coil; (*d*) the power-factor angle of the impedance coil; (*e*) the capacitance of the condenser in μf.; (*f*) the inductance of the impedance coil in henrys.

81. A series circuit consisting of a resistance of 10 ohms, an inductance coil (resistance negligible), and an unknown condenser is connected across 220-volt, 60-cycle mains. When 10-amp. leading current flow in the circuit, the voltage across the inductance is 160 volts. Determine the circuit power factor and the capacitance of the condenser.

82. A series circuit consisting of a resistance, a condenser, and an impedance coil (Fig. 82*A*) is connected across 200-volt, 60-cycle mains. An ammeter *A* indicates 2.0 amp. A voltmeter when connected across the resistance reads 120 volts; when connected across the capacitance, it reads

240 volts; when connected across both the resistance and capacitance, it reads 278 volts; when connected across the impedance coil it reads 170 volts. Draw a vector diagram to scale. Determine: (a) the power-factor angle and the power factor of the condenser; (b) the power-factor angle and the power factor of the entire circuit; (c) the power-factor angle and the power factor of the impedance coil; (d) the resistance of the impedance coil; (e) the inductance of the impedance coil.

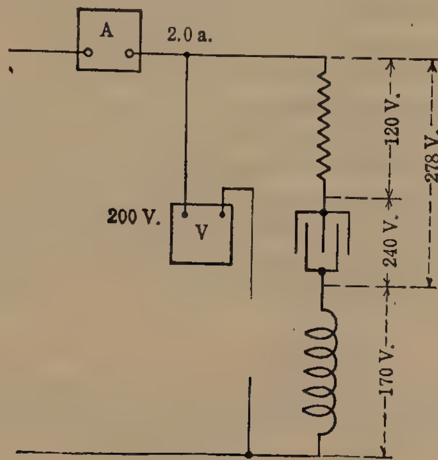


FIG. 82A.

83. A non-inductive resistance and an impedance coil are connected in parallel across 115-volt, 50-cycle mains. The resistance takes a current of 3.2 amp.; the impedance coil takes a current of 4.1 amp.; the total current is found to be 5.4 amp. Determine: (a) the power-factor angle and the power factor of the circuit; (b) the power taken by the circuit; (c) the power-factor angle and the power factor of the impedance coil; (d) the resistance of the impedance coil; (e) the inductance of the impedance coil.

84. A single-phase induction motor is connected across 220-volt, 60-cycle mains and takes a current of 8.5 amp.; a non-inductive resistance in parallel with the motor takes 6.8 amp.; the total current is measured and found to be 14.7 amp. Determine the power factor and the power taken by the motor.

85. The load on the motor (Prob. 84) is decreased. The motor current is now 7.0 amp.; the resistance still takes 6.8 amp.; the total current is 12.8 amp. Determine the power factor and the power taken by the motor.

86. A non-inductive resistance R_1 and a condenser having negligible loss in series with a resistance R_2 are connected in parallel across 110-volt, 60-cycle mains. The current taken by the resistance R_1 is 1.2 amp.; the current taken by the condenser and resistance R_2 is 0.82 amp.; the total current is 1.5 amp. Determine: (a) the power-factor angle and the power factor of the circuit; (b) the power taken by the entire circuit; (c) the power taken by resistance R_2; (d) the impedance of the branch of the circuit containing the condenser and R_2; (e) the capacitance of the condenser.

87. A parallel circuit consists of three branches—a non-inductive resistance, an impedance coil, and a condenser having negligible loss. This circuit is connected across 110-volt, 50-cycle mains. The current taken by the resistance is 2.4 amp.; the current taken by the impedance is 3.5 amp.; the current taken by the condenser is 1.0 amp.; the current supplied from the mains is 5.0 amp. Determine: (*a*) the power-factor angle and the power factor of the entire circuit; (*b*) the power taken by the entire circuit; (*c*) the power-factor angle and the power factor of the impedance coil; (*d*) the resistance of the impedance coil; (*e*) the inductance of the impedance coil.

88. Repeat Prob. 87 with the current taken by the impedance coil changed to 1.2 amp. and the current supplied by the mains changed to 3.50 amp.

89. A parallel circuit consisting of a non-inductive resistance, an impedance coil, and a condenser of negligible loss is connected across 120-volt, 60-cycle mains and takes 4.2 amp. lagging current at a power factor of 0.89. The resistance takes 3.2 amp., and the condenser takes 2.2 amp. Determine: (*a*) the power taken by the impedance coil; (*b*) the current taken by the impedance coil; (*c*) the power-factor angle and the power factor of the impedance coil.

90. A certain iron-cored impedance coil takes 8.2 amp. at 24-volts direct current. With 60-cycle alternating current, it takes 1.6 amp. at 100 volts, and the power is 13.6 watts. Determine: (*a*) the impedance; (*b*) the effective alternating-current resistance; (*c*) the inductance; (*d*) the ratio of effective to ohmic resistance.

91. The resistance of an alternator armature is measured with direct current. With 16 volts, the current is 52 amp. When 32-amp. alternating current flow in the armature, the power loss is found to be 430 watts. Determine the effective resistance of the armature and the ratio of effective to ohmic resistance.

92. A single-phase, 60-cycle induction motor takes 7.8 amp. at 220 volts at a power factor of 0.78. Determine: (*a*) the energy current; (*b*) the quadrature current; (*c*) the capacitance of a condenser which, if connected in parallel with the motor, would raise the circuit power factor to unity. Does it appear economical to raise the power factor in this manner?

93. A 100-kw., single-phase load is connected at the end of a feeder which has a resistance of 2 ohms per conductor. The voltage at the load is 2,300 volts, 60 cycles, and the power factor is 0.72. Determine: (*a*) the total current; (*b*) the line loss; (*c*) the efficiency of transmission; (*d*) the energy current; (*e*) the quadrature current; (*f*) the line loss due to (*d*) and to (*e*). The power remains constant.

94. The power factor of the load (Prob. 93) is raised to unity by connecting a static condenser in parallel with it. Determine: (*a*) the capacitance of the static condenser; (*b*) its kilovolt-ampere capacity; (*c*) the line loss and efficiency of transmission under these conditions.

95. A variable resistance is connected in series with a 40 μf. condenser across 110-volt, 60-cycle mains. Plot a curve with resistance R as abscissas and power as ordinates, from $R = 0$ to $R = 500$ ohms. For what value of R does it absorb the maximum power?

96. An adjustable resistance is in series with an inductance of 0.1 henry across 120-volt, 50-cycle mains. To what value must the resistance be adjusted in order that it may absorb the maximum power. The resistance of the inductance coil is negligible.

97. A variable resistance is in series with an impedance coil having an inductance of 0.04 henry and a resistance of 4 ohms. If this circuit is connected across 120-volt, 50-cycle mains, to what value must the variable resistance be adjusted in order that it may absorb the maximum power?

QUESTIONS ON CHAPTER III

1. What is the effect of operating on a vector with the quantity -1? Show that $\sqrt{-1}$ operating on a vector must rotate it through 90°. Define the *axis of reals;* the *axis of imaginaries.*

2. How are vectors in the complex plane designated? What is a *real component?* An *imaginary component?*

3. How are complex vectors added? subtracted?

4. What is meant by *rationalizing* a complex fraction?

5. Describe the polar notation of vectors. How are these vectors added and subtracted?

6. Show how polar vectors are multiplied and divided.

7. How are they raised to powers? How are roots extracted?

8. What operator will rotate a complex quantity through a positive angle α? a negative angle α?

9. Given the voltage and current of a circuit expressed in complex, show how the power is determined.

10. Why may impedance be expressed as a complex quantity?

11. Define *conductance; susceptance; admittance.*

Compare these quantities with resistance, reactance, and impedance.

Discuss the relation of the signs preceding the reactance and susceptance to the type of reactance.

PROBLEMS ON CHAPTER III

Note.—The instructor may require many of the following problems to be solved by rectangular and polar vectors.

98. Show graphically the positions of the following rectangular vectors, giving their absolute values and their real and imaginary components: $8 + j6$; $6 - j8$; $-4 + j10$; $12 - j2$; $-5 - j10$.

99. Repeat Prob. 98 for the following complex vectors: $0.3 - j0.7$; $-1.0 - j0.2$; $0.4 - j0.5$; $-0.4 + j0.5$; $+j0.8$.

100. Add the following complex vectors, and show graphically in the complex plane their individual positions and the positions of their resultants: $4 + j5$; and $-9 + j3$; $6 - j4$ and $-2 - j3$.

101. Add the following, as in Prob. 100: $-48 + j60$ and $-20 - j12$; $25 + j10$ and $-40 - j20$.

102. Subtract $148 - j86.9$ from $-42 - j12.8$; $-64 + j92$ from $42 - j12$. Show all quantities graphically.

103. Multiply $(25 + j10)$ by $(-40 - j20)$; $(14.4 - j20)$ by $(-3.6 + j8.4)$.

104. Multiply $(-124 + j12)$ by $(8.0 + j12)$; $(-16 - j24)$ by $(48 + j60)$.

105. Find $(3 + j5)(6 - j4)(-3 - j2)$.

106. Find $\dfrac{1}{4 + j5}$; $\dfrac{1}{18 - j24}$; $\dfrac{1}{-32 - j72}$. Show graphically the position of each vector and its reciprocal (use different scales).

107. Find $\dfrac{1}{-124 + j12}$; $\dfrac{1}{-40 - j72}$; $\dfrac{1}{-120 + j200}$.

108. Divide $(72 + j60)$ by $(6 + j5)$; $(72 + j60)$ by $(6 - j5)$.

109. Divide $(4 - j3)$ by $(12 + j28)$; $(54 - j14)$ by $(248 + j72)$.

110. Find $\dfrac{16 - j24}{(3 - j2)(8 - j6)}$.

111. Transform the following into polar vectors: $25 + j10$; $-40 - j20$; $14.4 - j20$; $-3.6 + j8.4$.

112. Transform the following into polar vectors: $72 + j60$; $6 + j5$; $72 - j60$; $6 - j5$.

113. Transform the following into rectangular vectors: $42/\underline{36^\circ}$; $24\backslash\underline{125^\circ}$; $16\backslash\overline{66^\circ}$; $28/\overline{150^\circ}$.

114. Transform the following into rectangular vectors: $127\backslash\overline{30^\circ}$; $150/\overline{170^\circ}$ $160\backslash\underline{150^\circ}$; $135/\underline{85^\circ}$.

115. Add together the four polar vectors of Prob. 113, and express the result as a polar vector.

116. Add together $24/\underline{30^\circ}$; $17/\underline{24^\circ}$; $27/\overline{120^\circ}$; and $30\backslash\overline{60^\circ}$. Express the result as a polar vector.

117. Multiply $12/\underline{20^\circ}$ by $8\backslash\overline{30^\circ}$; $6/\underline{45^\circ}$ by $10\backslash\underline{135^\circ}$.

118. Multiply $124\backslash\underline{115^\circ}$ by $12/\overline{160^\circ}$; $-22/\underline{30^\circ}$ by $64\backslash\underline{145^\circ}$.

119. Transform the following into polar vectors, and multiply: $(25 + j10)$ by $(-40 - j20)$; $(14.4 - j20)$ by $(-3.6 + j8.4)$. Compare results with those of Prob. 103.

120. Multiply together $(-4 - j6)(12\backslash\overline{72^\circ})(20/\underline{15^\circ})$.

121. Find $1/0.4/\underline{16^\circ}$; $1/84/\overline{115^\circ}$; $1/0.04\backslash\underline{130^\circ}$.

122. Find the following by conversion into polar vectors: $\dfrac{1}{4 + j5}$; $\dfrac{1}{18 - j24}$; $\dfrac{1}{-32 - j72}$. Compare with Prob. 106.

123. Divide $24\backslash\underline{120^\circ}$ by $6/\underline{60^\circ}$; $8/\overline{140^\circ}$ by $2|\underline{90^\circ}$.

124. Transform the following into polar vectors and divide: $(4 - j3)$ by $(12 + j28)$; $(54 - j14)$ by $(248 + j72)$. Compare results with those of Prob. 109.

125. Find $\dfrac{-6 - j6}{18/\underline{14^\circ} \times 12/\underline{54^\circ}}$.

126. Find $(32/\underline{22^\circ})^2$; $(14\backslash\underline{116^\circ})^2$; $(12/\underline{34^\circ})^3$.

127. Find $(2 + j3)^2$; $(2 - j3)^3$; $(3 - j2)^4$ by expanding as rectangular vectors and by first converting into polar vectors and then operating. Compare results.

128. Find $\sqrt{225\underline{/30°}}$; $\sqrt[3]{463\underline{/165°}}$; $\sqrt{0.0132\overline{\backslash 70°}}$.

129. Find $\sqrt{12.2 - j8.4}$; $\sqrt{-6.4 + j7.2}$; $\sqrt[3]{-22 - j64}$.

130. Rotate the vector $12.2 + j8.4$ in a positive direction through an angle of 62°, using the rectangular method (Eq. (45), p. 63). Rotate this same vector through a negative angle of $-110°$.

131. Rotate $-24 - j42$ in a negative direction through an angle of 80°.

132. A current $12 + j0$ flows through an impedance having resistance of 6.44 ohms and inductive reactance of 7.65 ohms. Find the complex expression for the voltage and also its magnitude.

133. Repeat Prob. 132 with the reactance given as $8.48 - j5.29$.

134. A 60-cycle voltage of 115 volts acts on a circuit whose resistance is 5 ohms and whose inductive reactance is 4 ohms. (*a*) Taking the voltage along the axis of reals, determine the current in complex. Find, also, (*b*) the conductance; (*c*) the susceptance.

135. Repeat Prob. 134 with the impedance given as $14 - j20$ ohms.

136. Express the impedance $10 + j14$ in polar form. If this impedance is connected across a 50-cycle voltage, $120\underline{/10°}$, find the current, expressing it both in polar and rectangular form.

137. A current $4 + j6$ amp. flows in a circuit whose impedance is given as $8 - j12$ ohms. Determine the voltage, the power, and the phase angle between voltage and current.

138. An e.m.f. of 220 volts, 60 cycles is impressed across a circuit in which a resistance of 12 ohms, a capacitive reactance of 40 ohms, and an inductive reactance of 23 ohms are in series. If the voltage is given by $220 + j0$, determine: (*a*) the complex expression for the current; (*b*) its magnitude; (*c*) the conductance; (*d*) the susceptance; (*e*) the power.

139. A voltage is given by $95.2 + j55$, and the current by $3.76 + j10.3$. Determine: (*a*) the complex expression for the impedance, stating whether it is inductive or capacitive; (*b*) the power; (*c*) the phase angle between voltage and current. (*d*) Draw a vector diagram.

140. A voltage of $600\overline{\backslash 120°}$ acts on a circuit whose impedance is given by $200 + j100$ ohms. Determine: (*a*) the current as a polar vector; (*b*) the power. (*c*) Draw a vector diagram.

141. A resistance of 22 ohms in series with an inductance of 0.1 henry is connected in parallel with a resistance of 18 ohms and a 50-μf. condenser in series (Fig. 141*A*). This parallel combination is connected across 115-volt, 60-cycle mains. Determine: (*a*) the complex expression for the admittance of the inductive branch and for the capacitive branch; (*b*) the complex expression for the total circuit admittance; (*c*) the absolute value of the circuit admittance; (*d*) the current in each branch; (*e*) the total current; (*f*) the power factor of the entire circuit; (*g*) the total power. (*h*) Draw a vector diagram.

142. If a variable condenser is substituted for the 50-μf. condenser (Prob. 141), to what two values may it be adjusted in order that the power factor of the entire circuit may be unity? Under these conditions, determine: (*a*) the total current; (*b*) the total power.

143. In Prob. 142, plot a locus of current as the capacitance is varied from zero to infinity. Plot energy current as abscissas and quadrature current as ordinates. Show on the locus the two values of the solution. Draw on this same diagram the vector giving the current in the inductive branch together with its energy and quadrature components.

144. Two impedances $4 + j5$ and $3 + j8$ are connected in parallel across 100-volt, 50-cycle mains. Determine for the circuit: (*a*) the admittance in complex; (*b*) the power factor; (*c*) the current; (*d*) the energy current; (*e*) the power.

145. Determine: the capacitance of a condenser, which, if connected in parallel with the circuit of Prob. 144, would make the circuit power factor unity.

146. Repeat Prob. 144 with an impedance $3 - j8$ substituted for $3 + j8$.

147. A resistance of 10 ohms is connected in series with the parallel circuits (Prob. 141). The entire circuit is connected across the 115-volt, 60-cycle mains. Determine: (*a*) the impedance of the entire circuit; (*b*) the total current; (*c*) the voltage across the 10-ohms resistance; (*d*) the voltage across the parallel circuit; (*e*) the current in each parallel branch; (*f*) the power in each circuit element; (*g*) the power factor of the entire circuit.

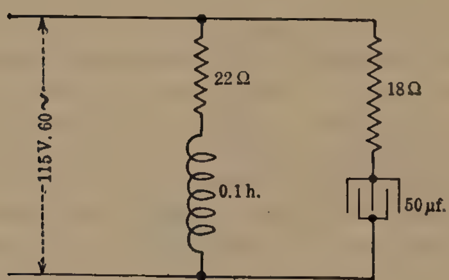


FIG. 141*A*.

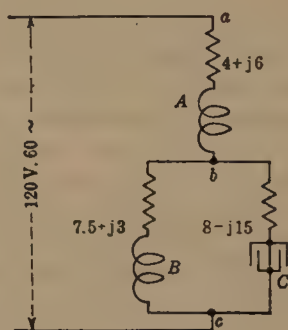


FIG. 148*A*.

148. A series-parallel circuit (Fig. 148*A*) consists of a parallel combination of two impedances: $B = 7.5 + j3$ and $C = 8 - j15$ in series with the impedance $A = 4 + j6$. The entire circuit is connected across 120-volt, 60-cycle mains. Determine: (*a*) the complex admittance of each parallel branch; (*b*) the equivalent impedance of the parallel portion *bc*; (*c*) the impedance of the entire circuit; (*d*) the total current; (*e*) the voltages across *ab* and *bc*; (*f*) the current in each parallel branch; (*g*) the power in each circuit element; (*h*) the total power; (*i*) the power factor of the entire circuit.

149. Repeat Prob. 148 with the impedances changed:

$$A = 12 - j20;\ B = 30 + j25;\ C = 20 - j50.$$

150. Solve Prob. 141, using the equivalent impedance method of Par. 46, and compare results.

151. A telephone induction coil having an impedance $40 + j940$ ohms at 1,000 cycles is connected in parallel with a 2-μf. condenser. Determine the equivalent impedance of the combination. If the current is 5 milliamp., what is the voltage across the circuit? Determine the power.

152. Determine the impedance Z at 796 cycles of the network shown in Fig. 152*A*. If a current of 5 milliamp. flows in the 80-Ω resistance, determine the current entering the network at *ab*.

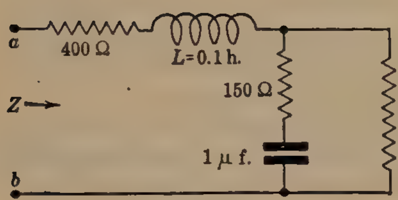


FIG. 152*A*.

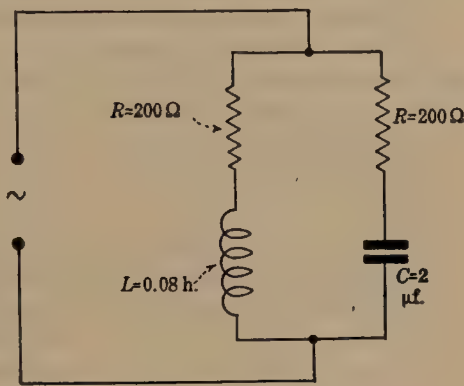


FIG. 153*A*.

153. In Fig. 153*A* is shown a parallel circuit, one branch of which consists of a 200-ohm resistance and an inductance of 0.08 henry in series; the other branch consists of a 200-ohm resistance and a 2-μf. condenser in series. Compute the equivalent impedance of this entire circuit at frequencies of 0, (direct current), 60, 100, 1,000, 20,000 cycles per second and at infinite frequency.

154. Figure 154*A* shows one station and its line of an intercommunicating telephone system. The line has a resistance of 40 ohms. At the

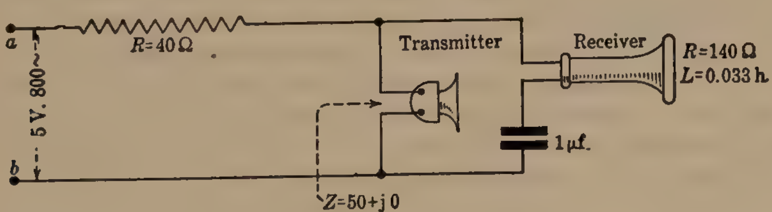


FIG. 154*A*.

subscriber's station, a transmitter having 50-ohms resistance and a receiver in series with a 1-μf. condenser are connected directly across the line. The resistance and inductance of the receiver at 800 cycles are 140 ohms and 0.033 henry. Determine at 800 cycles: (*a*) the impedance of the circuit measured between terminals *ab*; (*b*) the current flowing in the line when 5 volts at 800~ is impressed across terminals *ab*; (*c*) the power taken by the transmitter; (*d*) the power delivered to the receiver.

QUESTIONS ON CHAPTER IV

1. Describe the principle of the Siemens electrodynamometer. How are its coils connected, and what is the relation existing between the turning moment and the current? What are the disadvantages of this type of instrument?

2. In what way is the indicating electrodynamometer similar to the Siemens dynamometer? In what ways do the two instruments differ?

3. Explain how the electrodynamometer principle may be applied to a voltmeter. What is the general character of the scale divisions? Compare the magnitude of its current with that taken by a direct-current instrument of the same range. Discuss the accuracy of such an instrument when used with direct current.

4. Describe the inclined-coil voltmeter, giving the principle upon which it operates.

5. What difficulty arises when attempt is made to apply the dynamometer principle to the alternating-current ammeter?

6. Describe the construction of a wattmeter, and give the principle of its operation. Show how it is connected to a circuit. Give the best method of connecting the potential circuit, especially when the instrument is used in connection with considerable voltage.

7. Show two possible methods of connecting a wattmeter in circuit. Discuss the corrections that should be made in each case, if the exact value of the power be desired. What compensation for these errors is sometimes made in the construction of the instrument?

8. What precautions should be taken against overloading a wattmeter? How are wattmeters rated and why?

9. Give the advantages of a polyphase wattmeter over single-phase instruments. How is the polyphase wattmeter constructed?

10. How are wattmeters calibrated? Give a diagram of connections.

11. Describe how the Weston iron-vane type of voltmeter utilizes the principle of magnetized iron. Upon what fundamental electrical principle does this instrument operate? How is the instrument damped?

12. Show how the iron-vane principle has been adapted to an inclined-coil instrument. What two methods are used to damp this instrument?

13. What change should be made in the construction of the above two types of iron-vane voltmeters in order that they may be used as ammeters? What are the limitations of iron-vane instruments for direct-current measurements?

14. Discuss the use of direct-current watthour meters upon alternating-current circuits.

15. Describe the construction of the induction watthour meter. What should be the phase relation existing between the potential-coil flux and the circuit voltagė? How is this phase relation obtained?

16. How is friction compensation effected? Discuss this principle very carefully.

17. Show by simple sketches how the driving torque is developed. Why does the disc tend to rotate in the direction of the sliding field?

18. How is the induction watthour meter calibrated? What adjustments, not used for the direct-current type, are necessary? What are the advantages of this type of meter over the direct-current type?

19. Describe one common type of frequency meter. Upon what principle does it operate? Why are the vibrating reeds kept polarized?

20. Describe the Tuma phase meter. How is this instrument adapted for power-factor measurements? What control is exerted on the moving system? Why are the coils of the moving system not exactly 90° apart? What modifications of the instrument are necessary in order that it may be used on three-phase circuits?

21. For what purposes are synchroscopes used? Describe the construction of some one type. In what way is it related to the phase meter?

22. What are the commercial uses of the oscillograph? What is its principle of operation? In what way does the moving element differ from that of the ordinary galvanometer? How are the time abscissas obtained? Why is it desirable to immerse the moving element in oil?

23. Sketch the general arrangements of the laboratory type, giving the relative positions of the lamp, prisms, vibrators, lenses, rotating mirrors, film drum, etc.

24. Make a diagram of connections showing how the voltage vibrator and the current vibrator are connected in circuit.

25. Describe the impedance bridge, giving the equations for determining the unknown impedances.

QUESTIONS ON CHAPTER V

1. Give three reasons why polyphase power supply is superior to single-phase supply.

2. Why is it desirable at times to use symbolic notation in the solution of problems? Why is this system particularly applicable to polyphase systems?

State briefly the principles upon which one such system is based.

3. Describe an elementary three-phase generator. What relations exist among the three voltages of such a generator? How may three independent single-phase systems be obtained from such a generator?

4. What is meant by *Y-connection?* How many wires are necessary? What are the numerical relation and the phase relation of the line voltage to the coil voltage in this system? the line current to the coil current? What relation exists among the three-coil currents if there is no neutral wire?

5. At unity power factor, what is the total power generated in a Y-connected generator in terms of coil volts and coil current? if the power factor is other than unity? What is the line power in terms of line current, line voltage, and power factor?

6. Why is the line power factor the cosine of the *coil* power-factor angle? What significance has power factor in an unbalanced system?

7. Show that the delta-connection is not a short circuit for the three coil voltages. What is the numerical relation and the phase relation which exists between the coil current and the line current?

8. To what is the total power produced by a delta-connection equal in terms of coil voltage, coil current, and coil power factor? in terms of line voltage, line current, and coil power factor?

9. Sketch the connection of the three-wattmeter method of measuring power; (*a*) when the neutral of the system is accessible; (*b*) when the neutral is not accessible. To what is the total power equal in terms of the wattmeter readings?

10. Illustrate the principle of the Y-box, and state the conditions under which it can be used.

11. Sketch the connections of the two-wattmeter method of measuring power. Under what conditions do the wattmeters read the same? different? When does one wattmeter read zero?

12. Under what conditions does one wattmeter reverse? Give two methods of obtaining power factor from the two-wattmeter readings alone. Under what conditions can the two-wattmeter method not be used to measure power in a three-phase system?

13. What phase relations exist among the voltages of a two-phase system? Show the connections of four different types of two-phase system. What relations exist among all voltages in each of these systems?

14. Sketch two methods of connecting the coils of a two-phase generator. What relation exists between the coil voltage and the line voltage in each of the two systems? between coil current and line current?

PROBLEMS ON CHAPTER V

155. In the network shown in Fig. 155*A*, the current $I_{ab} = 8 - j2$; $I_{ed} = 4 + j5$; and $I_{fa} = -4 + j1$ amp. Determine the currents I_{cd} and I_{dg}.

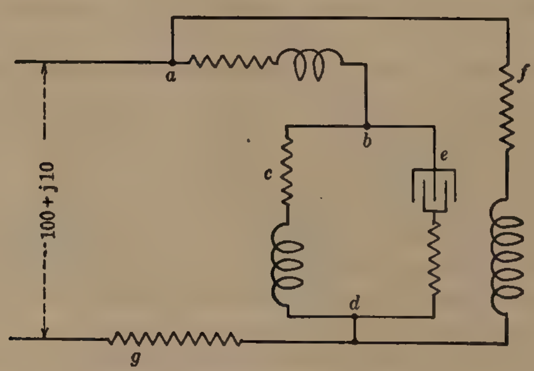


FIG. 155*A*.

156. The voltages in the network (Prob. 155) are as follows: $E_{ag} = 100 + j10$; $E_{ab} = 40 + j12$; $E_{gd} = -24 + j6$. Determine: (*a*) the voltages E_{bd} and E_{ad}; (*b*) the impedances Z_{cd}, Z_{ed}, Z_{fd}, Z_{dg}, all in complex. State whether the impedances are inductive or capacitive.

157. The equations for the three coil voltages of a three-phase system (see p. 107) are as follows:

$$e_{oa} = 155.5 \sin 314t; \; e_{ob} = 155.5 \sin (314t - 120°)$$
$$e_{oc} = 155.5 \sin (314t - 240°)$$

Find the instantaneous value of each of these three voltages for values of time corresponding to $t = \frac{1}{600}$ sec.; $\frac{1}{300}$ sec.; $\frac{1}{200}$ sec. Show that their algebraic sum at each of these three instants is zero.

158. A certain three-phase, Y-connected alternator is rated at 1,500 kv-a., 2,300 volts, at unity power factor. What is its current rating per terminal? What is the rated coil current and coil voltage?

159. What is the current and voltage rating of the machine in Prob. 158 if it is reconnected in delta?

160. A 290-hp., 2,300-volt, 60-cycle, three-phase, Y-connected synchronous motor has an efficiency of 93.2 per cent. (exclusive of field loss) at its rated load. (*a*) What line current does it take at rated load when its power factor is unity? (*b*) when the power factor is 0.80 leading current? (*c*) What are its coil voltages under these conditions?

161. If the windings of the synchronous motor (Prob. 160) were reconnected in delta, what would be its current and voltage ratings in (*a*) and (*b*)?

162. A Y-connected alternator delivers a balanced three-phase load of 75 amp. at 600 volts and 0.8 power factor to a delta-connected induction motor. Find the current and voltage per coil in both the generator and the motor. What power is involved? What kilovolt-amperes are involved?

163. Three resistances of 80 ohms each are connected in Y across a three-phase, 230-volt supply. How much current does each take, and what is the total power taken by the three?

164. If the three resistances of Prob. 163 are connected in delta, how much current does each take, and what is the line current? Determine the total power. What is the ratio of the delta to the Y kv-a.?

165. Three static condensers each have a capacitance of 50 μf. Determine the kilovolt-amperes that they take when connected in Y and then in delta across 2,300-volt, three-phase, 60-cycle mains. What is the ratio of the delta to the Y kv-a.?

166. A 2,000 kv-a., three-phase, 60-cycle, 6,600-volt alternator is delivering 1,500 kw. and 160 amp. at 6,600 volts. Determine: (*a*) the kilovolt-amperes which it is delivering; (*b*) its power factor.

167. A 100-hp., 550-volt, three-phase, Y-connected, 60-cycle, 870-r.p.m. squirrel-cage induction motor has a rated-load efficiency of 0.91 and a power factor of 0.90. Determine: (*a*) the line current at rated load; (*b*) the kilovolt-ampere input at rated load; (*c*) the voltage per coil; (*d*) the line current, terminal voltage, kilovolt-ampere input when this same motor is reconnected in delta. The coil voltage must remain unchanged.

168. The three e.m.fs. generated in the three coils of a three-phase, 400-kw., 600-volt, 60-cycle alternator are as follows: $E_{oa} = 519.6 + j300$; $E_{ob} = -j600$; $E_{oc} = -519.6 + j300$. Determine: (*a*) the vector and absolute

values of E_{ab}, E_{bc}, and E_{ca}; (*b*) the e.m.f. E_{ob} across the open ends *ob* when the coils *oa* and *ob* are connected so that terminal *a* of coil *oa* connects to terminal *o* of coil *ob*; (*c*) the e.m.f. E_{oo} across the open ends *oo* when the three coils are connected *oa-ob-co*; (*d*) E_{co} with the coils connected *oa-ob-oc*.

169. Each of the three coils (Prob. 168) delivers its rated current at a power factor of 0.80 lagging current. Give the values of these three currents in complex, expressed as rectangular vectors. Find the kilowatt output per coil. Show that, if the coils are connected in Y, the sum of these three currents is zero. If the coils are connected in delta, as in (*d*) (Prob. 168), find the complex and absolute values of the three line currents.

170. The two-wattmeter method is used to test a 10-hp., 220-volt, 60-cycle, three-phase induction motor. When the three line voltages are 220 volts, one wattmeter reads +5,920 watts, and the other, +2,080 watts. Determine the line current and the output in horsepower if the efficiency is 0.84. Use Eq. (65) (p. 119) and check with Fig. 108 (p. 120).

171. In Prob. 170, the load is reduced, and the two wattmeters then read +2,540 and −440 watts. Determine the line current and the output in horsepower if the efficiency at this load is 0.71.

172. The input to a 600-volt, 100-hp. synchronous motor is measured by the two-wattmeter method. One wattmeter reads +62.5 kw., and the other, +30.2 kw., when the motor takes a leading current. Determine the line current and the output if the efficiency at this load is 0.90 exclusive of the direct-current field loss.

173. Find the line currents and the power factor (Prob. 172) when the wattmeter readings become +52.0 and −40; +100.0 and −60.0; +42 and +52 kw. State whether the current lags or leads in each case.

174. Each of the two coils of a 500-kw., 60-cycle, two-phase alternator generates 2,300 volts, and these voltages differ in phase by 90°. If these two coils are connected together at their center points and this connection brought out with the others, indicate all the voltages that can be obtained. What is the current per terminal?

175. (*a*) If the two coils of Prob. 174 are connected together at one end, what is the voltage across the open ends? (*b*) If the current per coil is equal to its rated-load value, what will be the current in each of the three wires leading from the machine?

176. A two-phase generator rated at 2,000 kv-a., 2,300 volts, 60 cycles, has two coils. What is the current rating per coil? If this machine supplies a five-wire system, show how the wattmeters would be connected in order to measure its output.

177. Each of the coils of the alternator of Prob. 176 consists of two separate sections connected in series. The machine may be then connected in mesh. Determine its voltage and current per terminal under these conditions.

178. A two-phase, four-wire system has a potential difference of 115 volts between adjacent wires giving 163 volts, across diametrically opposite wires. Four resistances of 10 ohms each are connected between adjacent

wires, as shown in Fig. 178(*A*). Determine the total power and the current flowing in each line.

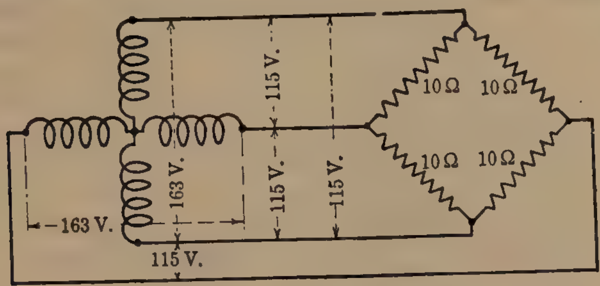


FIG. 178*A*.

179. How much current and how much power is supplied by the two alternator coils to the load in Prob. 178?

180. Across the three conductors a, b, and c of a 220-volt, 60-cycle, three-phase system, three impedances are connected, as shown in Fig. 180*A*. The sequence of phase rotation is E_{ab}, E_{bc}, E_{ca}. Taking E_{ab} along the axis of reals, find: (*a*) the current in each of the line conductors $a'a$, $b'b$, and $c'c$; (*b*) the readings of W_1 and W_2. (*c*) Check (*b*) by finding the power loss in the individual loads.

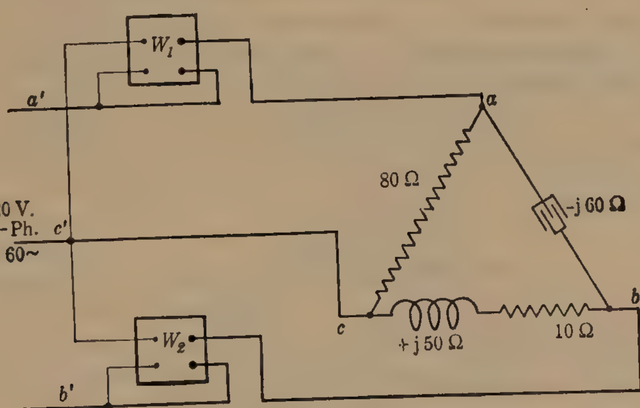


FIG. 180*A*.

181. Solve Example I (p. 126), taking the sequence of phase rotation as AC, CB, BA, and compare results. Take E_{BA} along the axis of reals.

182. Solve Prob. 180 with the 60-ohm impedance between a and b open circuited.

183. Solve Example I (p. 126) with the impedance between B and C changed to $15 + j15$.

184. In Fig. 184*A* are shown three impedances $10 + j12$, $20 - j30$, and $15 + j0$ connected between conductors a, b, c, and the neutral o of a balanced 230-volt, three-phase, 60-cycle, four-wire system. The direction of phase rotation is ao, bo, co. Choosing the position of E_{ao} along the axis of reals,

determine: (*a*) the current in each impedance; (*b*) the neutral current $I_{o'o}$; (*c*) the readings of wattmeters W_1, W_2, and W_3; (*d*) the power in each impedance determined by the method given on p. 66, Par. 43, and by the I^2R loss. (*e*) Show that the sum W_1, W_2, and W_3 in (*c*) equals the sum in (*d*).

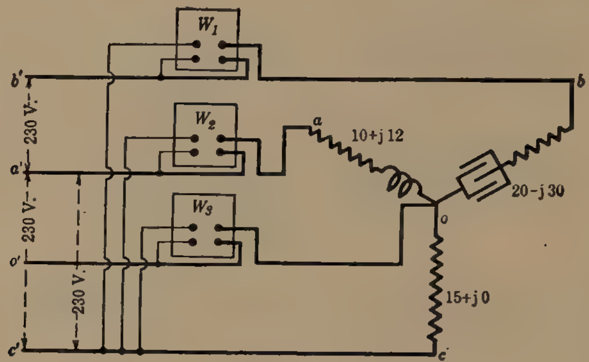


FIG. 184*A*.

185. Repeat Prob. 184, substituting for the impedances *ao*, *bo*, and *co*, $10 + j12$, $15 + j8$, and $20 - j25$, and using 600 volts, 60 cycles between outer conductors.

186. In Fig. 186*A* is shown a 200-volt, 25-cycle, two-phase, three-wire system. An impedance $10 + j12$ is connected across *ao*, and an impedance $8 + j0$ is connected across *bo*. Taking the voltage E_{ao} along the axis of reals and E_{bo} lagging 90°, determine: (*a*) the absolute currents $I_{a'a}$, $I_{o'o}$, and $I_{b'b}$; (*b*) the readings of the two wattmeters W_1 and W_2; (*c*) the power in circuits *ao* and *bo*. With (*c*) check (*b*).

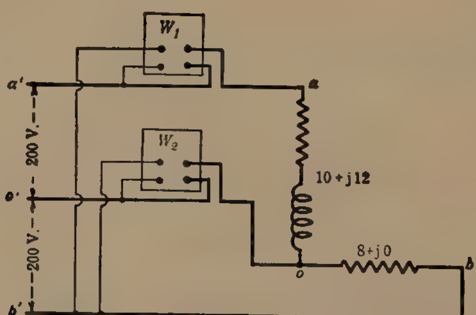


FIG. 186*A*.

187. Repeat Prob. 186, using for the impedances between *ao* and *bo*, $20 - j10$; $8 + j4$.

QUESTIONS ON CHAPTER VI

1. Why can a rotating field and a stationary armature be used for alternators where they cannot be conveniently used for direct-current machines?

Give two reasons why it is advantageous for alternators to be of the rotating-field type.

2. What two conditions must a coil of an alternator armature winding fulfill? Compare the wave and the lap winding of alternators with these same types of winding in direct-current machines.

3. Illustrate by a simple sketch the difference between a half-coil and a whole-coil winding. What is the difference between a single- and a two-layer winding? What are the objections to using one slot per pole?

4. In what way does the spiral winding differ from the lap and wave windings (barrel type) as regards mechanical disposition of the coils? Why is the interior coil usually omitted? What is meant by "single range"?

5. Show that a two-phase winding is an extended application of the single-phase winding. How may the chain winding be adapted to two-phase? Why is the two-range feature necessary? State the advantages and the disadvantages of this type of winding.

6. State the advantages of the lap winding. In the full-pitch lap winding, what relation exists between the two coil sides in any one slot?

7. Show that a three-phase winding consists of three single-phase windings properly spaced.

8. Under what conditions are coils of special shape required in three-phase, two-range, chain windings?

9. State the advantages and disadvantages of fractional-pitch windings.

10. Compare the types of stampings required for machines of large and machines of small diameter. Why are there perforations behind the slots in the stampings used for the larger machines?

11. How are the armature laminations held in position in engine-driven generators? Why is the frame usually of the hollow-box type?

12. Why is it necessary to brace very strongly the coil ends in alternators of large capacity?

13. Sketch the shape of two common types of alternator slot. State the advantages and disadvantages of each type.

14. Show that with large units, even when operating at very high efficiency, the amount of energy to be dissipated per minute is very large. What method is used to carry the resulting heat away from the machine?

15. Into how many classes are the rotating-field structures of alternators divided? Give reasons why different designs of field structure are necessary. What is the general construction of the field cores in all types of salient-pole alternators? How are these field cores held in position?

16. Describe the field spiders for (*a*) very slow-speed machines of large capacity; (*b*) moderately high-speed waterwheel generators. Why is it not possible to use salient poles in high-speed turbo-alternators?

17. Describe the construction of (*a*) the parallel-slot type of rotor; (*b*) the radial-slot type of rotor. Why must the end flanges be of non-magnetic material? Under what conditions are these types of rotor used?

18. Describe the method of conducting the field current into the field winding. What different methods are used for supplying excitation? What precautions are often taken to insure continuity of excitation?

19. Derive from a fundamental relation the equation of the induced e.m.f. in an alternator. What is meant by *breadth factor* and *pitch factor?*

20. What relation exists between the flux distribution and the shape of the e.m.f. wave per conductor? How may the shape of the actual e.m.f. wave of a generator be made nearly sinusoidal even though the e.m.f. wave in the individual conductors differs considerably from a sine wave?

21. Sketch the flux distribution curve for a distributed field winding. Explain why such machines usually have a better wave shape than machines of the salient-pole type.

22. State the procedure in phasing out the coils of a three-phase alternator so that they may be Y-connected.

23. Repeat for a machine which is to be delta-connected.

24. Upon what factors does the rating of an alternator depend? Why is a kilovolt-ampere rating more rational than a kilowatt rating? Upon which rating does the rating of the prime mover depend?

PROBLEMS ON CHAPTER VI

188. Draw a single-phase, full-pitch, four-pole, single-layer *lap* winding in which there are four slots per pole. The winding space occupies only 60 per cent. of the armature surface (see Fig. 121, p. 132).

189. Repeat Prob. 188, employing a *wave* winding. Compare the e.m.fs. in the two windings if both have the same number of series-connected conductors.

190. Draw a six-pole, single-layer winding, using half-coils. There are two slots per pole, which differ in position by 30 electrical space-degrees.

191. Using the same stator as in Prob. 190, design a two-layer whole coil, lap winding.

192. Draw a single-phase, single-range, spiral winding for a six-pole alternator in which there are eight slots per pole, the winding occupying but six of these slots.

In the following problems, show the windings of the different phases with different colors:

193. Draw a two-phase chain winding for the six-pole alternator (Prob. 192), using all the slots.

194. Repeat Prob. 193 for a similar machine having 12 slots per pole.

195. Repeat Prob. 194 for a three-phase, two-range winding.

196. An eight-pole alternator has 64 slots. Draw a two-phase seven-eighth-pitch, two-layer, lap winding for this machine.

197. A four-pole alternator has 48 slots. Design a three-phase, full-pitch, two-layer, lap winding.

198. Repeat Prob. 197 for a five-sixth-pitch winding.

199. Repeat Prob. 197 for a full-pitch wave winding.

200. Draw a four-pole, three-phase, two-layer, two-thirds-pitch, lap winding for an alternator having 15 slots per pole.

201. Determine the breadth factor in Probs. 188 and 190.

202. Determine the breadth factor for a two-pole, single-phase winding having 36 slots. The winding occupies the entire winding space.

203. Determine the breadth factor (Prob. 202) when the winding occupies only two-thirds of the winding space.

204. Determine the pitch factor in Probs. 196 and 200.

205. A four-pole, 60-cycle, single-phase alternator has a concentrated winding similar to that shown in Fig. 123 (p. 133). There are sixteen conductors per slot and 3,200,000 magnetic lines per pole. Assuming that the flux distribution under the pole is practically sinusoidal, determine the e.m.f. of this generator.

206. A six-pole, 25-cycle, two-phase alternator has one slot per pole per phase and 24 series-connected conductors per slot. Determine the induced volts per phase if there are 3,600,000 lines per pole. Assume that the flux distribution is sinusoidal.

207. A single-phase, four-pole, 1,800-r.p.m. alternator has eight slots per pole. Only half of these slots are occupied by the winding, so that the breadth factor 0.907 is the same as that of a two-phase winding having four slots per pole per phase. There are eight series-connected conductors in each slot, and the winding is full pitch. There are 4,000,000 lines per pole, and the flux may be assumed to be distributed sinusoidally. Determine the e.m.f. of this alternator.

208. A three-phase, 12-pole, 600-r.p.m. alternator has 12 slots per pole and a full-pitch winding having six series-connected conductors per slot. There are 3,500,000 lines per pole. What is the open-circuit terminal e.m.f., if the machine is Y-connected?

209. Repeat Prob. 208 for a five-sixth-pitch winding.

210. The following data are given for a two-pole, 25-cycle, 220-volt, Y-connected alternator:

Armature: 42 slots; 42 coils; three turns per coil. A two-circuit (two parallel paths for each phase) lap winding is used. Assuming a full-pitch winding, determine: (*a*) the breadth factor of this winding; (*b*) the induced e.m.f. across terminals when the flux per pole is 60,000,000 maxwells distributed sinusoidally along the gap.

211. Repeat Prob. 210 with the pitch such that when one side of a coil lies in slot 1, the other lies in slot 17.

212. The following data are given for a 12 kv-a., 220-volt, three-phase, 60-cycle alternator:

six poles; 72 slots; two coil sides per slot; 11 turns per coil; two-layer lap winding, Y-connected, three parallel circuits per phase; coil lies in slots 1 and 10; inside diameter of armature, 10 in.; axial length of armature iron, 5 in. The flux density along the air gap is a sine curve. The field current is adjusted until the maximum flux density in the air gap is 40,000 lines per square inch.

Determine: (*a*) the breadth factor of this winding; (*b*) the pitch factor; (*c*) the average flux density under a pole; (*d*) the total flux per pole; (*e*) the maximum e.m.f. induced in any single conductor; (*f*) the e.m.f. across the machine terminals.

213. An alternator is rated at 8,000 kw. at 75 per cent. power factor. (*a*) What is its kilovolt-ampere rating? (*b*) How many kilowatts can it safely deliver at unity power factor? (*c*) If it has an efficiency of 95 per cent. at 70 per cent. power factor, what should be the rating of its prime mover in horsepower? (*d*) If the prime-mover speed is 1,200 r.p.m., what torque does it develop?

214. A three-phase, 60-cycle, 13,200-volt alternator is rated at 25,000 kw. at 80 per cent. power factor. (*a*) What is the current per terminal? (*b*) What would be the safe current per terminal if the power factor were unity? (*c*) What would be its kilowatt rating at unity power factor? (*d*) At 80 per cent. power factor, what should be the approximate rating of its prime mover in horsepower?

QUESTIONS ON CHAPTER VII

1. Why is the question of the regulation of alternators more important than the regulation of direct-current generators? What factor other than the magnitude of the current determines the regulation of alternators? Why is it usually not desirable to determine the regulation of alternators by actual loading?

2. Show by a simple sketch that the inductors of an alternator armature have inductance. Compare the relative inductances, other conditions being equal, of (*a*) a smooth-core armature; (*b*) an iron-clad armature in which the slots are deep and narrow; (*c*) an iron-clad armature of the same number of slots but in which the slots are shallow and broad; (*d*) an iron-clad armature having semiclosed slots.

3. What is the effect of the number of conductors per slot upon the armature inductance? How does the reactance of a 25-cycle armature compare with that of a 60-cycle armature, other conditions being equal?

4. Give two reasons why the resistance of an alternator armature to alternating current is greater than its resistance to direct current. What is the order of magnitude of this increased resistance? How may this effective resistance be determined?

5. What is the effect of the current in an alternator coil upon the main field. (*a*) when the current is in phase with the induced e.m.f.? (*b*) when the armature current lags the induced e.m.f. by 90°? (*c*) when the current leads the induced e.m.f. by 90°? (*d*) when the current lags and leads the induced e.m.f. by an angle θ?

6. Compare the effects of 5 with corresponding effects in direct-current machines. Under what conditions does the armature m.m.f. for a given armature current have its greatest effect upon the main field of salient-pole alternators? Compare armature reaction in polyphase machines with that in single-phase machines.

7. Show by a vector diagram how the induced e.m.f. in an alternator armature may be calculated knowing the terminal voltage, the armature resistance drop, and the armature reactance drop, (*a*) when the power factor of the load is unity; (*b*) when the current lags the terminal voltage

by an angle θ; (*c*) when the current leads the terminal voltage by an angle θ. Give the trigonometric solution of the diagram in every case.

8. Repeat 7, using complex quantities.

9. Why is the induced e.m.f. in an alternator armature, when loaded, not equal to the no-load voltage? Why are open-circuit and short-circuit tests used in obtaining data for calculating alternator regulation rather than actually loading the machine?

10. How is the armature *reaction* taken into consideration in the synchronous-impedance method of determining regulation?

11. Show that when a coil has moved 90 electrical space-degrees from the point where the flux linking it is a maximum, the induced e.m.f. becomes a maximum. Distinguish between a *space* diagram and a *time* diagram. When can the two be combined?

12. Why is it rational to combine the space m.m.f. diagram of an alternator with the time-voltage and time-current diagrams of the same machine? What is the phase relation existing between (*a*) the armature current and the armature m.m.f.; (*b*) the resultant field and the induced e.m.f.; (*c*) the impressed field and the no-load e.m.f.?

13. Show that a fictitious voltage of the proper value leading the current by 90°, and, therefore, in phase with the voltage which balances the armature reactance drop, can be substituted for the effect of armature reaction and the no-load e.m.f., therefore, be determined. What armature constant may be increased to include this fictitious voltage, and what assumption is made in doing this?

14. What is meant by *synchronous reactance? synchronous impedance?* Describe carefully the method usually employed to determine these quantities.

What error occurs in the value of the synchronous reactance when it is determined under short-circuit conditions? How does this affect the regulation determined by using this value of synchronous reactance?

15. Why does the synchronous-impedance method of determining regulation give unsatisfactory results with single-phase machines? Why are results obtained with polyphase machines more in accord with the actual performance of the machine?

16. Describe the open-circuit test, giving the connections used. Repeat for the short-circuit test, giving two methods of connecting the ammeters. Compare the ammeter readings in each case with the line current and the coil current of a delta-connected machine.

17. How is the regulation of a Y-connected machine calculated? of a delta-connected machine? How do the respective coil resistances and reactances compare in the two cases for the same machine? What care should be taken when either method is used?

18. In what fundamental way does the m.m.f. method differ from the synchronous-impedance method? Show by a vector diagram the various voltages and the m.m.fs. which are substituted for voltages in this method. How is the resultant field obtained? the no-load e.m.f.?

19. How do results obtained by the synchronous-impedance method compare with those obtained by the m.m.f. method? Why do the two methods give different results? Which should be used?

20. Fundamentally, how does the A. I. E. E. method of determining regulation differ from the synchronous-impedance and the m.m.f. methods? What difficulty is encountered in this method which is not encountered in the other two methods? How do the results which it gives compare with the actual performance of the machine?

21. Compare the construction of the Tirrill regulator as applied to alternators with the regulator as used on direct-current machines (see Vol. I, p. 362, Fig. 308). What is the function of the *main contacts*, and how are they operated? What is the function of the *relay contacts*, and how are they operated? Explain the operation of the entire regulator from the time that the exciter voltage commences building up until the machine has reached rated voltage, and load then applied.

22. Explain why the prime-mover characteristics alone determine the kilowatt division of load between alternators. Why is this true of alternators and not true of direct-current generators? Why is it undesirable that the prime movers have flat speed-load characteristics?

23. What effect has a temporary change of speed of one prime mover on the phase relation of the e.m.f. induced in the two machines which they drive? What effect does the resultant voltage produce? Why is the resultant current called the *synchronizing* current? Show that the action of this current is such as makes the parallel operation of alternators a condition of stable equilibrium.

24. If the field of one of two alternators operating in parallel is strengthened, in what two ways is its internal e.m.f. affected? its current? why? How are the e.m.f. and the current of the other machine affected at this same time? Show that the reactions resulting from changing these field excitations cannot change the kilowatt division of load between the machines. What is the objection to having two alternators in parallel, one operating with a leading current and the other with a lagging current?

25. Sketch the connections of a simple method which may be used to show the proper time for switching alternators in parallel. How should the voltage rating of the synchronizing lamps compare with that of the system? How do such lamps indicate the relative phase relation of the incoming machine and the bus-bars? When should the line switch be closed?

26. State two disadvantages of the *three-dark* method of synchronizing. How may these disadvantages be, in part, eliminated by a different grouping of the lamps? Why is the use of a synchronism indicator superior to the foregoing methods, especially with certain types of alternators?

27. What types of prime mover have pulsating torques? How may the effect of these torque pulsations be magnified several times by direct-connected alternators? Why is it undesirable that pulsations of frequency be communicated to the system? State the general remedies which may be used to reduce *hunting* and the reason for the use of each of these.

PROBLEMS ON CHAPTER VII

(Problems marked * may be solved trigonometrically and with complex quantities.)

***215.** A 550-volt, 50-kv-a., single-phase alternator has an effective armature resistance of 0.18 ohm and an armature *reactance* of 0.65 ohm. What is the induced e.m.f. in this armature when the machine delivers its rated current at rated voltage and at unity power factor?

***216.** Repeat Prob. 215 for 0.8 power factor, lagging current; leading current.

217. The core loss, friction, and windage of the alternator of Prob. 215 is 1,700 watts. The field takes 15 amp. at 115 volts. What is the efficiency of this alternator at its rated load and unity power factor?

***218.** The synchronous reactance of the alternator of Prob. 215 is equal to 2.5 times the armature leakage reactance. If it is carrying full non-inductive load at rated voltage and at unity power factor, find the no-load voltage of the machine when this load is disconnected.

***219.** Find the no-load voltages in Prob. 216 when these loads are disconnected from the alternator.

220. What is the regulation of the machine under each of the conditions given in Probs. 218 and 219?

***221.** A 25-cycle, 25-kv-a., 550-volt, single-phase alternator has an effective armature resistance of 0.35 ohm and a synchronous impedance of 3.2 ohms. What is its synchronous reactance, and what is the regulation of this alternator at unity power factor; at 0.7 power factor, lagging current; at 0.7 power factor, leading current?

***222.** A three-phase, 100-kv-a., 600-volt, Y-connected, 60-cycle alternator has the following constants per phase or coil: effective armature resistance, 0.096 ohm; leakage reactance, 0.19 ohm; synchronous reactance, 1.2 ohms. With constant terminal voltage of 600 volts, determine: (*a*) the induced e.m.f. between terminals at unity power factor and 0.8 power factor lag; the regulation at (*b*) unity power factor; (*c*) 0.8 power factor lag; (*d*) 0.8 power factor lead.

***223.** A 3,000-kv-a., 2,200-volt, 25-cycle, two-phase alternator has the following constants for each phase: effective armature resistance, 0.190 ohm; synchronous impedance, 1.5 ohm. At unity power factor, the field current is 160 amp. at 125 volts; the friction and windage, 4.0 kw.; the core loss corresponding to the resultant field, 30 kw. Determine: (*a*) the regulation at power factors of unity, 0.70 lag and 0.70 lead; (*b*) the efficiency at unity power factor.

***224.** Following are the constants of a 250-kv-a., 60-cycle, 440-volt, three-phase, eight-pole alternator:

Average ohmic resistance between terminals, 0.038 ohm.

Field current adjusted to 42 amp., open-circuit terminal volts = 240 volts. Field current adjusted to 42 amp., generator short circuited, three line currents each equal to 350 amp. Ratio of effective to ohmic resistance = 1.30. Calculate the regulation of the machine with the power factor

0.8 lagging and 0.8 leading current. Assume that the machine is Y-connected.

***225.** Repeat Prob. 224, assuming the machine to be delta-connected.

226. The friction and windage loss (Prob. 224) is 1,900 watts; the core loss corresponding to the resultant field at 0.8 power factor lag is 5,800 watts; the exciting current is 79 amp.; and the excitation voltage is 125 volts. At rated load and 0.8 power-factor lag, determine: (*a*) the generator efficiency; (*b*) the horsepower input; (*c*) the torque on the driving shaft.

227. Below are given the open- and short-circuit data for a 15 kv-a., 220-volt, 60-cycle, six-pole, Y-connected alternator. The excitation voltage is 115 volts.

Field current	0	1	2	3	3.5	4	5	6	7
Terminal volts	0	78	144	198	220	237	265	284	296
Line current	0	11	22	34	40	46	57	69	80
Core loss	0	30	74	130	162	200	294	400	

The ohmic resistance between each pair of terminals is 0.25 ohm. When the short-circuit current is 40 amp., the input to the alternator shaft is 1,160 watts, of which 350 watts are friction and windage.

Plot the curves of terminal volts, line current, and core loss with field current as abscissas. Plot a curve of synchronous reactance and field current. Determine the ratio of effective to ohmic resistance with 40-amp. line current.

***228.** In Prob. 227, determine by the synchronous-impedance method and at rated current the regulation at power factors of unity, 80 per cent. lag, and 80 per cent. lead. Also, determine the excitation kilowatts for each of these conditions.

***229.** The armature leakage reactance per phase (coil) of the alternator (Prob. 227) may be taken as 0.6 ohm. Determine the *induced* e.m.f. at rated current and at power factors of unity, 0.8 lag, and 0.8 lead. Also, determine the resultant field, in terms of field current, for each of these conditions.

230. Determine the efficiency of the alternator (Prob. 227) under the three conditions given in Prob. 228. Use data from Prob. 228 for obtaining the field loss and data from both Probs. 227 and 229 for obtaining the core loss.

***231.** Repeat Prob. 228, assuming the machine to be delta-connected.

***232.** Determine the regulation and excitation kilowatts of the alternator (Prob. 227) at power factors of unity, 0.8 lag, and 0.8 lead, using the m.m.f. method.

233. Below are given data for the zero-power-factor curve of the alternator (Prob. 227) taken with a constant armature current of 39.4 amp.

Field current	3.5	4.0	5.0	6.0	7.0	8.0	9.0
Terminal volts	0	40	103	157	199	230	252

Plot both this saturation curve and the open-circuit saturation curve. Plot a curve of synchronous reactance and field current. Compare with values obtained in Prob. 227. Explain.

***234.** Determine, by the A. I. E. E. method, the regulation and excitation kilowatts of the alternator (Probs. 227 and 233) at power factors of unity, 0.8 lag, and 0.8 lead. Compare results with those obtained in Probs. 228 and 232.

***235.** A 1,000-kv-a., three-phase, 2,200-volt, 60-cycle alternator is tested for its regulation by the A. I. E. E. method. A load of practically zero-power-factor lagging current is applied, this load being adjusted until the rated current of the machine is flowing. The field current is then adjusted until the machine terminal voltage is 2,200 volts. Under these conditions, the field current is 250 amp. When the field current is 250 amp., the terminal voltage on open circuit is 2,900 volts. The machine is delta-connected and has an effective resistance of 0.34 ohm per coil. Find the synchronous reactance of this machine and its regulation at unity power factor and at 0.8 power factor, lagging current.

236. Two similar 1,500 kv-a. alternators operate in parallel. The speed-load characteristic of the first alternator is such that its frequency drops from 61 to 59 cycles from no load to 1,500-kilowatt load. The frequency of the second alternator drops from 61 to 58.5 cycles under the same conditions. When the combined load on the two alternators is 2,400 kw., how much load does each alternator supply? What is the maximum total unity-power-factor load that the two alternators can deliver without either machine's being overloaded?

237. The tension of the governor spring of the second alternator of Prob. 236 is so adjusted that both alternators have the same frequency when the load on each is 1,200 kw. This change in the speed-load characteristic of the second machine raises its speed-load characteristic the same number of cycles at every point. How much power does each alternator deliver (*a*) when there is no load on the system? (*b*) When the total combined load on the system is 2,800 kw.?

238. Two alternators are operating in parallel, supplying single-phase power at 2,300 volts to a load of 800 kw. whose power factor is unity. No. 1 alternator supplies 160 amp. at 0.8 power factor, lagging current. What power and what current does alternator 2 supply? What is its power factor? Draw a vector diagram.

239. Two three-phase alternators are operating in parallel to supply a 4,000-kw. load at 6,600 volts, this load having unity power factor. The current delivered by alternator 1 is 200 amp. at 0.85 power factor, leading current. What power and what current is alternator 2 delivering? What is its power factor?

QUESTIONS ON CHAPTER VIII

1. Define a *transformer*. What distinct advantages do transformers possess over most other types of electrical machinery?

2. By what means is energy transferred from one circuit to the other? Which winding is called the *primary?* the *secondary?*

3. Show that the induced e.m.f. of a transformer winding is proportional to the number of turns. To what three factors is the induced e.m.f. in any transformer winding proportional?

4. What current flows into a transformer primary when the secondary is open? What is its order of magnitude? What is the relation of the direction of primary current to the direction of flux in the core? of the secondary current? Explain.

5. Explain the sequence of reactions which cause the primary to take more power from the line when load is applied to the secondary.

6. Why is the mutual flux in a transformer nearly constant from no-load to rated load? What is the magnitude of the variation of the magnetizing current under these conditions?

7. What relation exists between primary ampere-turns and secondary ampere-turns? What relation exists between primary current and secondary current?

8. Discriminate among primary leakage flux, secondary leakage flux, and mutual flux. Which of the foregoing depend upon the voltage and which depend upon the current?

9. What effect have the two leakage fluxes upon the operation of the transformer?

10. In the complete vector diagram of the transformer, why are the primary and the secondary induced voltages shown equal in magnitude and in phase with each other? Why is a voltage equal and opposite to the primary induced voltage necessary in order to find the voltage at the terminals of the primary?

11. Show that the total primary current is not equal and opposite to the secondary current even when both windings have the same number of turns. Resolve the primary current into two components, explaining why one of these components varies with the load on the transformer secondary.

12. What is meant by *transformer regulation*, and what assumptions are usually made in obtaining it?

13. Explain what approximation is made in obtaining the simplified transformer diagram. What advantages result in so making the diagram?

14. What is the relation ordinarily existing between the primary and secondary resistances in a transformer? What is meant by *equivalent resistance referred to the primary*, and how is this quantity used?

15. Discuss the relation usually existing between the primary and the secondary reactance, giving reasons for the existence of this relation. What is meant by *equivalent reactance referred to the primary?* How is this quantity used in determining the transformer characteristics?

16. What relation exists between the equivalent reactance referred to the primary and that referred to the secondary?

17. Show that if one side of a transformer is open and the other side is connected to the line, practically the entire input goes to supply the core loss. How does this core loss vary with the voltage? Why?

18. Why do both the magnetizing current and the core loss increase very rapidly after the rated voltage of the transformer has been reached? Why is it practically impossible to operate transformers at voltages very much in excess of those for which they are rated? How is the true magnetizing current found?

19. When one side of a transformer is short circuited and the other side is connected to an alternating supply, show that the input goes almost entirely to supplying the copper losses of the primary and secondary coils. How are the equivalent impedance and the equivalent reactance referred to either side determined from the short-circuit test?

20. What losses exist in a transformer operating under load? How may these losses be computed for different loads? Indicate the method of calculating the efficiency over the working range of the transformer? What are the advantages of this method over direct measurements of output and input?

21. In what way does the core type of transformer differ in construction from the shell type? Compare the dimensions of the electrical and magnetic circuits in the two cases. Which type is better adapted for high voltage and why? How is the leakage flux reduced to a minimum in the two types?

22. What advantage is claimed for the type-H transformer of the General Electric Company? What provisions are made for keeping this type of transformer cool?

23. Describe one method of keeping transformers cool. Apart from its cooling properties, what other advantage is obtained by using oil?

24. Describe two other methods used to dissipate the heat from self-cooled transformers when the surface of the case itself becomes inadequate.

25. What are the advantages and disadvantages of air-cooled transformers? How is the circulation of air maintained?

26. Describe the method ordinarily used for artificially cooling oil-filled transformers. What care should be taken when this method is used?

27. Explain the principle upon which three-phase transformers operate. What are the advantages and the disadvantages of this type of transformer? From the operating standpoint, in what ways do the shell type and the core type of three-phase transformer differ?

28. In what way does the auto-transformer differ from a resistance drop wire? From an ordinary transformer?

29. Under what conditions is it advantageous to use an auto-transformer? Under what conditions should an auto-transformer not be used? How may an ordinary transformer be connected to operate as an auto-transformer?

30. Indicate the different connections that can be used for three-phase transformer banks. State the conditions for which each connection is best adapted.

31. What is meant by a *floating neutral*, and by what connection is it produced? How may it be eliminated?

32. Under what conditions cannot three-phase transformer banks be operated in parallel, even although the ratios between line voltages are alike for the several banks?

33. Give the reasons why, at no load, the three three-phase voltages existing across delta-delta-connected transformer secondaries are not in any way disturbed by the removal of one of the transformers, if the voltages are balanced.

34. What is the ratio of the kilovolt-ampere capacity of the delta-connected bank to the V-connected bank? Under what conditions is the V-connection used?

35. Make a diagram of the T-connection when used for transforming three-phase to three-phase. How does the total three-phase kilovolt-ampere capacity of the T-bank compare with the sum of the kilovolt-ampere capacities of the individual transformers?

36. Show how the T-connection may be used for obtaining a two-phase, three-wire system. What connection is necessary if the three-wire system is to have equal voltages on both legs?

37. How may a two-phase (or four-phase) four- or five-wire system be obtained from the T-connection? Where is the neutral of the T?

38. How does the construction of a constant-current transformer differ from that of a constant-potential transformer? Assuming a change of load, analyze the reactions which cause the transformer to maintain the current constant. Why is the power factor of this type of transformer usually low?

39. Sketch the connections of a constant-current transformer as used with a mercury-arc tube to obtain unidirectional current for magnetite arcs. Why is reactance necessary?

40. For what reasons is it necessary to use instrument transformers for measuring power on high-voltage, alternating-current circuits?

41. Describe potential transformers. What is the usual voltage rating of their secondaries? Why should the secondaries always be well grounded at one point?

42. Describe the construction of a current transformer. What prevents it from giving a ratio of transformation that is exactly proportional to the ratio of secondary to primary turns? Why should the secondary always be kept closed? In what ways does a current transformer differ from a constant-potential transformer?

PROBLEMS ON CHAPTER VIII

240. A 10-kv-a., 60-cycle transformer, rated at 2,200/220 volts, has 880 turns on the primary or high-side winding. How many secondary turns are there if the secondary no-load voltage must be increased 4 per cent. to allow for the 4 per cent. voltage drop in the transformer when under load?

241. A 2200/110-volt transformer has 60 turns in the low-side winding. How many turns are there in the high-side winding if compensation is made for a 4 per cent. voltage drop in the transformer when rated load is applied to the low side?

242. What voltage is induced in a transformer winding of 1,200 turns if the frequency is 60 cycles per second and the maximum value of the flux is 3,200,000 lines (assume sine wave)?

243. A certain 100 kv-a., 60-cycle, 13,200/660-volt transformer is to have a primary winding of 2,000 turns and a secondary winding of 108 turns. What must be the cross-section of the core if the maximum flux density in the iron is 60,000 lines per square inch? The ratio of net iron to the total volume of the core is 0.9.

244. Repeat Prob. 243 for a 25-cycle transformer operating at 75,000 lines per square inch. Which transformer has the more iron and the more copper? Why? Explain why the iron in a 25-cycle transformer can be operated at a higher flux density than in a 60-cycle transformer.

245. A 2,200/550-volt transformer is rated at 50 kw. at unity power factor. What is the current rating of each winding? What is the current rating of each winding at 0.8 power factor, lagging current?

246. A 30 kv-a., 2,400/240-volt transformer at no load takes 3.20 amp. and 206 watts at 240 volts. What is the energy component and what is the magnetizing component of this no-load exciting current? If a load is applied to the transformer so that an additional 30 amp., substantially in phase with the line voltage, is taken from the line, what does the total primary current become? What is the power factor of the primary? If the transformer steps the voltage up 1 to 10, what is the approximate value of the secondary voltage?

247. A 25 kv-a., 2,200/600-volt transformer has the following constants:

High-side resistance, 1.42 ohms; low-side resistance, 0.10 ohm; high-side reactance 3.2 ohms; low-side reactance, 0.22 ohm. Determine: (*a*) the equivalent resistance referred to both the high and low sides; (*b*) the equivalent reactance referred to both the high and low sides.

***248.** Determine the regulation of the transformer (Prob. 247) at unity power factor and at 0.8 power factor, lagging and leading current. Use both the high- and the low-side equivalent resistances and reactances.

249. Determine the efficiency of the transformer of Prob. 247 at $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{2}$, rated, and $\frac{5}{4}$-kv-a. load at unity power factor. Plot a curve with current as abscissas. The no-load core loss is 137 watts.

250. Repeat Prob. 249 for 0.8 power factor, lagging current.

***251.** A 200-kv-a., 13,800/2,300-volt transformer has the following constants:

High-side resistance, 6.2 ohms; low-side resistance 0.145 ohm; equivalent reactance referred to the high side, 26.0 ohms. Determine the regulation of the transformer at unity power factor; at 0.707 power factor, lagging and leading current.

252. The core loss (Prob. 251) is 1,200 watts. Determine the efficiency at one-fourth, one-half, three-fourths, and full load, 0.8 power factor, lagging current.

(Problems marked * may be solved trigonometrically and with complex quantities.)

***253.** The following data are taken from open- and short-circuit tests of a 30-kv-a., 2,400/240-volt, 60-cycle transformer:
High-side open, instruments on low-voltage side:

$E = 240$ volts $\quad I = 2.20$ amp. $\quad P = 230$ watts

Low-side short circuited, instruments on high side:

$E = 70 \quad I = 18.8 \quad P = 1,050$

Ohmic resistance: High side, 1.19 ohms; low side, 0.0154 ohm. (*a*) Determine the regulation of this transformer at unity power-factor and at 0.8 power factor, lagging and leading current. (*b*) Determine the transformer efficiency under each of the above conditions. (*c*) What is the ratio of effective to ohmic resistance in this transformer?

254. Determine the all-day efficiency of the transformer (Prob. 247) under the following loading: five-fourths load, 1 hr.; full load, 4 hr.; one-half load, 4 hr.; one-fourth load, 2 hr.; no load, 13 hr. Power factor = 1.0.

255. Determine the all-day efficiency of the transformer (Prob. 253); five-fourths load, 1 hr.; full load, 6 hr.; three-fourths load, 4 hr.; one-half load, 3 hr.; one-fourth load, 4 hr.; no load, 6 hr. Power factor = 1.0.

256. A 20-kv-a., 1,100/110-volt transformer is connected as shown in Fig. 256*A* so that it acts as a booster to raise the line voltage from 1,100 to 1,210 volts. Neglecting magnetizing current and voltage drops in the transformer, determine: (*a*) the power received by the system; (*b*) the power delivered by the system; (*c*) the power transformed; (*d*) the power which flows through without transformation. (*e*) If the efficiency of the transformer is 97 per cent., what is the efficiency of the entire system? In the above problem, the transformer currents must not exceed the values given by the transformer rating.

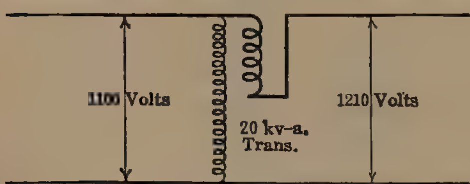


FIG. 256*A*.

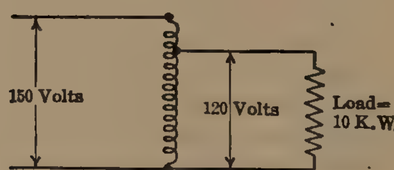


FIG. 258*A*.

257. Repeat Prob. 256 with the secondary reversed so that the ultimate voltage is 990 volts. Which coil is the primary and which is the secondary under these conditions?

258. Figure 258*A* shows a compensator used for reducing the voltage from 150 volts to 120 volts for a 10-kw. non-inductive load. Indicate all the currents and all the voltages existing in the system. How much power is transformed and how much passes through without transformation? Neglect losses.

259. An auto-transformer boosts the voltage of a feeder from 2,300 to 2,500 volts. When the load on the 2,500-volt side is 100 kw., power factor = 1.0, determine the current in each portion of the transformer, indicating primary and secondary. How much power is transformed, and how much flows through conductively?

260. Power generated in a station, by a three-phase, 10,000 kv-a., 50-cycle alternator at 6,600 volts, is transformed for transmission at 70,000 volts by a delta-Y-connected transformer bank. The voltage is then stepped down by a delta-delta bank to 13,800 volts for distribution. What is the current, voltage, and kilovolt-ampere rating of each transformer? Give the current and the voltage rating of the primary and secondary of each transformer. Neglect losses.

261. Power is generated at 6,600 volts, 60 cycles, three-phase, stepped up to 140,000 volts for transmission with a delta-Y transformer bank, stepped down to 26,400 volts for distribution with a Y-delta transformer bank. If 15,000 kv-a. are transmitted, determine the current and voltage at each portion of the circuit; the current, voltage, and kilovolt-ampere rating of each transformer. Neglect losses.

262. A certain substation receives three-phase power at 13,800 volts, and this power is stepped down to 2,300 volts by two 250-kv-a., V-connected transformers. If the power factor of the secondary load is unity, what is the maximum power which the transformers can deliver without exceeding their ratings? What is the current and voltage rating of each winding?

263. Repeat Prob. 262 for three 250-kv-a. transformers connected delta-delta. Compare the percentage increase in capacity with the percentage increase in investment.

264. Power is received at a certain factory at 2,300 volts, three-phase. It is desired to transform it to 230 volts, four-phase, five-wire. If the total power is 150 kw. at 0.8 power factor, what should be the ratings of the transformers if the Scott connection is used? Make a sketch showing the method of connecting these transformers. What is the primary current in each transformer? the secondary current?

265. If the transformers of Prob. 264 were used to obtain a 230-volt, two-phase, three-wire system, indicate the currents and voltages in each part of the system. Make a sketch.

266. A single-phase line delivers 100 kw. at 2,200 volts, 0.8 power factor. An ammeter, a voltmeter, an indicating wattmeter, and a watthour meter are all necessary in operating this circuit. Sketch the connections of the instrument transformers and the instruments. Give the ratios of the transformers and the factor by which each instrument reading must be multiplied in order to obtain the corresponding value of the current, voltage, power, etc., existing in the high-voltage circuit. The ammeter and the wattmeter should have 5-amp. ratings at rated load. Allow for 50 per cent. overload.

QUESTIONS ON CHAPTER IX

1. Describe a simple experiment illustrating the underlying principle of induction-motor action. Show that the tendency of the rotor to follow the inducing magnetic field is another illustration of Lenz's law. Why cannot the rotor attain the speed of the inducing magnetic field?

2. Make a sketch of a two-phase gramme-ring winding and sketch the position of the magnetic field for three or four different values of the currents. Repeat for a three-phase drum winding. What is the relation between the space advance of the magnetic field and the time change of the currents?

3. What is meant by *revolutions slip? percentage slip?* Show how the rotor frequency is related to the slip.

4. Upon what three factors does the torque developed by an alternating-current motor depend? Plot a sine wave of current and a sine wave of flux about 45° out of phase, and then plot the resulting torque curve.

5. Describe the construction of a squirrel-cage rotor, stating the various methods of making the end-ring connections.

6. Why does the rotor slip increase with increased load? What is the order of magnitude of the slip in commercial motors?

7. What direct-current motor characteristics do the squirrel-cage motor characteristics resemble? Why do the power factor and the efficiency of the induction motor increase rapidly with load?

8. State one very serious objection to the squirrel-cage motor for certain types of service. Analyze carefully the reasons why this type of motor develops but little torque at starting, although it takes an unusually large current. Under what conditions is the torque a maximum when the current and flux are fixed in magnitude?

9. Sketch a typical slip-torque curve of an induction motor. What is meant by the *breakdown torque?* Upon what three factors does it depend?

10. Name several industrial applications to which the squirrel-cage motor is particularly well adapted.

11. Sketch the special connections used for starting squirrel-cage motors when no starting compensator is necessary. Why are starting compensators necessary? Make a diagram of two different types.

12. What is the effect upon the slip of an induction motor of introducing resistance into the rotor circuit? Explain. What is the distinct disadvantage of controlling the speed of the motor by inserting resistance into the rotor circuit?

13. Why are wound-rotor induction motors often necessary? Compare their starting characteristics, their operating characteristics, and their cost with those of squirrel-cage motors. State a few of the industrial applications of the wound-rotor motor.

14. What is the effect upon the operation of the induction motor (*a*) of increasing the length of the air-gap; (*b*) of using open slots; (*c*) of using semiclosed stator slots; (*d*) of using semiclosed rotor slots? Discuss the mechanical construction of the motor with special reference to air-gap requirements.

15. What three quantities determine the speed of the induction motor? State briefly the underlying principle of a method of speed control which does away with some of the disadvantages of introducing resistance into the rotor circuit. Make a diagram of connections and discuss the efficiency of this method. Where would such methods of speed control be used?

16. Give an example where speed may be controlled by change of frequency, stating the limits of such speed control.

17. State how the number of poles of an induction motor may be changed in order to give different speeds. What are the limitations of this method?

18. What is meant by *concatenation?* Discuss this method of speed control, giving a diagram of connections when two similar motors are used. To what direct-current method of speed control does this correspond?

19. Under what conditions will an induction machine develop electrical energy? State (*a*) the rotor reactions which cause the reversal of electrical energy in the rotor; (*b*) the effect of these reactions upon the stator. (*c*) How is the load of the induction generator controlled? (*d*) Whence does the machine obtain its exciting current? (*e*) What determines its frequency and voltage?

20. State the advantages and the disadvantages of the induction generator as compared with the synchronous generator. Why is the machine power factor determined by the machine itself and not by the load? Illustrate with a vector diagram. To what type of work is the induction generator particularly well adapted?

21. What measurements are necessary in order to obtain data for the construction of the circle diagram? Why is reduced voltage used in the blocked run? How is the diameter of the semicircle determined? What construction is necessary in order to separate the primary and secondary copper losses?

22. How are the following factors determined for any given value of primary current: (*a*) secondary current; (*b*) power input; (*c*) core and friction losses; (*d*) primary copper loss; (*e*) secondary copper loss; (*f*) output; (*g*) efficiency; (*h*) torque; (*i*) slip; (*j*) power factor?

23. Why is it inaccurate to determine the slip by measuring the rotor and synchronous speeds and then subtracting? Describe the principle of the stroboscope method. How may slip be measured mechanically?

24. What type of common alternating-current machinery does the induction regulator resemble? What windings has the regulator, and where are they placed? Why is a tertiary winding necessary in the single-phase regulator, and where is it placed?

25. How is the regulator operated? How is it connected to the circuit? Compare the three-phase with the single-phase regulator.

PROBLEMS ON CHAPTER IX

267. A three-phase, 60-cycle induction motor has eight poles. Through how many space-degrees will the rotating field advance during one cycle? What is the speed of the rotating field in r.p.s.? in r.p.m.?

268. Repeat Prob. 267 for a 25-cycle motor of the same number of poles.

269. It is desired to obtain an induction motor which shall have a speed in the neighborhood of 400 r.p.m. How many poles should this motor have if 60-cycle power is available? 25-cycle-power?

270. The rotor of a six-pole, three-phase, induction motor rotates at 1,160 r.p.m. when the motor is connected across 60-cycle mains. What is the slip of the motor in per cent.? What is the frequency of the currents in the rotor when it is running at this speed?

271. A 25-cycle, four-pole induction motor has a speed of 715 r.p.m. What is the frequency of its rotor currents?

272. A certain squirrel-cage induction motor develops a starting torque of 60 lb.-ft., when it is connected to the 40 per cent. taps of a three-phase starting compensator, and the line current is 80 amp. What are the approximate starting torque and the line current when it is connected to the 50 per cent. taps? 60 per cent. taps?

273. What is the approximate ratio of the breakdown torques of two similar induction motors, one of 25-cycles and the other of 60-cycles, if the rotor inductance, the flux, and the currents are the same in each?

274. A 10-hp., 230-volt, three-phase, 60-cycle induction motor, when connected directly across 230-volt, three-phase mains, takes 130 amp. at the instant the circuit is closed. What current will the motor take and what will be the line current if a compensator similar to the one in Fig. 262 (p. 291) is used and the motor is connected to the 50 per cent. taps? 40 per cent. taps?

275. Repeat Prob. 274 for the 60 per cent. taps. What will be the relation of the starting torque in the three cases 50, 60, and 40 per cent. taps to the starting torque at full voltage?

276. A six-pole, 60-cycle, three-phase wound-rotor, induction motor is taking 10,000 watts from the line. The core loss plus friction losses is 700 watts and the total I^2R loss of the stator is 300 watts. The total I^2R loss of the rotor is 400 watts. What is the motor efficiency under these conditions? At what speed does it run, and what torque does it develop? (*Hint.*—The ratio of the rotor I^2R loss to the rotor input is proportional to the slip.)

277. Find the motor efficiency in Prob. 276 if resistance is introduced in the rotor circuit so that the motor runs at (*a*) 900 r.p.m. (*b*) 600 r.p.m.

278. A 150-hp., 2,200-volt, 60-cycle, six-pole induction motor at no load takes 4.0 kw. from the line. The stator is Y-connected and has an effective resistance of 0.9 ohm per phase; the rotor is Y-connected and has a resistance of 0.07 ohm per phase. The ratio of transformation, stator to rotor, is $4/1$. When the motor is taking its rated current of 36 amp. from the line at 0.91 power factor, determine approximately: (*a*) the stator copper loss; (*b*) the rotor copper loss; (*c*) the output; (*d*) the efficiency; (*e*) the slip; (*f*) the torque.

279. A-10 hp., 230-volt, six-pole, 60-cycle, 1,140-r.p.m., induction motor is tested by means of a prony brake. The data are as follows:

Volts	Average amperes per line	Kilowatts		Balance, pounds	Revolutions slip	Frequency
		P_1	P_2			
229	11.3	−0.97	1.40	1.85	1.2	58.0
230	11.6	−0.38	1.96	4.15	9.75	58.5
229	14.8	+0.57	3.20	10.90	27.5	58.0
229	18.0	+1.20	4.00	16.00	40.0	58.5
229	20.6	+1.65	4.70	19.00	48.0	58.5
228	24.0	+2.45	5.42	23.00	56.0	58.0
228	26.6	+2.95	6.12	26.00	60.0	58.0
227	29.4	+3.40	6.75	29.00	76.0	58.5

The brake arm is 2 ft. long, and its dead weight is 1.85 lb. The two-wattmeter method is used. From the above data, compute the following: torque; percentage slip; speed; horsepower output; efficiency; power factor. Plot the above data with horsepower as abscissas. Why does the efficiency increase to a maximum and then decrease? Why does the power factor increase with the load? From the two wattmeter readings, determine the power factor, using either Eq. (65) (p. 119) or Fig. 108 (p. 120). Compare these values with those obtained from dividing the total power by the volt-amperes.

280. The following data were taken with a 5-hp., 220-volt, 60-cycle, six-pole, three-phase induction motor for the purpose of making a circle diagram:

No load: $E = 220$ volts; $I = 5.94$ amp.; $W_1 = +960$ watts; $W_2 = -2.94$ watts. Blocked test: $E = 53$ volts; $I = 15.3$ amp.; $W_1 = +750$ watts; $W_2 = +140$ watts.

Stator ohmic resistance per phase, 0.82 ohm; rotor resistance per phase, 0.38 ohm. Both stator and rotor are Y-connected, and the ratio of transformation is $\frac{1}{1}$. With the foregoing data, draw a circle diagram similar to Fig. 274 (p. 309). Divide the line *HF* proportionately to the rotor and stator resistances. Use a scale of at least 1 in. = 2 amp. From the circle diagram, determine, for a current of 15.3 amp., (*a*) the no-load losses; (*b*) stator copper loss; (*c*) rotor copper loss; (*d*) output in horsepower; (*e*) slip; (*f*) power factor; (*g*) torque.

281. In Prob. 280, determine: (*a*) maximum power factor; (*b*) breakdown torque; (*c*) maximum output.

QUESTIONS ON CHAPTER X

1. What suggests that both the shunt and the series direct-current motors might possibly be used with alternating current? Why is it not possible to use the shunt motor effectively with alternating current? What characteristic of the series motor makes it possible for this type of motor to operate effectively with alternating current?

2. In what way does the field structure of an alternating-current series motor differ from that of the direct-current series motor? How does the number of series turns of the alternating-current motor compare with the number ordinarily used with the direct-current motor of corresponding rating? Why are the poles short and of comparatively large cross-section? Why is the air-gap short? Why is low frequency necessary?

3. Why does the alternating-current series motor have a large number of armature turns? Give two reasons why armature reaction must be compensated. Show two methods of compensating.

4. What commutating difficulty exists in the alternating-current motor, which is not present in the direct-current motor? How is this difficulty met? Why is there a large number of commutator segments?

5. Sketch a vector diagram of the motor. How is the speed controlled? Where is this type of motor used?

6. What is the nature of the induced e.m.fs. in a gramme-ring armature having a commutator, when the armature is placed in a single-phase, alternating-current field? What occurs when the brushes are short circuited and placed in the geometrical neutral? when these brushes are placed parallel to the pole axis? Why is no torque developed in either case?

7. Why is torque developed when the brush axis makes some angle greater than zero and less than 90° with the pole axis? How is the direction of rotation controlled? How may the field structure be wound so that the brushes may be left in the geometrical neutral?

8. Why are repulsion motors made with uniform air-gaps rather than with salient poles? What is the nature of the speed and torque curves of the repulsion motor?

9. Show that a single-phase, alternating-current field can be replaced by two fields rotating around the air-gap in opposite directions. Sketch the slip—torque curve due to each of these two fields. How may the fact that the single-phase induction motor has no starting torque be explained by these curves? How do they explain the fact of the motor's accelerating in the direction in which it is started?

10. By means of a sketch, show the position of the rotor ampere-turns of a single-phase induction motor when the transformer currents alone are considered. Show the direction of the magnetic field which these ampere-turns produce.

11. Show that a speed e.m.f. in time phase with the single-phase flux is produced by the rotation of the armature conductors. What flux is due to the current produced by this speed e.m.f., and what is its space position? Why do the combined effects of this field and of the speed field produce a rotating magnetic field? How does this, in part, explain the operation of the single-phase induction motor?

12. What is the approximate ratio of weight of single-phase to polyphase induction motors of the same ratings? How may a three-phase induction motor, be operated single-phase? What is often the cause of a polyphase motor's overheating when carrying its normal load?

13. Describe the manner in which the initial starting torque of a single-phase motor may be obtained by splitting the phase. What is the order of

magnitude of this starting torque? Show how the phase may be split by the use of resistance and inductance; by the use of resistance and capacitance.

14. Discuss the operation of the *shaded pole* as a method of starting single-phase motors. How is the repulsion-motor principle utilized in starting the Wagner single-phase induction motor? What operation converts the motor from a repulsion motor to an induction motor?

15. Upon what principle does the phase converter operate? What advantage is derived by its use on railway locomotives? Make a diagram of connections showing how the single-phase power received at high voltage from the trolley is converted into low-voltage three-phase power for use in the motors of the railway locomotive.

16. Make a diagram of connections showing the relation of field windings to armature, etc., in the General-Electric repulsion-induction motor. How is this motor reversed?

17. What unique feature is embodied in the armature of the Wagner, type-BK motor? What excellent operating characteristics are claimed for this motor?

QUESTIONS ON CHAPTER XI

1. Compare the design of the alternator with that of the synchronous motor.

2. Show that at standstill the average torque of the single-phase synchronous motor is zero and that, in order to develop a continuous torque, either the moving conductor or the moving pole must cover a distance equal to one pole pitch every half-cycle. What is the relation among speed, frequency, and number of poles?

3. What reaction occurs in the direct-current shunt motor which enables it to take more current when additional load is applied? Show that the reaction in the synchronous motor under similar conditions cannot be exactly the same as that of the shunt motor.

4. What is the first reaction which occurs when load is applied to any motor? What resulting reaction follows in the case of the synchronous motor? Show that the current taken by a synchronous motor when the angular position of the rotor is slightly retarded is mostly *energy* current.

5. What are the reactions which follow an increase of the excitation of a direct-current shunt motor? Why cannot these same reactions occur in a synchronous motor?

6. What two reactions permit the synchronous motor to operate when its field current is increased above the normal value? Show that the induced armature voltage can be numerically greater than the terminal voltage. When the synchronous motor is overexcited, what must be the phase relation between its current and its terminal voltage? Illustrate by a vector diagram.

7. What effect is noted when the field of a direct-current shunt motor is weakened? Why cannot these same reactions occur when the field of the synchronous motor is weakened?

8. What is the effect of a lagging current upon the field of a synchronous motor? Upon the relation of the induced to terminal voltage? Make a vector diagram for the motor when operating underexcited.

9. Why is a salient-pole synchronous motor able to operate, even without direct-current excitation? Whence does it obtain its excitation under these conditions?

10. Sketch a synchronous-motor V-curve. Show the point of unity power factor, the region of lagging current, and the region of leading current. Sketch another V-curve in which the power is twice that of the original curve. How is the position of this curve determined? What is meant by *normal* excitation, and how does this vary with the motor load?

11. Give two reasons for building squirrel-cage or *amortisseur* windings around the poles of a synchronous motor. Analyze the reactions by which an amortisseur winding stabilizes the operation of the synchronous motor.

12. Describe the method of starting a synchronous motor by means of an auxiliary motor. What types of motor are used for this purpose? What are the objections to their use?

13. What is the sequence of operations in starting a synchronous motor by means of its direct-current generator? What objection is there to starting a motor in this way?

14. By what process may the synchronous motor start of itself? Why is a compensator used? Of what order of magnitude is the starting torque? When should the direct-current field be closed?

15. Analyze closely the method by which the synchronous motor, when starting as an induction motor, is able to pull into synchronism even without direct-current excitation.

16. What happens at the time of closing the field switch if the direct-current excitation opposes the field built up by armature reaction? What should be the position of the starting device when the field switch is closed, and why?

17. How may correct polarity of the field poles be insured so that little or no disturbance results when the field switch is closed?

18. Why is it necessary to insulate the field coils of a synchronous motor for a voltage several times the normal operating voltage? How may the e.m.f. induced in the field be reduced?

19. Describe two methods which permit synchronous motors to develop full-load torque while bringing the motors from standstill to synchronous speed.

20. Why are synchronous motors, running without load, often installed at various points of power systems? What is the motor called when operating under these conditions? What is the distinct advantage of using a synchronous-motor drive in certain instances?

21. Show by a vector diagram how it is possible to control the voltage at some point on a system by means of a synchronous motor or synchronous condenser. What condition is necessary in order that the voltage at the motor may be raised to a value higher than that of the rest of the system? What degree of excitation is necessary in order that the voltage may be

raised? Sketch the connections of a motor, together with the necessary instruments for making tests when the motor is used as a voltage-controlling device.

22. Why are single-phase synchronous motors not common?

23. What are the advantages of the polyphase synchronous motor over the polyphase induction motor? What are its disadvantages? Under what conditions should it be used?

24. Describe two simple types of synchronous motor which are of very small size. Upon what property of the magnetic circuit do they depend for their operation? From where do they obtain their field excitation? For what purposes are such motors used?

PROBLEMS ON CHAPTER XI

(Problems marked * may be solved trigonometrically and also with complex quantities.)

282. Determine the speeds in r.p.m. of the following synchronous motors: (*a*) 25-cycle, 2-pole; (*b*) 60-cycle, 12-pole; 60-cycle, 8-pole; (*c*) 50-cycle, 72-pole; (*d*) 20-cycle, 24-pole.

283. When a 500-hp., 12-pole, 2,300-volt, three-phase, 60-cycle, Y-connected synchronous motor is running at no load, its counter e.m.f. is practically equal numerically to its terminal voltage and in phase with it. If load is applied to the rotating-field structure, its angular position is retarded 0.80 mechanical space-degree. (*a*) What net e.m.f. per coil is developed causing the motor to take current? (*b*) If the armature impedance is 1.4 ohms per phase, what current per phase flows into the armature as a result of the application of load? (*c*) Draw an approximate vector diagram. Neglect the flux distortion caused by armature reaction.

284. Through how many space-degrees must the rotor (Prob. 283) be retarded in order that the motor may take 110 amp. under the conditions given?

***285.** A three-phase, 6,600-volt, 1,200-hp., 25-cycle, Y-connected synchronous motor has an effective resistance of 0.4 ohm per phase (coil) and a leakage reactance of 4.0 ohms per phase. (*a*) Determine the induced e.m.f. of the motor at unity power factor when it takes rated current. The efficiency, exclusive of field loss, is 0.94. (*b*) What mechanical power (horsepower) does the rotor develop? (*c*) Draw a vector diagram.

***286.** In Prob. 285, determine (*a*) and (*b*) with the power factor equal to 0.80, leading current. The efficiency is 0.93. Draw a vector diagram.

***287.** In Prob. 285, determine (*a*) and (*b*) with the power factor equal to 0.80, lagging current. The efficiency is 0.93. Draw a vector diagram

***288.** A 100-kv-a., 600-volt, Y-connected, eight-pole, 60-cycle, three-phase synchronous motor has the following constants: armature resistance per coil = 0.08 ohm; armature reactance per coil = 1.0 ohm.

Determine the counter e.m.f. of the motor when the current is 100 amp.: (*a*) when the power factor of the motor is unity; (*b*) when the power

factor is 0.80 lagging current; (c) when the power factor is 0.80 leading current.

*289. At unity power factor, the motor of Prob. 288 requires a field current of 30 amp. at 110 volts, and the core and friction losses are 3,000 watts when the motor is operating at unity power factor at its rated load. Determine (a) the mechanical power developed within the armature; (b) the horsepower output; (c) the torque at the pulley; (d) the efficiency.

290. Figure 290A shows four V-curves for a 200-kv-a., 600-volt, synchronous motor. Find the kilowatt input at which each curve was obtained. Draw

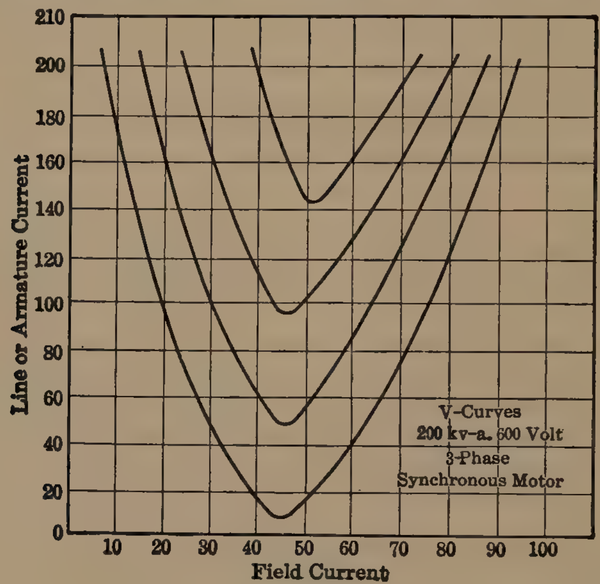


Fig. 290*A*.

lines through points of unity power factor and of 0.75 power factor, leading and lagging current.

291. The synchronous motor of Prob. 290 is connected in parallel with a load of 120 kilowatts at 600 volts and 0.6 power factor, lagging current. The motor runs light without load. To what value must its excitation be adjusted in order to bring the system power factor to unity (use lowest curve, Fig. 290*A*)?

292. Repeat Prob. 291, except that the power factor is brought to 0.9. How many kilovolt-amperes must the motor take in each case?

293. A 600-kw., 2,300-volt, three-phase load has an average power factor of 0.50, lagging current. How many leading quadrature kilovolt-amperes are necessary to raise its power factor to 0.6? 0.8? 0.9? 1.0? Plot a curve with power factor as abscissas and kilovolt-amperes as ordinates.

294. What size synchronous motor would be necessary to raise the power factor of Prob. 293 to unity and at the same time take 300 kw. from the line in order to carry mechanical load?

295. Repeat Prob. 294 except that the power factor is raised to 0.9.

296. How much current must the motor (Prob. 290) take to raise the *system* power factor to unity if it is in parallel with a 120-kw. load, 0.6 power factor lagging current, and at the same time takes 98.8 kw. itself from the line to carry mechanical load. To what value must the field current be adjusted?

297. Repeat Prob. 296 except that the power factor is raised to 0.9 lagging current.

298. How many leading kilovolt-amperes can the motor (Prob. 290) supply and at the same time take 150 kw. from the line? The motor current must not exceed 200 amp. What is the maximum power factor which can be obtained with this motor when in parallel with a 250-kv-a., 0.707 power-factor load, lagging current? What is the value of the field current?

QUESTIONS ON CHAPTER XII

1. Enumerate some of the applications of electrical energy in which it is impossible to employ alternating current.

2. Make a sketch showing the method of operation of the rectifying commutator. What are the disadvantages and limitations of this type of rectifier?

3. Upon what principle does the mercury-arc rectifier operate? Why are two or more anodes usually employed? Why is it necessary to have reactance in circuit? Sketch the diagram of connections which would be used for charging a low-voltage storage battery, and trace the current flow, explaining carefully the operation of the auto-transformer. Why is a starting anode desirable?

4. Describe the construction of mercury-arc rectifiers in large power sizes. Compare with synchronous converters as regards efficiency.

5. What is the underlying principle of the tungar rectifier? Why are the electrons repelled from the incandescent filament during one half-cycle and attracted during the other half-cycle? What are the approximate efficiency and maximum capacity of this type of device?

6. What property has aluminum, when immersed in certain salt solutions, which makes it possible to utilize it in the rectification of alternating currents? Sketch a wiring diagram of such a rectifier which rectifies every half-cycle. What are the disadvantages and the limits of capacity for this type of rectifier?

7. What types of rotating machinery may be used to convert alternating to direct current on a large scale? Name the disadvantages of each type of apparatus.

8. Name the machines whose principles are embodied in the synchronous converter. Just how is the converter armature connected? How is power supplied to the ordinary converter armature? What power is taken from the armature? Name the different types of familiar machines for which the converter may be used.

9. Under what operating conditions is the synchronous converter called *direct? inverted?*

10. Indicate the points at which the slip-ring taps connect to the winding in a four-phase, two-pole converter; a four-phase, four-pole converter.

11. Repeat 10 for a three-phase, two-pole converter and a three-phase, four-pole converter. How many taps will an eight-pole, six-phase converter have? What special restriction, not necessary with the ordinary direct-current winding, is imposed on the converter winding? Why?

12. Compare the number of active conductors between brushes with the number between slip-ring taps in the single-phase converter. How is the resulting voltage between direct-current brushes obtained? Between slip-ring taps? What is the relation between the two?

13. Show by a circle and an inscribed polygon how the individual inductor voltages of a converter add. Indicate the method of obtaining (*a*) the single-phase voltage; (*b*) the three-phase voltage; (*c*) the four-phase voltage; (*d*) the six-phase voltage.

14. Knowing the voltage relations in a converter armature, derive the ratio of the direct-current to the alternating current per terminal in (*a*) the single-phase converter; (*b*) the three-phase converter; (*c*) the four-phase converter; (*d*) the six-phase converter.

15. Sketch the variation of the direct current in a single conductor midway between slip-ring taps, as the conductor takes successive positions in its rotation. Sketch the alternating current in this same conductor for corresponding positions when the current is in phase with the induced e.m.f. Find the resultant current for each position of the conductor.

16. Repeat 15 for a conductor at one of the slip-ring taps.

17. Repeat 16 for a power factor considerably less than unity. What is the effect on the resultant current curve of increasing the number of phases?

18. Why does increasing the number of phases materially increase the rating of a converter? Why does the efficiency of a converter decrease more rapidly with a decrease in power factor than is the case in most other types of apparatus?

19. Compare commutation in a converter when operating as such and when operating as a direct-current generator carrying the same load. Why does the very materially increased armature current resulting from low power factors have little distorting effect on the main fields? What is its effect on commutation?

20. Why are commutating poles desirable in synchronous converters, even although the main field is not distorted to any considerable extent by armature reaction?

21. Why are the voltage ratios in a converter almost constant under operating conditions? Why is it possible to modify the ratio of the direct to the alternating voltage a small amount by changing the excitation?

22. Explain how a series reactance may be used to control the direct-current voltage. When may a separate reactance be omitted? State the disadvantages of this method of voltage control.

23. Explain the use of the induction regulator as a means of controlling the direct-current voltage. What is the objection to the use of the regulator?

24. Explain the operation of the series booster. What are its advantages and disadvantages?

25. Why is it impracticable to control the direct-current voltage by changing to different transformer taps when the converter is in operation?

26. Explain the underlying principle of the split-pole method of control.

27. Sketch a diagram of connections, including all instruments, which would be used in determining the various characteristics of the converter. What characteristics is it instructive to determine? How should they be plotted?

28. Why are transformers almost always necessary with synchronous converters? Sketch the connections of the six-phase star-secondary connection, showing the primaries in either Y or delta. Indicate the voltage at each point, assuming 220 volts between the three-phase lines on the primary side. What is the advantage of this system?

29. Repeat 28 for the double-delta connection of secondaries.

30. How does the rating of a synchronous converter, when operating inverted, compare with its rating when operating direct? Why? How do the speed relations in the two cases compare? Show by careful analysis the sequence of reactions which may cause an inverted converter to race. What means are used to prevent racing?

31. By what reactions does a synchronous converter armature start rotating when polyphase currents are supplied to its slip-rings? What is a field sectionalizing switch, and what should be its position when starting the converter from the alternating-current side? Why is it necessary to open the series-field shunt, etc.? When starting, why does sparking take place under the brushes even with no direct-current load? Why are brush-lifting devices necessary?

32. How does the armature pull into synchronism? What effects occur if the shunt-field current opposes the field built up by armature reaction? How may the continual "slipping of a pole" be stopped?

33. Give two methods by which the direct-current polarity may be reversed, if necessary.

34. Describe how the speed of rotation in space of the field produced by the armature currents becomes less and less, during starting, as the armature speed approaches synchronism. How does this affect commutation? Describe the behavior of a direct-current voltmeter connected across the brushes during the starting period. When should the field switch be closed?

35. How may the armature be induced to build up the field poles to the right polarity and so insure the correct direct-current polarity at the brushes?

36. Describe briefly the procedure of starting a synchronous converter by means of an auxiliary machine.

37. Give the connections of both the shunt- and the series-field circuits of a synchronous converter when it is started from the direct-current side. Why should the switch between the transformer secondaries and the slip-rings be opened during the starting period? What difficulty is encountered in synchronizing?

38. Discuss the operation of synchronous converters in parallel. How many equalizers may be required? How are the loads between machines

adjusted? Why is it preferable that each converter have its own transformer bank?

39. Why may synchronous converters operating in parallel show a tendency to run away under some circumstances? Describe methods which are used to prevent synchronous converters from thus running away.

40. What is the principle by which a neutral is obtained in the three-wire converter? Why is it undesirable to use three single secondaries connected in Y when obtaining a neutral? How may a Y-connection be used and at the same time direct-current magnetization of the core be prevented?

41. Sketch the complete connections of a three-wire, six-phase synchronous converter having two series fields, where the transformer secondaries are connected six-phase star.

PROBLEMS ON CHAPTER XII

299. A railway synchronous converter is delivering direct current at 600 volts. The converter has six slip-rings. Determine: (*a*) the diametrical voltage of the armature; (*b*) the voltage between adjacent slip-ring taps; (*c*) the voltage between alternate slip-ring taps.

300. The converter (Prob. 299) is delivering 100 kw. Determine the approximate current per slip-ring tap when the alternating-current power is supplied (*a*) single-phase; (*b*) three-phase; (*c*) six-phase. Assume unity power factor and an efficiency of 0.93.

301. It is desired to secure a 200-kw. synchronous converter and its transformers for changing three-phase, 6,900-volt power into 230 volts direct current. The converter has three slip-rings, and the transformers are connected with primaries in delta and secondaries in Y. The converter has an efficiency of 93 per cent., and the transformer bank an efficiency of 97.8 per cent. When the converter operates at 95 per cent. power factor, determine: (*a*) the direct-current rating of the converter; (*b*) the alternating current per slip-ring; (*c*) the rating in kilovolt-amperes, amperes, and volts of the transformer secondaries; (*d*) the power input, the current, and the voltage of each transformer primary when the converter is delivering rated output.

302. A converter similar to that of Prob. 301 has six slip-rings and is operated six-phase. This increases its efficiency to 94 per cent. The transformers are connected with primaries in delta and secondaries in six-phase diametrical. When the converter is delivering 200 kw. and operating at 95 per cent. power factor (see Fig. 302*A*), determine: (*a*) the voltages $E_{1\text{-}2}$, $E_{2\text{-}3}$, etc.; (*b*) the voltages $E_{1\text{-}3}$, $E_{3\text{-}5}$, $E_{5\text{-}1}$; (*c*) the diametrical voltages $E_{1\text{-}4}$, $E_{2\text{-}5}$, etc.; (*d*) the current in each transformer secondary. (*e*) What is the probable rating of the converter under these new conditions, and have the transformers now the proper rating?

303. A 1,000-kw., 25-cycle, 500-r.p.m., 250-volt synchronous converter is designed to supply a three-wire, direct-current system. The transformers are connected delta-six-phase star and receive power at 26,800 volts, three-phase. The converter has a full-load efficiency of 0.955, and the trans-

formers have full-load efficiencies of 0.973. The power factor is unity. Determine: (*a*) the direct current; (*b*) the diametrical slip-ring voltage; (*c*) the slip-ring voltage to neutral; (*d*) the six-phase slip-ring voltage; (*e*) the current per slip-ring brush; (*f*) the primary current per transformer; (*g*) the incoming line current. Make a diagram of the connections showing the method of obtaining the direct-current neutral.

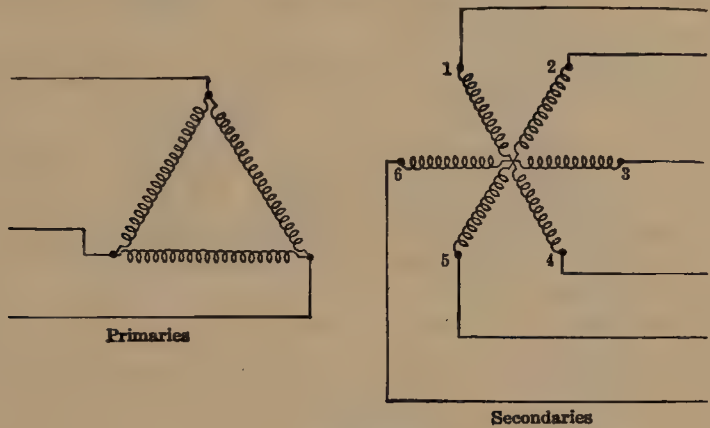


Fig. 302*A*.

304. A synchronous converter is to be used to obtain 1,500 kw. at 600 volts for an electric railway. The power available at the substation is 33,000 volts, three-phase, 60 cycles. The transformer primaries are Y-connected, the secondaries being six-phase diametrical. The converter has a rated-load efficiency of 0.945, and the transformers have a rated-load efficiency of 0.98. The converter is operating at unity power factor. Determine: (*a*) the direct-current line current; (*b*) the current per slip-ring; (*c*) the diametrical slip-ring voltage; (*d*) the voltage between adjacent slip-rings; (*e*) the ratings in volts and amperes of the transformer primaries. Make a complete diagram of connections.

QUESTIONS ON CHAPTER XIII

1. Why is alternating current particularly well adapted for transmitting power over considerable distances? What difficulties are encountered when direct current is similarly used?

2. State the advantages of polyphase transmission. Which of the polyphase systems is most commonly used and why? Under what conditions is single-phase occasionally used?

3. Why are 6,600- and 13,200-volt generators commonly used when the transmission voltage is high? What rough basis can be used for determining the transmission voltage? What economic considerations are involved in determining this voltage?

4. Give the principal links in a power system which distributes power to large and small consumers located at a considerable distance from the point of generation of power. State the considerations which govern the selection of each of these links.

5. Why are the voltages ordinarily selected for power and for lighting purposes usually different? Why should the secondaries of lighting transformers be grounded?

6. Name the various types of apparatus which may be installed in a substation, giving the type of service which each supplies.

7. Make a sketch of the magnetic field existing between the two parallel conductors of a single-phase transmission line. What effect does this field have on the operation of the line?

8. On what two factors does the inductance of such a line depend? Distinguish between the inductance of the circuit loop and that of a single wire.

9. Sketch the magnetic field which may exist at some particular instant in the region between the three conductors of a three-phase transmission line, these conductors being symmetrically spaced. What is the general nature of the field existing in this region, and what is its effect on the operation of the transmission system?

10. On what three factors does the reactance per conductor of a three-phase system depend?

11. Sketch the electrostatic field which exists between the two conductors of a single-phase transmission system. On what two factors does the capacitance existing between two such wires depend?

12. Show that a thin, fictitious plane may be inserted midway between two parallel wires and perpendicular to their plane without disturbing the electrostatic field between these conductors. With this as a basis, replace the capacitance between conductors by two series-connected condensers. What is the ratio of the capacitance of each of these condensers to the capacitance between the line conductors?

13. Replace the actual capacitance which exists between symmetrically-spaced three-phase lines by two different arrangements of condensers. Which of these two arrangements is ordinarily considered and why?

14. What close approximation as to wire spacing may be used in a three-phase system when transmission conductors are not located at the corners of an equilateral triangle?

15. State some of the advantages of splitting a single-phase transmission line along a fictitious neutral and using the quantities to neutral when working out the line characteristics.

16. Why can the ground be considered as having no resistance and no inductance, although such is actually not the case?

17. Given the line resistance and reactance, the load voltage, current and power factor, show by means of a vector diagram the method of obtaining the voltage at the sending end of the line. Derive the complex equation which gives the solution of such lines.

18. Show that a three-phase line may be split into three single lines, any one of which may be used for purposes of calculation. Why can the ground be considered as having zero resistance and zero reactance under these conditions?

19. How is the capacitance of a line actually distributed? For purposes of calculation, how may this total capacitance be considered as being distributed? What effect does the current taken by each condenser have on the line behavior? on the generator load?

20. Derive the equation which gives by complex quantities the solution of transmission lines and draw the corresponding vector diagram.

21. What is the general nature of *corona?* Upon what factors does its appearance depend? Upon what parts of a conductor does it first appear? How may corona loss on transmission lines be minimized?

22. What factors may cause abnormal voltage rise in a power system? What is the purpose of a lightning arrester? What five properties should lightning arresters possess?

23. Describe the multigap arrester, discussing its operation. What is the weak point in this type of arrester? On what principle does the horngap operate? Why is it necessary to use resistance or reactance in series with such a gap? What are the disadvantages of the horngap arrester?

24. What is the underlying principle of the aluminum-cell arrester? Why can an alternating current flow into such an arrester when a direct current cannot, even though the film is intact in both cases? Describe the characteristics which make such a device an excellent lightning arrester.

25. What is the general arrangement of the individual cells in the actual arrester? Why is oil used? Why is there a short horngap in series with each arrester? Why is it necessary to "charge" such arresters at intervals?

26. Describe the principle of operation of the oxide-film lightning arrester. the autovalve lightning arrester. What are the advantages of these two types over the aluminum-cell type?

27. Sketch the connections of an arrester designed to protect a three-phase ungrounded system. Sketch the connections showing the position of the arrester, the choke coil with relation to the incoming (or outgoing) line, and the apparatus which it is designed to protect.

28. State the advantages of pin-type insulators for low and moderate voltages. What are their limitations at the higher voltages? What materials are used for these insulators, and what are their relative advantages and disadvantages? Why are the larger units made up in sections?

29. In what manner does the suspension type of insulator support the line conductors? What are the advantages of this type of insulator over the pin type?

30. Under what conditions are wooden poles employed as line supports? steel poles? steel towers? Compare steel towers and steel poles.

31. What is meant by *flexible-tower* construction? Under what conditions are flexible towers used, and what are their advantages?

32. What is the function of the substation? Sketch roughly the connections of a transformer substation.

33. By what types of apparatus is direct current obtained from alternating-current supply? Compare the advantages and disadvantages of these different types.

34. Why is it difficult to break a high-voltage power arc in air? Discuss briefly the construction of an oil switch for high voltages. Why is it necessary that the case be designed to withstand high pressures?

35. What special care must be used in carrying high-voltage lines into stations? What care should be taken in the location of the various types of apparatus?

36. What are the economic necessities which have developed the outdoor substation? In what way does the apparatus for such a station differ from that of an indoor station?

PROBLEMS ON CHAPTER XIII

(Problems marked * may be solved both trigonometrically and with complex quantities.)

305. Determine the loop inductance of a 6,000-ft. single-phase line of No. 0 solid conductor spaced 12 in. between centers. Repeat for 18 in. between centers.

306. What is the reactance at 60 cycles of the loop circuit (Prob. 305)? at 25 cycles? What is the impedance in each case? What is the line impedance drop in each case, if the current is 90 amp.?

307. A 6-mile, single-phase, distribution line consists of solid No. 000 copper conductor spaced 42 in. between centers. Determine: (*a*) the inductance per wire of the line; (*b*) the reactance per wire at a frequency of 60 cycles; (*c*) the impedance per wire at this same frequency.

308. Determine: (*a*) the capacitance between conductors of the line (Prob. 305); (*b*) the capacitance to neutral; (*c*) the charging current of the line if the voltage is 2,200 volts, 50 cycles.

309. Determine: (*a*) the capacitance between conductors of the line (Prob. 307); (*b*) the capacitance to neutral; (*c*) the charging current if the voltage is 2,300 volts, 60 cycles between conductors. Compare this current with the load current determined in Prob. 310.

***310.** A load of 150 kw. is connected to the end of the line (Prob. 307). The voltage at this load is maintained constant at 2,300 volts. Determine: (*a*) the voltage at the sending end when the load power factor is 0.8 lagging current; (*b*) the efficiency of transmission under these conditions. (*c*) Repeat (*a*) for unity power factor. (*d*) Repeat (*b*) for unity power factor.

***311.** Repeat (*a*) and (*b*) (Prob. 310) with the load power factor equal to 0.8, leading current.

312. A 60-mile, 66,000-volt, three-phase transmission line consists of three 250,000-C.-M. stranded copper conductors spaced symmetrically, 6 ft. between centers. The power delivered is 18,000 kw. at 66,000 volts. Determine: (*a*) the reactance per conductor at 60 cycles; (*b*) the resistance per conductor; (*c*) the voltage to neutral at the load; (*d*) the current at unity power factor; (*e*) the current at 0.75 power factor.

313. Determine the capacitance to neutral (Prob. 312). What is the charging current per conductor?

***314.** In Prob. 312, determine: (*a*) the voltage at the sending end of the line when the power factor is unity; (*b*) the line regulation; (*c*) the efficiency of transmission. Neglect the line charging current.

***315.** In Prob. 314, determine (*a*), (*b*), and (*c*) when the power factor is 0.75 lagging current.

***316.** In Prob. 314, determine (*a*), (*b*), and (*c*) when the power factor is 0.75 leading current.

***317.** Given the transmission distance 25 miles; load, 5,000 kw.; load voltage, 33,000 volts between conductors; system, three-phase; frequency, 60 cycles per second; load power factor of 0.80, lagging current; spacing of conductors, 60 in.; permissible line loss, 10 per cent. of receiver power. Find: (*a*) line regulation; (*b*) generator power factor. Neglect the line charging current.

***318.** Repeat Prob. 317 for a power factor of 0.80 leading current.

319. It is desired to transmit 38,000 kw. a distance of 140 miles at 132,000 volts, three-phase, 60 cycles. The line loss shall not exceed 10 per cent. of the transmitted power when the load power factor is 0.85. Copper conductors are to be used. The spacing is 13 ft. between centers. Determine: (*a*) the load current at 0.85 power factor; (*b*) the smallest permissible size of copper conductor (Appendix H); (*c*) the line loss at 0.85 power factor, due to the load current alone; (*d*) the line reactance per conductor; (*e*) the charging current of the line, assuming that the potential difference between the line conductors is 132,000 volts the entire length of the line.

***320.** In Prob. 319, determine: (*a*) the *total* line current when the load is 38,000 kw. and the power factor is unity, using the "split-condenser method"; (*b*) the line loss; (*c*) the efficiency of transmission; (*d*) the voltage at the sending end of the line; (*e*) the line regulation.

***321.** Repeat Prob. 320 for 0.85 power factor, lagging current.

***322.** Repeat Prob. 320 for 0.85 power factor, leading current.

QUESTIONS ON CHAPTER XIV

1. Discuss the nature of electrons, their mass, their relation to the atom, etc.

2. What conditions are necessary for a free emission of electrons? What is meant by *critical velocity?*

3. State and analyze Richardson's law. Discuss thermionic efficiency.

4. Describe the two-electrode tube and the conditions necessary for electron emission.

5. State and analyze Child's $3/2$ power law, with particular reference to the emission characteristics of two-electrode tubes.

6. Explain the *Edison effect.*

7. Describe the operation of the Fleming valve and two-electrode rectifiers.

8. Describe and analyze the operation of the three-electrode tube, with particular reference to its characteristics.

9. Analyze the operation of the three-electrode tube as an amplifier.

10. What is the fundamental principle of the oscillator? Make a diagram of connections of two types and discuss their operation.

11. What is meant by *modulation?* Discuss *carrier frequency; side bands.*

12. Make a diagram of connections which shows the usual method of plate-circuit modulation.

13. Analyze detection with the two-electrode tube and state the limitations of this method.

14. Analyze detection with the three-electrode tube, using a polarized grid.

15. Analyze detection with the three-electrode tube, using grid resistance combined with regeneration.

16. Analyze beat or heterodyne reception and make a diagram of connections of a circuit which is used in this method.

QUESTIONS ON CHAPTER XV

1. How may light be described? What is illumination? Photometry?

2. What is luminous intensity? In what units is it measured? What is the objection to the use of the candle as a photometric standard? What photometric standards are used at the present time?

3. Define a unit solid angle or *steradian.* What are its geometrical properties? How many solid angles exist about a point?

4. In what way may light flux be considered? In what way is it comparable to magnetic flux? What is a *lumen?* Why is a given cone of light flux confined to the solid angle? How many lumens would a standard candle emit if its intensity in all directions were the same as its horizontal intensity?

5. What is the objection to rating lamps in terms of mean horizontal candle-power? Why is it more rational, particularly at the present time, to rate lamps in lumens? What is the relation between lumens and mean spherical candlepower?

6. What is illumination? What is the unit? To what does it correspond in magnetism?

7. What is the law of inverse squares? How is this law proved geometrically? What assumption may introduce error into the application of this law? What is the magnitude of this error, and how may it be made negligible?

8. What is meant by *absorption?* What types of surfaces reflect the largest proportion of incident light? What types reflect the least? What determines the color of a surface? What is meant by the *coefficient of absorption?*

9. Why does the intensity of light emanating from light sources usually vary in different directions? How does light intensity, in general, vary in horizontal planes or zones? In what way may this distribution be represented? In what regions is the light from an incandescent lamp of the greatest intensity? of the least intensity? Of what commercial use are these distribution curves?

10. What is meant by *incandescence?* How does the light emitted by an incandescent substance vary with its temperature? Does this bear any relation to the efficiencies of various illuminants?

11. In what way does luminescence differ from incandescence? Give examples of luminescent sources of light.

12. Name two essential requirements for an incandescent-lamp filament. Why was a carbon filament practically the only satisfactory filament for a number of years? How does a GEM lamp filament differ from the ordinary carbon filament? What are its advantages? What are the efficiencies of these two filaments?

13. What are the objections to a carbon filament? How may the efficiency of a carbon-filament lamp be increased, and what is the objection? What is the approximate life of a carbon-filament lamp? When does it become more economical to throw away a lamp rather than continue its use?

14. Why does a tantalum lamp develop a higher efficiency than a carbon lamp? About what efficiency does the tantalum lamp develop? What is the average life of such a lamp? What is its peculiarity when used with alternating current?

15. Why is tungsten particularly well adapted to being used for lamp filaments? Compare the drawn and the pressed filament. What are the approximate efficiencies of Mazda-B lamps? What is the guaranteed life of such a lamp? Why does the tungsten lamp take an excessive current when it is first switched on?

16. What factor limits the operating temperature of the vacuum tungsten lamp? What two effects are produced? How may one of these factors be minimized?

17. What is the basic principle of the gas-filled lamp? Why is an inert gas necessary? Why does this type of lamp have a long, narrow neck? Why does the filament differ in shape from that of the vacuum lamp? Why are the efficiencies higher in the larger units? How may daylight be approximated with these lamps?

18. What is the basic principle of the arc light? Why is the arc a high-efficiency illuminant? What is the reason that the arc itself cannot be connected directly across the line? What is the *ballast,* and approximately what percentage of the power is lost in the ballast?

19. What is the principal source of light in the direct-current arc? What determines the relative positions of the two electrodes? Which is consumed the more rapidly?

20. Compare the alternating-current arc with the direct-current arc. Why is a reflector very desirable in the alternating-current lamp? What advantage has the alternating-current multiple arc over the direct-current multiple arc?

21. What is the effect upon its efficiency and upon the cost of operation of enclosing the arc? What are the advantages of the enclosed over the open arc? Where are open direct-current arcs now commonly used?

22. In what way does the flame arc differ from the ordinary direct-current arc as regards the light source? How may the color of the light be controlled?

23. Of what materials do the positive and negative electrodes in the magnetite arc consist? Why must the copper always be the anode?

In general, how does the regulation of this type of lamp differ from that of other arcs? Why is it different? What is the number of hours that the lamp burns per trim? What is the efficiency of the 4.4-amp. arc? of the 6.6-amp. arc?

24. What is the principle of the mercury arc? To what is the light due? What is the color effect produced by this type of illuminant? Is it injurious? What is its physiological effect? How may the illumination be made more pleasing when these lights are used? What is the efficiency of this type of lamp?

25. What is the source of light in the Moore tube? What are its advantages? Why are these tubes not in more common use?

26. What is the principle of the Nernst lamp? What can be said of its light and its efficiency? Why is it practically obsolete? Discuss neon tubes.

27. Why are there excellent opportunities for still further increasing the efficiencies of illuminants?

28. What is *photometry?* In what way are the measurements generally made? Name a very marked source of error in photometric measurements.

29. Upon what principle is the Bunsen photometer screen based? How is the position of balance determined? How is the candlepower calculated after a balance has been obtained?

30. Why is a candle itself seldom used at the present time as a working photometric standard? What are used as standards? Sketch the connections which facilitate voltage adjustment when a photometric measurement is being made.

31. Describe the Lummer-Brodhun comparison photometer screen. How is the condition of balance determined with this screen?

32. Describe the Lummer-Brodhun contrast photometer screen. How is the condition of balance determined?

33. Where is the use of a portable photometer often necessary? Compare portable photometers with the laboratory type.

34. Describe the Sharp-Millar photometer. In what way is the standard accurately adjusted? How is brightness or illumination measured? How is the candlepower of a lamp measured? How may the range of the instrument be greatly increased?

35. How is the mean horizontal candlepower of a lamp determined with one photometer setting? How may the candlepower in other zones be measured in a similar manner? How does the candlepower in a vertical direction of an incandescent lamp compare with that in the horizontal direction?

36. Sketch the Rousseau diagram, showing how the mean spherical candlepower may be determined from the polar distribution diagram. What is meant by *spherical reduction factor?* Of what use is a knowledge of its value?

37. Why are reflectors useful even although they absorb a considerable amount of light? When may it be desirable to throw light upward? downward? How is this accomplished?

38. What are the illuminants most generally used in interior illumination? What important factors must be considered when illuminating an interior?

39. What illumination in foot-candles is required for reading, writing, etc.? When should such illumination be furnished entirely by overhead fixtures? by individual desk lamps?

40. When a room is to be illuminated by a single unit, how should the reflector differ from those required when a number of units are used?

41. What is *indirect lighting?* What is its chief advantage? What are its disadvantages? How are these disadvantages overcome by the use of semi-indirect fixtures?

42. Where are individual lamps required in factory illumination? How do overhead belts and traveling cranes affect the placing of the lighting units?

43. In what way does street illumination differ from interior illumination? How do the lighting intensities compare in the two cases? What can be said of reflected light in comparing the two?

44. Why may uniform street illumination be objectionable? Where is street lighting of uniform intensity used? Why is its use considered good engineering under these conditions?

45. How does the road surface assist in illumination? What is meant by *specular reflection?* How does automobile traffic affect street illumination problems? How may the improper placing of lights on curves, etc., be the cause of accidents?

46. What is meant by *floodlighting,* and where is this type of illumination used?

PROBLEMS ON CHAPTER XV

323. A sphere has a radius of 1.2 ft. (*a*) How many steradians or unit solid angles will an area of 2 sq. ft. on its surface subtend? (*b*) 3 sq. ft.?

324. Repeat Prob. 323 for a sphere whose radius is 2 ft.

325. A 115-volt Mazda-C lamp emits a total light flux of 800 lumens. Determine its mean spherical candlepower (m.s.cp.).

326. A 150-watt Mazda-B lamp has a mean spherical candlepower of 132. Determine its total light-flux emission in lumens.

327. The 150-watt lamp (Prob. 326) has a mean horizontal candlepower (m.h.cp.) of 160. The light intensity is sensibly uniform over a zone extending a considerable distance above and below the horizontal plane (see Fig. 436, p. 503). Determine: (*a*) the number of lumens intercepted by a spherical zone 10° above and below the horizontal plane; (*b*) the lumens intercepted per square foot in this zone when the zone is 1 ft. distant from the light source; (*c*) the illumination in foot-candles on this area of 1 sq. ft.

328. Repeat (*b*) and (*c*) (Prob. 327) when the zonal area is 4 ft. from the light source.

329. A light source has a mean spherical intensity of 120 cp. Determine: (*a*) the lumens that would be intercepted by a sphere of 3-ft. radius, having

the light source at its center; (b) the lumens per square foot in the horizontal zone if the spherical reduction factor of the lamp is 0.78 (see p. 521).

330. A certain lamp alone emits 500 lumens into the upper hemisphere and 700 lumens into the lower hemisphere. It is equipped with a reflector which deflects to the lower hemisphere the light which the lamp itself emits to the upper hemisphere. The efficiency of the reflector is 0.80. (a) How many lumens does the lamp with the reflector emit? (b) What is the resulting mean lower hemispherical candlepower?

331. The candlepower of the lamp in Prob. 330 at a 45° angle downward is 120. What is the illumination in foot-candles in this downward direction on a surface 10 ft. from this lamp when the surface is perpendicular to the direction of the light?

332. A certain lamp placed 20 ft. above the street has an intensity of 420 cp. in a vertically downward direction. What is the illumination in foot-candles on the street directly below the lamp?

333. What does the illumination in Prob. 332 become if the lamp is raised 4 ft.?

334. A gas-filled lamp has a rating of 6,700 lumens. If it takes 3.5 amp. at 115 volts, what is its efficiency in watts per mean spherical candlepower? in lumens per watt?

335. If the lamp (Prob. 334) has a spherical reduction factor of 0.85, determine its rating in watts per mean horizontal candlepower.

336. A tungsten vacuum lamp takes 0.52 amp. at 115 volts and is rated at 58.8 m.h.cp. What is the efficiency in watts per mean horizontal candlepower? In lumens per watt if the spherical reduction factor is 0.78?

337. A lighting company furnishes 100-watt tungsten vacuum lamps at 20 cts. each; 75-watt gas-filled lamps are supplied at 50 cts. each. Assuming that the candlepower of each is practically the same and that energy costs 7.5 cts. per kilowatt-hour, how many hours must the lamps burn before the total cost of the two is the same?

338. Assuming that each lamp of Prob. 337 has a life of 1,000 hr., at what cost of energy per kilowatt hour will the total cost of the lamps be equal?

339. In a series direct-current enclosed arc lamp, the arc itself takes 490 watts, and the current is 6.6 amp. The mean spherical candlepower is 260, and the maximum candlepower downward in an oblique direction (see Fig. 440, p. 509) is 600 cp. Determine: (a) the efficiency in watts per mean spherical candlepower; (b) the lumens per watt; (c) the voltage across the lamp.

340. In Prob. 339, determine (a) and (b) when the arc lamp is supplied from 115-volt multiple mains.

341. A multiple, direct-current, 115-volt, enclosed-arc lamp takes 715 watts at 115 volts and has a mean spherical candlepower of 240. Determine: (a) the efficiency in watts per mean spherical candlepower; (b) the lumens per watt.

342. In Prob. 341, the voltage across the arc is 74 volts. If the lamp were operating in a series circuit where no ballast is necessary, what values would be obtained for (a) and (b)?

343. An enclosed alternating-current arc takes 6.6 amp. at 115 volts, and the mean spherical candlepower is 140. The voltage across the arc is 75 volts, and that across the ballast is 78 volts. Determine: (*a*) the power factor of the lamp; (*b*) the power taken by the lamp; (*c*) the efficiency in watts per mean spherical candlepower; (*d*) the efficiency in lumens per watt; (*e*) the power taken by the ballast.

344. A multiple alternating-current arc which takes 5.0 amp. at 115 volts and a power factor of 0.70 has a mean spherical candlepower of 145. The voltage across the arc itself is 70 volts. Determine: (*a*) the efficiency of this lamp in watts per mean spherical candlepower; (*b*) the lumens per watt; (*c*) the power consumed by the ballast. (*d*) Draw a vector diagram of the lamp circuit.

345. A series magnetite lamp takes 6.6 amp. at 75 volts and has a rating of 8,400 lumens. What is the efficiency in watts per mean spherical candlepower? in lumens per watt?

346. Assume that a lamp similar to that of Prob. 345 is arranged to operate in multiple across 110-volt, direct-current mains and that the arc itself takes the same volts and the same watts and has the same candlepower. What is the overall light efficiency of this lamp in lumens per watt?

347. A lamp trimmer receiving $5.00 per day can trim 75 500-watt magnetite lamps in that time. Allowing 1.1 cts. per kilowatt hour for energy, 125 burning hr. per trim, 10 cts. per electrode, and $45.00 per year maintenance, compute the approximate operating cost per year of such a lamp, assuming that a lamp is in service 4,000 hr. during that time.

348. In a photometric measurement, the standard lamp has a candlepower of 25.8, and the photometric balance is obtained when the photometer screen is 73 in. from the test lamp. The photometer screen is 120 in. long. What is the candlepower of the test lamp?

349. Compute the candlepower of another test lamp in Prob. 348 when the screen is 62 in. from the standard lamp. The same photometer screen is used.

350. The test lamp in Prob. 348 has a spherical reduction factor of 0.82. What is its mean spherical candlepower, and what is its output in lumens?

351. The lamp, whose polar-candlepower diagram is given in Fig. 436 (p. 503), takes 0.87 amp. at 115 volts. Determine (without reflector): (*a*) the mean spherical candlepower; (*b*) the watts per mean spherical candlepower; (*c*) the spherical reduction factor; (*d*) the watts per mean horizontal candlepower; (*e*) the lumens per watt.

352. Determine the percentage of light absorbed by the reflector (Prob. 351, see Fig. 436).

353. A 40-watt Mazda-B lamp when tested in a rotator takes 0.352 amp. at 110 volts. The working standard has a mean horizontal candlepower of 36.2. The photometer is 600 cm. long. Below are given the settings of the photometer screen. The first column gives the degrees (see Fig. 436, p. 503), and the second column, the distances from the test lamp to the photometer screen.

Degrees	Centimeters	Degrees	Centimeters	Degrees	Centimeters
0	169.3	60	291.7	120	289.0
10	204.6	70	297.2	130	277.8
20	237.7	80	299.9	140	259.1
30	259.9	90	301.9	150	225.3
40	273.8	100	300.1	160	158.0
50	283.5	110	295.4	170	

(*a*) Plot a polar candlepower and Rousseau diagram (see Fig. 447, p. 521). Determine: (*b*) the mean horizontal candlepower; (*c*) the mean spherical candlepower; (*d*) the spherical reduction factor; (*e*) the watts per mean horizontal candlepower; (*f*) the watts per mean spherical candlepower; (*g*) the lumens; (*h*) the lumens per watt.

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